Asset Pricing under Information-Processing Constraints*

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January 2008

Abstract

This paper studies the implications of limited information-processing capacity (also called "rational inattention") for asset pricing in a linear-quadratic permanent income model. We have two main results. First, RI increases the size of the risk adjustment to asset prices by increasing the volatility and persistence of consumption growth. Second, RI increases the expected excess return. Thus, RI has the potential to play an important role in resolving extant asset pricing puzzles.

JEL Classification Codes: *C61, D81, E21.*

Keywords: Rational Inattention, Asset Pricing, Permanent Income.

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* We thank the helpful comments of Chris Otrok. All errors remain our responsibility.

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1. Introduction

The rational expectations hypothesis assumes that agents are endowed with infinite information-processing capacity, allowing them to respond immediately and completely to changes in the economy. However, individuals do not seem to have unlimited mental capacity, as mounting evidence suggests that they do not respond swiftly or thoroughly to all available information. Sims (2003) proposes rational inattention (RI) to capture this fact by assuming that agents only have finite processing capacity for information on the state of the economy. He shows that RI can introduce realistic features such as sluggishness, randomness, and delays into the responses of economic variables to shocks.

This paper considers a simple rational inattention version of the permanent-income model as studied in Hall (1978) and examines the implications of RI for the pricing of multi-period securities. The focus of the PIH model is on the relationship between aggregate consumption and aggregate income in an environment in which output can be stored in the form of capital to smooth consumption over time. In contrast, the focus of the intertemporal asset pricing model is on the relationship between aggregate consumption and equilibrium asset prices. Following Hansen (1987) and Cochrane (chapter 2, 2005), we construct a model that combines Hall (1978) – a permanent income model – and Lucas (1978) – an asset pricing model. We use the approach from Sims (2003) and Luo (2007) to solve the model with RI in closed-form and then explore how RI affects asset prices within this framework.

Following Cochrane (2005), we decompose the price of a risky asset into a risk-neutral perpetuity and a risk adjustment component. RI alters asset pricing by increasing the size of the risk adjustment relative to the risk-neutral component, leading to a decline in the price of the asset. This decline is driven by two key effects – RI introduces persistence and increases the volatility of consumption growth; both of these features are undesirable from the perspective of agents, so asset prices decline. The size of the price decline is negatively related to the channel capacity of the agents; agents with low channel capacity will require large premia to hold risky assets relative to those with unlimited capacity. We then show that the resulting price decline generates an increase in the expected excess return (the risk premium). Thus, RI can potentially play an important role in resolving some extant asset pricing puzzles, such as the equity premium puzzle, where the behavior of excess returns is hard to replicate in standard models.

2. The Model

The model is a simplified version of Hansen (1987)'s model, in which Hall's permanent income model and Lucas's asset pricing model are combined to examine the asset pricing implications of exogenous endowment shocks. In this section, we first derive the expression of (optimal) aggregate consumption in terms of the state variables by solving an otherwise standard PIH model with RI; we then price assets by treating the process of aggregate consumption that solves the RI-PIH model *as though it were an endowment process.* Because we adopt the representative agent setup, equilibrium prices are shadow prices that leave the representative agent content with that endowment process.

2.1. Permanent-Income Model

A standard rational expectations (RE) version of the PIH model can be formulated as follows

$$\max_{\{c_t\}} E_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right] \tag{2.1}$$

subject to

$$k_{t+1} = Rk_t + e_t - c_t, \tag{2.2}$$

where $u(c_t) = -\frac{1}{2}(c_t - \overline{c})^2$ is the utility function, $\overline{c}$ is the bliss point, Equation (2.2) represents a linear production technology, c_t is consumption, k_t is capital, e_t is an exogenous endowment with Gaussian white noise innovations, β is the discount factor, and R is the rate of return on capital.[1] Let $\beta R = 1$; then this specification implies that optimal consumption is determined by permanent income:

$$c_t = (R-1)\, s_t \tag{2.3}$$

where

$$s_t = k_t + \frac{1}{R} \sum_{j=0}^{\infty} \left(\frac{1}{R} \right)^j E_t \left[e_{t+j} \right] \tag{2.4}$$

is the expected present value of lifetime resources, consisting of physical capital plus human wealth. As noted in Cochrane (chapter 2, 2005), it is not a partial equilibrium result – it is a general equilibrium model with a linear production technology and an endowment process. As shown in

[1] Beyond the assumption of normality with iid innovations, we do not need to restrict the process for e_t.

Luo (2007), the above PIH model can be reduced to the univariate model with iid innovations to permanent income s_t that can be solved in closed-form after introducing RI. Specifically, if s_t is defined as a new state variable, we can rewrite the evolution equation of s_t as

$$s_{t+1} = Rs_t - c_t + \zeta_{t+1} \tag{2.5}$$

where the $t+1$ innovation ζ_{t+1} is

$$\zeta_{t+1} = \frac{1}{R} \sum_{j=t+1}^{\infty} \left(\frac{1}{R}\right)^{j-(t+1)} \left(E_{t+1} - E_t\right) \left[e_j\right]. \tag{2.6}$$

This reduction is critical because multi-dimensional RI problems do not remain within the linear-quadratic-Gaussian class that can be solved analytically.[2]

Consumption growth can be written as

$$\begin{aligned} \Delta c_t &= \frac{R-1}{R} \left(E_t - E_{t-1}\right) \left[\sum_{j=0}^{\infty} \left(\left(\frac{1}{R}\right)^j e_{t+j}\right)\right] \\ &= (R-1)\,\zeta_t. \end{aligned}$$

2.2. Optimal Consumption

Following Sims (2003) and Luo (2007), the consumer's information-processing constraint can be characterized by the equation

$$\mathcal{H}\left(s_{t+1}|\mathcal{I}_t\right) - \mathcal{H}\left(s_{t+1}|\mathcal{I}_{t+1}\right) = \kappa, \tag{2.7}$$

where $\mathcal{I}_t$ is the consumer's currently processed information, κ is the consumer's channel capacity, $\mathcal{H}\left(s_{t+1}|\mathcal{I}_t\right)$ denotes the entropy of the state prior to observing the new signal at $t+1$, and $\mathcal{H}\left(s_{t+1}|\mathcal{I}_{t+1}\right)$ is the entropy after observing the new signal. (2.7) implies that the reduction in the uncertainty about the state variable gained from observing a new signal is bounded by κ. As shown in Sims (2003), $\mathcal{D}_t$ is a normal distribution $N\left(\widehat{s}_t, \sigma_t^2\right)$; as a result, (2.7) can be reduced to

$$\log|\psi_t^2| - \log|\sigma_{t+1}^2| = 2\kappa \tag{2.8}$$

[2] The operator $(E_{t+1} - E_t)$ generates the difference between an expectation taken with information at time $t+1$ and that taken with information at time t.

where $\sigma_{t+1}^2 = \operatorname{var}[s_{t+1}|\, \mathcal{I}_{t+1}]$ and $\psi_t^2 = \operatorname{var}[s_{t+1}|\, \mathcal{I}_t]$ are the posterior and prior variances of the state variable, respectively.

In the univariate case (2.8) has a steady state $\sigma^2 = \frac{\omega_\zeta^2}{\exp(2k)-R^2}$, in which the consumer behaves as if observing a noisy measurement of permanent income $s_{t+1}^* = s_{t+1} + \xi_{t+1}$, where ξ_{t+1} is the endogenous noise with variance $\lambda_t^2 = \operatorname{var}\left[\xi_{t+1}|\, \mathcal{I}_t\right]$; in the steady state $\lambda^2 = \left(\sigma^{-2} - \psi^{-2}\right)^{-1}$. As shown in Luo (2007), the consumption function under RI is

$$c_t = (R-1)\,\widehat{s}_t, \tag{2.9}$$

where the conditional mean $\widehat{s}_t$ evolves according to the Kalman filter equation

$$\widehat{s}_{t+1} = (1-\theta)\widehat{s}_t + \theta(s_{t+1} + \xi_{t+1}). \tag{2.10}$$

$\theta = 1 - 1/\exp(2\kappa) \in [0,1]$ is the constant optimal weight on any new observation. Straightforward calculations imply that

$$\Delta c_t = \theta\,(R-1)\left[\left(\frac{\zeta_t}{1-(1-\theta)RL}\right) + \left(\xi_t - \frac{\theta R \xi_{t-1}}{1-(1-\theta)RL}\right)\right]. \tag{2.11}$$

3. Equilibrium Asset Prices

R is the return on technology and is not yet the interest rate (the equilibrium rate of return on one-period claims to consumption). As proposed in Cochrane (chapter 2, 2005), we first find optimal consumption and then price one-period claims using the equilibrium consumption stream. Denoting the risk free rate by R^f, we have the following Euler equation:

$$\frac{1}{R^f} = E_t\left[\beta\frac{u'(c_{t+1})}{u'(c_t)}\right] = \beta E_t\left[\frac{\overline{c}-c_{t+1}}{\overline{c}-c_t}\right] = \beta = \frac{1}{R},$$

where $E_t[\cdot]$ is the consumer's expectation operator conditional on his processed information at time t.

We can now use the basic pricing equation, $p = E[mx]$,[3] to compute the price of the stream of

[3] As argued in Cochrane (2005), this equation tells us only what the price should be given the joint distribution of consumption (the discount factor) and the asset payoff. We know $E[mx]$ after solving the PIH model given the state variables and can use them to determine the asset price p.

aggregate consumption (treated as the stream of endowments) as

$$\begin{aligned} p_t &= E_t\left[\sum_{j=1}^{\infty}\left(m_{t,t+j}c_{t+j}\right)\right] \\ &= \sum_{j=1}^{\infty}\left(\beta^j\frac{\overline{c}c_t - c_t^2 - \mathrm{var}_t\left[c_{t+j}\right]}{\overline{c}-c_t}\right) \\ &= \frac{1}{R-1}c_t - \frac{1}{\overline{c}-c_t}\Xi, \end{aligned} \tag{3.1}$$

where

$$\Xi = \sum_{j=1}^{\infty}\left(\beta^j\,\mathrm{var}_t\left[c_{t+j}\right]\right); \tag{3.2}$$

we exploit the facts that

$$\begin{aligned} m_{t,t+j} &= \beta^j\frac{u'\left(c_{t+j}\right)}{u'\left(c_t\right)} = \beta^j\frac{\overline{c}-c_{t+j}}{\overline{c}-c_t}, \\ E_t\left[c_{t+j}^2\right] &= \mathrm{var}_t\left[c_{t+j}\right] + c_t^2. \end{aligned}$$

Denoting the risk-neutral component by p_t^{rn} and the risk-adjusted component by p_t^{rc}, we have

$$p_t^{rn} = \frac{1}{R-1}c_t \tag{3.3}$$

and

$$p_t^{rc} = \frac{1}{\overline{c}-c_t}\sum_{j=1}^{\infty}\left(\beta^j\,\mathrm{var}_t\left[c_{t+j}\right]\right). \tag{3.4}$$

(3.1) yield the following implications. The first term in (3.1) is the p_t^{rn}, the value of a perpetuity paying c_t. The second term is a p_t^{rc}; it lowers the asset price relative to the risk-neutral level because $c_t \leq \overline{c}$. The following proposition states our key result; the proof is contained in the appendix.

Proposition 1. *The risk-neutral component p_t^{rn} of the asset price is independent of the degree of inattention, while the risk-adjusted component in the asset price p_t^{rc} increases with the degree of inattention. The ratio of the risk-adjusted component under RI to that under RE is*

$$F \equiv \frac{p_t^{rc}\left(\theta<1\right)}{p_t^{rc}\left(\theta=1\right)} = \left(1+2\frac{1-\theta}{\theta}\right)\frac{\theta}{1-R^2\left(1-\theta\right)}. \tag{3.5}$$

(6.3) shows that RI reduces the asset price by two channels. First, RI increases the volatility of consumption growth σ_c^2; second, RI introduces persistence into consumption growth ρ_c.[4] Given the expression for σ_c^2, the higher the volatility of the innovation to permanent income ω_ζ^2 the lower the price. Asset prices are also affected by both the persistence and volatility of the fundamental endowment shocks since ω_ζ^2 depends on them. The following corollary holds under the mild parameter restriction $\theta > \frac{R^2-1}{R^2}$.[5]

Corollary 2. *F is decreasing and convex in θ and increasing and convex in R. The cross-partial is negative.*

Interpretation of these partial derivatives is straightforward; for example, $\frac{\partial F}{\partial \theta} < 0$ implies that economies where households have high channel capacity will have lower equity prices. Figure 1 plots the relationship between the degree of attention and F; when $\theta = 0.67$ and $R = 1.01$ we obtain $F = 2$, meaning RI doubles the relative size of the risk-adjusted component of asset prices (since $F = 1$ if $\theta = 1$).[6] R also affects asset prices; as seen in Figure 1, the effect of R is stronger when θ is small.

4. Equilibrium Asset Returns

Given (3.1), (6.3), and (3.5), the expected asset return is

$$\begin{aligned} E_t\left[R_{t,t+1}\right] &= E_t\left[\frac{p_{t+1}+c_{t+1}}{p_t}\right] \\ &= R + \frac{\frac{R}{\bar{c}-c_t} + E_t\left[\frac{1}{c_{t+1}-\bar{c}}\right]}{\frac{1}{R-1}c_t - \frac{1}{\bar{c}-c_t}\Xi}\Xi, \end{aligned} \tag{4.1}$$

[4] By direct calculation

$$\frac{\partial \sigma_c^2}{\partial \theta} = \frac{R^2-1}{\left(1-R^2\left(1-\theta\right)\right)^2} > 0$$

and

$$\rho_c = 0$$

iff $\theta = 1$.

[5] Luo and Young (2007) impose a similar condition to guarantee an agent with higher channel capacity always chooses a signal distribution with lower variance.

[6] We choose this numerical parametrization because it is consistent with a 1 percent annual risk-free rate and Luo and Young (2007) found that $\theta = 0.67$ implies a relative volatility of consumption to income roughly consistent with US data.

since $c_t = E_t[c_{t+1}]$. To evaluate $E_t\left[\frac{1}{c_{t+1}-\bar{c}}\right]$ we approximate the concave function $\frac{1}{c_{t+1}-\bar{c}}$ around $E_t[c_{t+1}]$ to obtain

$$\frac{1}{c_{t+1}-\bar{c}} \simeq \frac{1}{E_t[c_{t+1}]-\bar{c}} - (E_t[c_{t+1}]-\bar{c})^{-2}(c_{t+1}-E_t[c_{t+1}]) + (E_t[c_{t+1}]-\bar{c})^{-3}(c_{t+1}-E_t[c_{t+1}])^2;$$

taking $E_t[\cdot]$ of both sides gives

$$\begin{aligned} E_t\left[\frac{1}{c_{t+1}-\bar{c}}\right] &= \frac{1}{E_t[c_{t+1}]-\bar{c}} + (E_t[c_{t+1}]-\bar{c})^{-3}\,\mathrm{var}_t[\Delta c_{t+1}] \\ &\leq \frac{1}{c_t-\bar{c}}, \end{aligned} \tag{4.2}$$

since $c_t \leq \bar{c}$. We can establish an upper bound for the expected return as

$$E_t[R_{t,t+1}] \leq R + \frac{(R-1)^2\,\Xi}{c_t(\bar{c}-c_t)-(R-1)\,\Xi} = \overline{R}_{t,t+1}. \tag{4.3}$$

$\overline{R}_{t,t+1}$ is increasing with the degree of inattention because $\frac{\partial \Xi}{\partial \theta} = \frac{\partial F}{\partial \theta} R\omega_\zeta^2 < 0$.

The expected return on the asset is given by

$$\begin{aligned} E_t[R_{t,t+1}] &= R + \frac{\frac{R}{\bar{c}-c_t} - E_t\left[\frac{1}{\bar{c}-c_{t+1}}\right]}{\frac{1}{R-1}c_t - \frac{1}{\bar{c}-c_t}\Xi}\Xi \qquad (4.4) \\ &= R + \frac{R-1}{(\bar{c}-c_t)^2}\frac{(\bar{c}-c_t)^2 - \frac{R-1}{R}\frac{\theta}{2-\theta}\Xi}{c_t(\bar{c}-c_t)/(R-1)-\Xi}\Xi, \qquad (4.5) \end{aligned}$$

since

$$\sigma_c^2 = \frac{\theta}{2-\theta}\frac{(1-\beta)^2}{\beta}\Xi.$$

(4.5) shows that the expected excess return is increasing with the degree of inattention:

$$\frac{\partial E_t[R_{t,t+1}-R]}{\partial \theta} < 0. \tag{4.6}$$

because $\frac{\partial \Xi}{\partial \theta} < 0$ and $\frac{\partial}{\partial \theta}\left(-\frac{\theta}{2-\theta}\right) < 0$. Thus, (4.4) implies that RI raises the expected excess return holding R and σ_c^2 constant; if agents have low channel capacity, consumption is more volatile and therefore claims to that volatile stream must pay a higher excess return.

5. Conclusion

In this paper we examine the implications of limited information-processing capacity – rational inattention in the sense of Sims (2003) – for asset prices in an otherwise standard permanent-income model. We find that RI raises the size of the risk-adjusted component of the asset price by increasing both the relative volatility and persistence of consumption growth; this increase leads to a decline in asset prices and an increase in the expected excess return.

6. Appendix

This appendix contains the proof of Proposition 1.

Proof. (2.11) can be written

$$\Delta c_{t+1} = \rho_c \Delta c_t + \theta (R-1) \left(\varepsilon_{t+1} + \xi_{t+1} - R\xi_t \right), \tag{6.1}$$

where $\rho_c = (1-\theta)R$. The variance of Δc_t is

$$\sigma_c^2 = \operatorname{var}[\Delta c_t] = \frac{\theta}{1 - R^2 (1-\theta)} \omega_\zeta^2. \tag{6.2}$$

Substituting (6.1) into $\sum_{j=1}^{\infty} \left(\beta^j \operatorname{var}_t [c_{t+j}] \right)$ yields

$$\begin{aligned}
\Xi &= \sum_{j=1}^{\infty} \left(\beta^j \operatorname{var}_t [c_{t+j}] \right) \\
&= \sum_{j=1}^{\infty} \left(\beta^j \operatorname{var}_t [c_{t+j} - c_t] \right) \\
&= \beta\sigma_c^2 + \beta^2 \left(2\sigma_c^2 + 2\operatorname{cov}[\Delta c_{t+2}, \Delta c_{t+1}] \right) + \beta^3 \begin{pmatrix} 3\sigma_c^2 + 2\operatorname{cov}[\Delta c_{t+2}, \Delta c_{t+1}] \\ +2\operatorname{cov}[\Delta c_{t+3}, \Delta c_{t+1}] + 2\operatorname{cov}[\Delta c_{t+2}, \Delta c_{t+3}] \end{pmatrix} + \cdots \\
&= \left[\sum_{j=1}^{\infty} \left(j\beta^j \right) + 2 \frac{\beta\rho_c}{1-\beta\rho_c} \frac{\beta}{(1-\beta)^2} \right] \sigma_c^2 \\
&= \left(\frac{2-\theta}{\theta} \right) \left(\frac{\theta}{1 - R^2(1-\theta)} \right) R\omega_\zeta^2,
\end{aligned} \tag{6.3}$$

since $\beta R = 1$. If $\theta = 1$, $\Xi = R\omega_\zeta^2$. Hence,

$$F = \frac{2-\theta}{\theta} \frac{\theta}{1 - R^2(1-\theta)}.$$

■

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