Optimal Attention Allocation and Heterogeneous Consumption Responses[*]

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Abstract

Consumers often face both labor and capital income risks when making consumption-saving decisions. In this paper, we study how individuals allocate their limited attention between capital income and labor income risks in a two-period consumption-saving model with recursive utility. Specifically, we examine how the optimal attention and consumption-saving decisions are influenced by individuals' attention capacity, wealth endowments, income risks, and preferences for risk and time. We show that our model can generate results that are consistent with several novel empirical facts regarding how differences in individuals' wealth levels and beliefs about their unemployment risks influence their consumption behavior. Furthermore, we find that the welfare loss due to limited attention is significantly larger for households with lower wealth; allowing these households to flexibly allocate their attention can significantly reduce this welfare loss.

Keywords: Capital Income and Labor Income Risks; Optimal Attention Allocation; Consumption and Saving Decisions.

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1 Introduction

Consumers often face various income shocks when making consumption-saving decisions. How people adjust their consumption in response to these shocks depends on the size of the shocks as well as their assessments of the associated risks. Intuitively, when the magnitudes of the shocks are large or when individuals are less prepared for them (due to limited information), their consumption responses are likely to be large. How well individuals prepare for these shocks in the face of limited information depends on their information-processing capacity and how they allocate that capacity to assessing various types of risks, such as labor and capital risks. Although many studies have examined how household consumption responds to income shocks, relatively few studies have explained consumption responses in the presence of both capital and labor income shocks under limited information-processing capacity.[1] In this paper, we therefore investigate, both empirically and theoretically, the implications of limited attention and the optimal attention allocation for the consumption and saving decision when there exist both labor and capital income risks.

On the empirical side, we document three novel facts regarding households' consumption changes using a unique dataset of Health and Retirement Study (HRS) with detailed information about individuals' prior beliefs about unemployment risks as well as ex-post changes in their employment status, consumption, and financial assets. First, individuals who have more uncertainty about their future labor income – that is, a larger prior variance – experience a smaller consumption decline in percentage terms when they become unemployed.[2] Second, individuals who have a larger prior variance on their labor income experience a larger consumption decline in percentage terms when they experience the same loss in capital income (in percentage terms). Third, wealthier individuals experience a larger consumption drop in percentage terms when they become unemployed.

On the theoretical side, to explain the empirical findings documented above, we develop a tractable two-period model with the following key elements: (i) rational inattention (RI) due to limited information capacity in the vein of Sims (2003), (ii) recursive utility, and (iii) multiple risks (*additive* labor-income risk and *multiplicative* capital-income risk) to study how individuals' optimal consumption responses are jointly determined by their attention capacity, endogenous

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[1] It is worth noting that even under full-information, the effects of uninsurable labor-income risk on consumption-saving dynamics at both individual- and aggregate-levels have been the subject of a large literature. In contrast, the effects of uninsurable investment, or capital-income risk on consumption-savings are far less well understood and studied. Clearly, such risks are paramount and relevant for both developed and developing countries. For example, in the U.S., the private businesses account for about one half of aggregate capital, and the "representative" investor has a poorly diversified portfolio.

[2] Our analysis uses data at both the individual and the household levels, with more details provided in the data section. Individual consumption and wealth are based on household levels, while unemployment and prior information are at the individual level. In the model, when we link changes in labor income at unemployment to consumption, we make corresponding adjustments to make consumption and income data comparable.

attention allocation (to monitor each risk), initial wealth endowments, and the magnitude of the shocks.[3] We model attention allocated to risks as in Van Nieuwerburgh and Veldkamp (2010), and agents face risks in labor and capital income tomorrow but know their current wealth status perfectly. To account for the different effects of individuals' risks and time preferences on the optimal attention and consumption allocation, we introduce a Kreps-Porteus-Selden type recursive utility to fully separate the elasticity of intertemporal substitution (EIS) from the coefficient of relative risk aversion (CRRA).[4]

This paper extends the literature on optimal consumption-saving decisions under RI by introducing both labor and capital income risks rather than focusing on the labor income risk (see, for example, Sims (2003, 2006) and Luo (2008)).[5] It also complements the literature on consumption-saving under RI, which assumes (approximated) linear-quadratic (LQ) or constant-absolute-risk-averse (CARA) preferences (see, for example, Mondria (2010), Van Nieuwerburgh and Veldkamp (2010), Maćkowiak and Wiederholt (2015)) by employing the Epstein-Zin recursive utility, which allows us not only to study the precautionary saving behavior but also to disentangle effects of relative risk aversion degree and elasticity of intertemporal substitution (EIS) on optimal attention-consumption allocation.

Specifically, our analysis contributes to the literature in three dimensions. First, we construct a model to study the joint optimal attention-consumption allocation problem and quantitatively show how households' optimal attention allocation and consumption-saving decisions are driven by their prior beliefs on risks, their attention capacity and wealth endowments, and their time and risk preferences. We find that an individual's optimal attention allocation between labor and capital income risks depends on the relative size of their prior variances in these risks. This behavior is unsurprising and in line with existing results in the RI literature. What is more surprising is that individuals allocate more attention to capital income risk (which has a multiplicative form) than to labor income risk (which has an additive form) even when the prior variance of the labor income risk is higher than that of the capital income risk.[6] For the effects of wealth endowment and attention capacity, we find that richer households pay more attention to capital income risks and save at higher rates than less wealthy households; we find that the labor income risk becomes more important to households when they have a more limited attention capacity, and the average saving rate decreases accordingly due to the precautionary saving motive. For the effects of time preferences, we find that more patient individuals save more and pay more attention to the capital

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[3] Leland (1968), Sandmo (1970), Kimball and Weil (2009), Selden and Wei (2018), and Kubler, Selden, and Wei (2020) also adopt the two-period setting to examine the effects of income uncertainty on savings and/or investment.

[4] Angeletos (2007) and Wang, Wang, and Yang (2016) find that these two parameters have different effects on optimal consumption-saving allocation.

[5] As shown in Bertaut and Starr-McCluer (2002) about half of Americans invest directly or indirectly in stocks and investment funds, which exhibit highly non-diversified risks.

[6] Note that a common conclusion in the previous studies on rational inattention, such as Maćkowiak and Wiederholt (2009), is that when prior variances of different risks are the same, agents pay the same amount of attention to each risk.

return risk. Furthermore, we find that an increase in the CRRA (the EIS) decreases (increases) the optimal attention allocated to the labor (capital) income risk and leads to higher (lower) savings.[7]

Second, we show that our model is consistent with the three novel facts empirically documented during the Great Recession period. The first fact – that individuals with a larger prior variance on their labor income experience a smaller consumption decline in percentage terms when they become unemployed – is actually the interactive outcome of the three distinct effects on individual households' savings: i) a larger prior variance on the labor income that raises individuals' savings due to the precautionary motive; ii) an increase in attention allocated to the labor-income risk which reduces the posterior variance of the labor-income risk and therefore reduces individuals' saving; and iii) a reduction in the amount of attention allocated to the capital risk, which increases the posterior variance of the capital risk and thus reduces individuals' savings.[8] In our calibrated model, the first effect dominates the other two, leading to an overall increase in savings and a smaller decline in consumption when an individual becomes unemployed. In comparison, in a model with zero attention, a change in the prior variance of the unemployment shock has too large effects on savings and consumption responses, which is inconsistent with the data as well. It is worth noting that in a model with infinite attention capacity (i.e., perfect foresight), a change in the prior variance of the unemployment shock has *no* effects on savings and thus consumption responses, which is inconsistent with the empirical couterpart.

The second fact documented during the Great Recession period – that a higher prior variance on labor income leads to a larger consumption decline in response to a financial loss – can be understood through the impact of the prior variance on savings. In particular, a larger prior variance on labor income is associated with higher savings (which has been explained in the previous paragraph), increasing an individual's exposure to financial risks and thus leading them to make larger consumption cuts in response to a negative financial shock than individuals with a smaller prior variance on labor income. In contrast, in a model with no attention, changes in the prior variance of the unemployment shock has too large effects on savings and consumption responses, which is inconsistent with the empirical counterpart.[9]

The third fact documented during the Great Recession period – that wealthier individuals experience a larger consumption drop in percentage terms when they become unemployed – can be understood through two competing channels: i) individuals with a higher wealth endowment save

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[7] Luo, Nie, Wang, and Young (2017) also discuss the effects of the EIS and the CRRA on the optimal attention amount. However, since they only consider one-type risk and have no attention allocation problem, their results cannot be compared with the results in this paper.

[8] The increase in the posterior variance of the capital risk leads to a reduction in saving because the income effect dominates the substitution effect in our calibrated model. As shown in Weil (1990), when the EIS is small, a mean-preserving increase in the capital return risk (specifically, a lower certainty equivalent capital return) leads to increased saving.

[9] Similar to the previous fact, in a model with infinite attention, the saving rate is deterministic and thus the prior variance of labor income has no effect on the change in consumption when there is a loss in capital income.

more, and thus cut their consumption less proportionally after they become unemployed because they can draw on their savings; and ii) unemployment benefits are relatively less important to wealthier households, suggesting a larger decline in consumption when they become unemployed. We show that which effect dominates depends on households' attention capacity. In particular, a larger attention capacity makes the first channel more important by increasing the sensitivity of the saving rate to wealth changes. In other words, to explain the data, the model requires households to have a smaller attention capacity, which makes the second channel dominate the first one. When the information capacity is sufficiently large, such as in the infinite-attention case, these two effects are exactly canceled out; thus, the consumption change is the same across wealth groups, contradicting the fact we document.

Finally, our model also yields several important implications for households' welfare. As highlighted in the existing literature, raising households' information capacity improves their welfare (see, for example, Luo (2008) and Maćkowiak and Wiederholt (2015)). Our model shows that the welfare gain is heterogeneous across individuals with different wealth endowments. Specifically, we find that lower-wealth individuals experience larger welfare gains from increasing their information capacity. One possible explanation is that poor individuals have lower consumption and higher marginal utility, implying that an additional increase in attention may help the poor make more efficient consumption-saving decisions and thus has a larger impact on welfare improvement.[10] In addition, we find the welfare gains increase with the coefficient of relative risk aversion: more risk-averse people benefit more from increasing their attention capacity, as they can use this increased attention capacity to reduce the uncertainty about their consumption and welfare. Furthermore, we find the welfare gains decrease with the EIS: because people with a low EIS dislike consumption fluctuations across periods, a larger attention capacity allows them to predict their future incomes more precisely and thus better smooth their consumption profile. One policy implication of these results is that providing more information about the risks to households who have less wealth, a lower attention capacity, lower EIS, and who are more averse to risk is particularly welfare improving.

Literature Review. Our paper is related to three strands of literature. First, our paper is related to literature on the optimal consumption-saving theory. The idea that consumers smooth consumption and accumulate financial wealth when facing unexpected fluctuations in income dates back to Leland (1968) and Sandmo (1970). Later studies by Bhamra and Uppal (2006), Kimball and Weil (2009), and Selden and Wei (2018) extend the expected utility specification to the recursive utility specification within the two-period framework and further explore how risk aversion and intertemporal substitution affect the optimal consumption and saving decisions. Skinner (1988), Zeldes (1989), Caballero (1990), Weil (1990), Kaplan and Violante (2014), and others

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[10] Consider an extreme case, if a consumer has a sufficiently high wealth level such that the marginal utility is close to zero, limited attention or infinite attention makes no big difference.

extend the two-period framework to multi-period models and further explore how consumption responds to anticipated and unanticipated income shocks, or how the amount of precautionary savings is related to an increase in the persistence and volatility of the income process.[11]

Second, our paper is also related to the literature on the rational inattention theory proposed by Sims (2003, 2010). Most of the recent studies on optimal consumption-saving-investment or optimal monetary policy under rational inattention consider (approximated)linear-quadratic preferences.[12] In contrast, our paper is based on a recursive utility framework with constant elasticity of intertemporal substitution and constant relative risk aversion.

Third, our paper is also related to empirical studies on how consumption responds to income shocks. In the literature on consumption responses to income shocks, economists focus on different types of labor income shocks, such as the asymmetric responses to positive and negative shocks, or different responses to temporary and permanent shocks. (See Jappelli and Pistaferri (2010) for a survey and Deaton (1993) for a textbook treatment on this issue.) In addition, some empirical studies examine how consumption responds to capital income shocks. The results based on microdata are mixed, with some studies finding large consumption responses to house and stock price shocks, and others finding smaller effects.[13]

However, very few studies have examined consumption responses to both capital and labor income shocks. One exception is Christelis, Georgarakos, and Jappelli (2015) who estimate the separate impacts of three different shocks, shocks to stocks, housing, and unemployment, on households' expenditures during the Great Recession using recently available micro data (the 2009 Internet Survey of the Health and Retirement Study). The main focus of their paper is to estimate the separate effects of unemployment and financial-loss shocks on households expenditures and the role of heterogeneous expectations on the persistence of stock losses in consumption responses. More precisely, they find that for every loss of 10% in housing and financial wealth, the estimated drop in household expenditure was about 0.56% and 0.9%, respectively, and those who became unemployed reduced spending by 10%. In this paper, we use the same data set as in Christelis, Georgarakos, and Jappelli (2015). But, the empirical analysis in our paper has a different focus, and we aim at studying the role of prior variance of labor income shock and the role of wealth heterogeneity in consumption responses to financial loss and unemployment shocks. We find that conditional on prior uncertainty of the capital return, those who are more uncertain about labor income experience a smaller consumption reduction when hit by unem-

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[11] In a recent paper by Lian (2022), the author shows that inefficient responses of future consumption to saving changes lead to high marginal propensities to consume in current period although consumers have no liquidity constraints.

[12] For consumption-saving-investment studies under RI, see e.g., Sims (2003, 2006), Luo (2008), Mondria (2010), Van Nieuwerburgh and Veldkamp (2010), Maćkowiak and Wiederholt (2015), Miao, Wu, and Young (2022) etc.; for monetary policy studies under RI, see e.g. Paciello (2012), Afrouzi and Yang (2021) etc.

[13] Sinai and Souleles (2005), Campbell and Cocco (2007), and Attanasio, Blow, Hamilton, and Leicester (2009) find that the consumption response to changes in capital income is quite heterogeneous across the population.

ployment shock, but experience a larger consumption decline when hit by a negative shock to financial wealth. In addition, we find that wealthier individuals experience a larger consumption drop when became unemployed. These empirical findings work as a nice motivation on our model of attention allocation to different income shocks.

The remainder of this paper is organized as follows. Section 2 provides an empirical motivation for this paper. Section 3 describes our baseline model by introducing key elements step by step. Section 4 presents main results of our model with recursive utility and discusses the implications of the joint optimal attention-consumption/savings allocation. Section 5 discusses the testable implication on the heterogenous consumption responses to the two income shocks, and show how our model fits the data in these aspects. Section 6 examines the welfare implications of limited attention optimal attention allocation. Section 7 further discusses the implications for the relative dispersion of consumption to income as well as the role of the bequest motive. Section 8 concludes.

2 Heterogeneous Consumption Responses: Empirical Evidence

In this section, we provide empirical evidence that the response of individuals' consumption to labor and capital income shocks depends on their prior knowledge on the unemployment risk and their financial wealth.

2.1 Data

Our empirical analysis is based on two micro-data surveys from HRS. The first one is the HRS main survey in 2006 and 2008, which is a longitudinal, nationally representative survey interviewing respondents aged 50 and above in the U.S. economy. The survey has been conducted on a biannual basis since 1992 and provides information on households demographic characteristics, income, and asset holdings.[14] The second source is the HRS internet survey, which was conducted from March 2009 to August 2009, and contains $4,415$ respondents belonging to $3,438$ households. To reduce the possibility that estimates are affected by outliers, we delete observations for which the absolute value of the percentage change in consumption is larger than 0.8 and our sample contains 3279 respondents.

For the purpose of this paper, an important feature of the 2006 wave of the HRS main survey is that respondents are asked about their expectations regarding the likelihood that they will lose their jobs in the future. On the prior variance of the labor income, we follow Lusardi (1998) to define it as $p(1-p)(1-\eta)^2Y^2$, where p is an individual's subjective probability (i.e. prior) of losing the job, η is the replacement rate of unemployment benefits, and Y is the labor income. In the HRS, we have a direct measure of p that allows us to construct the prior variance.[15] It is

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[14] The details about the survey can be found in Hauser and Willis (2005).

[15] In the HRS, the survey question is "On the same scale from 0 to 100 , where 0 equals absolutely no chance and

worth noting that a fundamental problem in the empirical studies on consumption and income is about how to measure the subjective uncertainty of future income fluctuations since this variable is unobservable, and the literature usually relies on indirect proxies for risk (or uncertainty). For example, Guiso, Jappelli, and Terlizzese (1992) use the 1989 Survey of Household Income and Wealth (SHIW) to infer information on the probability distribution of household earnings one year ahead. Hence, our assumption on using the likelihood of losing a job in the future as a proxy for the prior variance of the labor income shock is consistent with what is used in the existing literature.

An important feature of the 2009 internet survey is that respondents are asked to report percentage changes in their total spending compared to the previous year, i.e., 2008, changes in financial assets since September 2008, and detailed timing (year and month) of changes in their employment status.[16] This helps us to identify whether respondents were hit by an unemployment shock between the survey date and a year before. Detailed definitions and descriptions about main variables and be found in Appendix 9.1 and Table 1 provides summary statistics on these variables.

2.2 Estimation Results

Our estimation approach is similar to that in Christelis, Georgarakos, and Jappelli (2015), but extends it to better focus on the prior variance of labor income and households' financial wealth. Specifically, our benchmark estimation links households' consumption to changes in financial wealth, changes in unemployment status, their interactions with individuals' priors on labor income variance, as well as other control variables:

$$\frac{\Delta C_{it}}{C_{i,t-1}} = \alpha + \beta \frac{\Delta FW_{it}}{FW_{i,t-1}} + \delta \Delta U_{it} + \gamma_1 pvar_i + \gamma_2 pvar_i \cdot \frac{\Delta FW_{it}}{FW_{i,t-1}} + \gamma_3 pvar_i \cdot \Delta U_{it} + \xi X_{it} + \epsilon_{it}, \quad (1)$$

where i denotes individual households and the term on the left-hand side of the equation is percentage change in consumption. On the right side of the equation, the second term is the percentage change in the values of financial wealth; ΔU indicates whether an individual becomes unemployed between the survey date and a year before; $pvar_i$ is individual i's prior variance of the labor income (which is defined above); the next two terms measure the interactions between the prior variance and financial and unemployment shocks; X is a vector of demographic and

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100 equals absolutely certain, what are the chances that you will lose your job during the next year?" In addition, Guariglia (2001) measures earnings uncertainty by using a similar question in the British Household Panel Survey.

[16] Respondents also report the amount of change in the value of their house compared to its value in the summer of 2006. For each assets owners of employer retirement saving plans (incl. 401k's), individual retirement accounts (IRAs) or Keogh plans, investment trusts, mutual funds, directly held stocks, they are asked to report the percentage decline of the asset value since September 2008, which was the month in which Lehman Brothers collapsed. The discussion regarding biased estimation due to measurement error in Christelis, Georgarakos, and Jappelli (2015) also holds here in our analysis.

economic variables, including gender, age, marital status, retirement status, percentage change in value of the main residences, the prior mean of labor income,[17] and individuals' expectation about the probability of an increase in Dow Jones Industrial Average; ϵ_{it} is an error term.

Columns 1 and 2 of Table 2 report our benchmark estimation results.[18] First, as the first two rows show, not surprisingly, household's consumption declines at unemployment or when they experience a decline in financial wealth. Second, the positive coefficient of the interaction term of prior variance and unemployment shock implies that, on average, those who had higher prior variance in the labor income, experience a smaller decline in their spending conditional on a loss in financial wealth and labor income. From the positive coefficient of the interaction term of prior variance and the change in financial assets, we find that conditional on a certain drop in financial assets, those with higher prior variance of labor income experience a larger decline in spending. Together, these show that for those who have a larger prior variance on their labor income, their consumption declines less after becoming unemployed but declines more when they experience financial losses.[19] In Column 2, we also find that adding more control variables (including the logarithm of income and household size) does not change the main results.

Columns 3 and 4 of Table 2 report another interesting heterogeneity in the consumption response to changes in the value of financial asset and the unemployment shock. In these exercises, we add an interaction term of net financial asset and the percentage drop in financial asset and an interaction term of net financial asset and the unemployment shock. From these results, we can see that rich people, i.e. those with high net financial asset (normalized by average income), tend to react more strongly while unemployed. This finding appears to be inconsistent with the finding in the literature of a positive correlation between the level of lifetime income and the saving rate (see Dynan, Skinner, and Zeldes (2004)) as a higher saving rate may lead to a smaller decline in consumption. However, as we will explain in section 5, our model can rationalize both facts by explaining how an increase in wealth can raise savings but also lead to a decline in consumption in the presence of attention allocation.[20]

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[17] We control the prior mean of the labor income because those who have higher subjective probability of unemployment (p) also have lower expectation of labor income, and therefore, the subjective probability p affects both the first and the second moment.

[18] Among respondents who reported consumption change, 1509 of them also reported percentage change of financial wealth. After deleting outliers with too large wealth-to-income ratio (over 500), we have 838 observations. When control for demographic characteristics we have 747 observations left.

[19] As discussed in our model below, prior variance affects consumption changes through attention allocation and precautionary savings. Especially, the relationship between prior variance and precautionary savings have been studied in previous empirical works such as Lusardi (1998). The author also used HRS data and and subjective belief of being unemployed as a measure of prior variance and show a positive correlation between prior variance and net worth to income ratio. This is consistent with the prediction of the precautionary-saving model: the higher the uncertainty, the higher is the accumulation of wealth due to precautionary motive.

[20] We check the robustness of our baseline estimations as follows: in Table 1 of the online appendix we add one extra dummy variable that indicates if respondent's partner is working and partner's prior variance as control variables; in Table 2 of the online appendix, we set different values of replacement rate of unemployment benefit for people in different income groups. It takes value 0.6 if an individual's income is below 67% of the average

One thing we want to mention is that a standard life cycle consumption model with full information or no attention allocation cannot explain these empirical facts simultaneously. For example, in a two-period model with infinite attention, the saving rate and the consumption change do not react to various prior variances of income shocks. In addition, as shown later in this paper, a model with no attention (allocation) leads to the saving rate too large to match the empirical facts shown above. However, the following sections will show that, by introducing optimal attention allocation into an otherwise standard consumption-saving model, we can potentially explain these facts.

3 An Optimal Attention-Allocation Model with Both Labor and Capital Risks

To fully examine how the optimal attention allocation impacts consumption saving decisions with both labor and capital income shocks, we build a two period model that is rich enough to explore a series of fundamental factors in driving the optimal decisions but still tractable. We start with households' preferences, budget constraints, and two fundamental shocks they face: shocks to capital income and labor income. We then discuss how to incorporate rational inattention due to information-processing constraints into an otherwise standard two-period consumption-saving model.

3.1 Households' Preferences and Budget Constraints

To capture how risk aversion coefficient and intertemporal elasticity of substitution affect the optimal attention-consumption/savings decisions, we follow Kimball and Weil (2009); Bommier and Le Grand (2019); Selden and Wei (2018); and Kubler, Selden, and Wei (2020) in assuming a general KPS (Kreps and Porteus (1978) and Selden (1978)) preference structure as well as the two-period specification. Specifically, in the model economy, households live for two periods: $t \in \{0, 1\}$, and have the following recursive utility:

$$U = u(C_0) + \beta u\left(v^{-1}(\mathbb{E}v(C_1))\right), \tag{2}$$

where $\beta \in (0, 1)$ denotes the households' subjective discount factor and C_0 and C_1 are consumption in periods 0 and 1, respectively.[21] The functions, $u(\cdot)$ and $V(\cdot)$, that govern the preferences for

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income; it takes value 0.5 if an individual's income is above 67% of the average income but smaller than the average income; it takes value 0.3 if an individual's income is above the average level.

[21] It is worth noting that (2) is equivalent to the following recursions:

$$\begin{aligned} U &= u(C_0) + \beta W^{-1}(\mathbb{E}\left[W\left(U_1\right)\right]), \text{ or} && (3) \\ \widetilde{U} &= u^{-1}\left[u(C_0) + \beta u\left(v^{-1}\left(\mathbb{E}\left[v\left(\widetilde{U}_1\right)\right]\right)\right)\right], && (4) \end{aligned}$$

intertemporal substitution and risk aversion are characterized by the CES certainty and constant relative risk aversion risk preferences functional forms, respectively. Specifically, we assume that:

$$u(x) = \frac{x^{1-1/\psi}}{1-1/\psi} \text{ and } v(x) = \frac{x^{1-\gamma}}{1-\gamma}, \tag{5}$$

where γ is the CRRA, whereas ψ is the EIS (i.e., $1/\psi$ is the relative resistance to intertemporal substitution.) This recursive utility specification rules out any possible time inconsistency problem. When $1/\psi = \gamma$, this specification reduces to the standard expected utility case. In this specification, U represents the time preference over certain $(C_0, \widehat{C}_1)$ pairs, where $\widehat{C}_1$ is the period-2 certainty equivalent associated with the random period-2 consumption, C_1: $\widehat{C}_1 = v^{-1}\left(\mathbb{E}v(C_1)\right)$.

We assume that households make consumption and saving decisions for a given initial endowment in period 0, and receive both capital income from this risky saving behavior and a risky labor income in period 1. Specifically, the households' budget constraints in periods 0 and 1 can be written as:

$$C_0 + K_1 = Y_0, \tag{6}$$

$$C_1 = A_1 K_1 + Y_1, \tag{7}$$

respectively, where Y_0 is initial wealth which is strictly positive, and $K_1 > 0$ is the total savings/investment in period 0.

As a note, for simplicity, we do not explicitly model the risk-free asset and optimal asset allocation between the risky asset and the risk-free asset in this model. Such an assumption is not unrealistic from a macroeconomic perspective. In the literature, the real risk-free interest rate in the U.S. economy is calculated based on U.S. Treasury yields adjusted for inflation. Although U.S. Treasuries are usually viewed as risk-free assets in terms of credit risk, they are still subject to the interest rate risk and inflation risk. In addition, some economists also argue that the global economy could be faced with a shortage of safe assets. For example, during the $2007-2009$ financial crisis, many of the private assets which were perceived as safe as they were bestowed with an AAA rating lost their quality and then disappeared.[22] As a result, the strains associated with the financial crisis quickly caused concerns about the safety of sovereign debts, leading to a further shrinkage in the global supply of safe assets.[23]

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where $W = v \circ u^{-1}$, U_1 is future uncertainty utility, and $U = u\left(\widetilde{U}\right)$. Using (3) or (4) is just a matter of normalization.

[22] During $2002-2007$, the US and European financial markets created large amounts of private safe assets through the securitization of riskier assets.

[23] In a recent study, Liu, Schmid, and Yaron (2020) show both theoretically and empirically that expanding safe asset supply can be risky because it creates a new risk channel that leads to depressed growth prospects, rising Treasury yields, and elevated consumption risks.

3.2 Shocks and Information Structure

The capital return (A_1) and labor income (Y_1) processes are assumed as follows:

$$A_1 = \exp(\epsilon_a) \text{ and } Y_1 = \exp(\epsilon_y), \tag{8}$$

where ϵ_a and ϵ_y are exogenously iid shocks.[24] For simplicity, in this paper we assume that the labor income risk is uncorrelated with the risky asset return. It is worth noting that estimating the correlation between the individual labor income risk and the capital return risk is complicated due to the lack of data on household portfolio choices that has both time-series and panel dimensions. In addition, identifying individuals' unanticipated income shocks is also challenging. Viceira (2001) and Haliassos and Michaelides (2003) show that varying the correlation between the transitory earnings shock and the stock return does not affect the portfolio choice allocation. Given our income specification and the available empirical evidence to date, we view the zero correlation as a reasonable hypothesis and use it as the benchmark correlation.

Next, we assume that households are endowed with prior beliefs about the distributions from which these shocks are drawn: $\epsilon_a \sim N(\mu_a - 0.5\sigma_a^2, \sigma_a^2)$ and $\epsilon_y \sim N(\mu_y - 0.5\sigma_y^2, \sigma_y^2)$.[25] However, the realizations of these two period-1 shocks are unobservable in period 0 due to households' limited information-processing ability. It is well-known that economists have no consensus on what is the best empirical measure of labor income uncertainty. Some studies use the income variability or the expenditure variability as proxies for labor income uncertainty. As argued in Guiso, Jappelli, and Terlizzese (1992), Lusardi (1998), Carroll, Dynan, and Krane (2003), variability measures may be poor proxies for income uncertainty because they could contain large controllable elements, while the estimated probability of job loss represents a potential major interruption to labor income over which individuals cannot directly control. We therefore use the probability of job loss to measure labor income uncertainty.

We then assume that households learn exogenous income shocks by observing the following noisy signals:[26]

$$S_0 = \begin{bmatrix} S_a \\ S_y \end{bmatrix} = \begin{bmatrix} \epsilon_a + \zeta_a \\ \epsilon_y + \zeta_y \end{bmatrix}, \tag{9}$$

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[24] As discussed in Haliassos and Michaelides (2003), the estimated correlation between labor income and asset returns for different population groups could be different. For example, Davis and Willen (2000) estimated that the correlation is between 0.1 and 0.3 for college-educated males, and is only about −0.25 for male high school dropouts.

[25] This implies that the unconditional mean of capital return and labor income $\mathbb{E}[Y_1] = \exp(\mu_y)$ and $\mathbb{E}[A_1] = \exp(\mu_a)$.

[26] Sims (2010) provides two ways to solve models with limited information-processing capacity. The first way is to solve the optimal joint distribution of the control variable and the unobservable state variable. The second way is to assume a signal structure, and then solve for the optimal policy as a function of signal. However, as argued by Sims (2010), the optimal joint distribution can be characterized by many different combinations of signal structure and policy function.

where the signals are noisy but unbiased. $\zeta_a \sim N(0, \eta_a^2)$ and $\zeta_y \sim N(0, \eta_y^2)$ are the endogenous noises induced by limited-information processing capacity. The variance of signal regarding capital income shock is $\sigma_a^2 + \eta_a^2$, and therefore, the precision of the signal is $1/\left(\sigma_a^2 + \eta_a^2\right)$. Similarly, the variance of signal regarding labor income shock is $\sigma_y^2 + \eta_y^2$, and therefore, the precision of the signal is $1/\left(\sigma_y^2 + \eta_y^2\right)$.

Households now use Bayes' Law to combine their prior beliefs on the two shocks and the observed noisy signals in (9) to update their beliefs about the shocks such that $\epsilon_a|S_a \sim N(\hat{\epsilon}_a, \hat{\sigma}_a^2)$ and $\epsilon_y|S_y \sim N(\hat{\epsilon}_y, \hat{\sigma}_y^2)$, where $\hat{\epsilon}_a$ and $\hat{\epsilon}_y$ are the posterior means and $\hat{\sigma}_a^2$ and $\hat{\sigma}_y^2$ are the posterior variances determined by the following updating rules:

$$\hat{\epsilon}_a \equiv \mathbb{E}[\epsilon_a|S_a = s_a] = \frac{(\mu_a - 0.5\sigma_a^2)\eta_a^2 + \sigma_a^2 s_a}{\sigma_a^2 + \eta_a^2}, \tag{10}$$

$$\hat{\sigma}_a^2 \equiv \mathbb{V}[\epsilon_a|S_a = s_a] = \frac{\sigma_a^2 \eta_a^2}{\sigma_a^2 + \eta_a^2}, \tag{11}$$

$$\hat{\epsilon}_y \equiv \mathbb{E}[\epsilon_y|S_y = s_y] = \frac{(\mu_y - 0.5\sigma_y^2)\eta_y^2 + \sigma_y^2 s_y}{\sigma_y^2 + \eta_y^2}, \tag{12}$$

$$\hat{\sigma}_y^2 \equiv \mathbb{V}[\epsilon_y|S_y = s_y] = \frac{\sigma_y^2 \eta_y^2}{\sigma_y^2 + \eta_y^2}. \tag{13}$$

Given the prior beliefs, (11) and (13) imply that the signal precision can be uniquely determined by the posterior variance. We can now define information sets before and after observing the signals, which are called Stages 1 and 2 of period 0.

Definition. $\mathbb{I}^1$ *and* $\mathbb{I}^2$ *are the information sets in Stages 1 and 2, respectively:*

$$\begin{aligned} \mathbb{I}^1 &= \left\{Y_0, \epsilon_a \sim N\left(\mu_a - 0.5\sigma_a^2, \sigma_a^2\right), \epsilon_y \sim N\left(\mu_y - 0.5\sigma_y^2, \sigma_y^2\right)\right\}, \\ \mathbb{I}^2 &= \mathbb{I}^1 \cup \{S_a, S_y\}. \end{aligned}$$

Following Sims (2003, 2010), we assume that households face a limited information-processing capacity, κ:

$$\kappa_a + \kappa_y \leq \kappa, \tag{14}$$

where $0 < \kappa < \infty$, κ_a and κ_y are capacity levels devoted to monitoring the capital and labor income shocks, respectively. For simplicity, we assume that the two signals are independent such that:

$$\mathbb{I}(\epsilon_a, S_a) = \mathbb{H}(\epsilon_a) - \mathbb{H}(\epsilon_a|S_a) = \frac{1}{2}\log\left(\frac{\sigma_a^2}{\hat{\sigma}_a^2}\right) = \kappa_a, \tag{15}$$

$$\mathbb{I}(\epsilon_y, S_y) = \mathbb{H}(\epsilon_y) - \mathbb{H}(\epsilon_y|S_y) = \frac{1}{2}\log\left(\frac{\sigma_y^2}{\hat{\sigma}_y^2}\right) = \kappa_y, \tag{16}$$

where κ_a and κ_y are measured in nats,[27] $\mathbb{H}(\cdot)$ is the entropy of productivity shock, $\mathbb{H}(\cdot|\cdot)$ is the conditional entropy of productivity shock given signal observation, and $\mathbb{I}(\cdot,\cdot)$ is called the mutual information between the fundamental shock and signal observation and can be interpreted as how much information about the fundamental shock is contained in the corresponding noisy signal.

We make two comments at the end of this section. First, although in reality there are more than two types of information as modeled in this paper, as the first step to study the rational inattention implications for consumption and saving decisions with multiple shocks, we try to capture two major risks a typical household faces in their consumption-saving decisions. Second, to help understand the attention allocation problem studied here, we may think of the attention allocation as the efforts a household exerts to learn the labor market or the financial market information. Therefore, there likely exists a trade-off between the two types of learning efforts. For example, an increase in learning effort in the financial market, say, by reading articles on the financial conditions, may reduce the effort they can use to learn labor market conditions, say, by reading news about labor markets.

3.3 Households' Optimization Problem

We follow Van Nieuwerburgh and Veldkamp (2010) and model the optimization problem as follows: At period 0, rationally inattentive consumers know their current wealth (Y_0) perfectly but face future period-1 income risks, ϵ_a and ϵ_y.[28] In this model, households need not only solve an optimal consumption-saving problem but also solve an optimal attention allocation problem. The whole optimization problem can be formalized as follows:

$$V = \max_{\{\kappa_a,\kappa_y\}} \mathbb{E}_{\mathbb{I}^1}\left[u(C_0^*) + \beta u\left(v^{-1}(\mathbb{E}[v(C_1^*)|S_0])\right)\right] \tag{17}$$

subject to:

$$C_0^* = \arg\max_{C_0}\left\{u(C_0) + \beta u\left(v^{-1}(\mathbb{E}_{\mathbb{I}^2}[v(C_1)])\right)\right\}, \tag{18}$$

$$C_1^* = A_1(Y_0 - C_0^*) + Y_1, \tag{19}$$

$$\kappa_a + \kappa_y \leq \kappa, \tag{20}$$

where Equation (17) is the objective function for the household, $\mathbb{E}_{\mathbb{I}^2}[\cdot]$ is the expectation operator conditional on the information set $\mathbb{I}^2$, $\mathbb{E}_{\mathbb{I}^1}[\cdot]$ is the expectation over all possible signals, the budget constraints are incorporated into Equations (18) and (19), and Equation (20) displays the

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[27] Sims (2003) states that the logarithm in the formula can be to any base, because the base only determines a scale factor for the information measure, but conventionally it takes the logarithm to base 2, and as a result the entropy of a discrete distribution with equal weight on two points is 1 or $-0.5\log(0.5) - 0.5\log(0.5)$, which is the unit of information called a "bit." When the base is e, the unit of information is a "nat."

[28] In Van Nieuwerburgh and Veldkamp (2010), investors know their initial wealth and face risks in asset returns that are revealed at the end of the period.

attention capacity constraint.

4 Model's Implications for Attention Allocation and Consumption-Saving Decisions

In this section, we first briefly describe how we solve the model numerically and then explore intensively the model's implications for households' optimal consumption and saving decisions under the limited attention.

4.1 Solution Method

As illustrated in Figure 1, we decompose the optimization problem proposed above into two stages: (i) attention allocation and (ii) consumption-saving choice. In the first stage, before observing the noisy signals about capital return and labor income, households decide how much attention to allocate to learning capital return and labor income, respectively. This procedure determines how precise these two signals are. In the second stage, after observing the signals, households then decide how much to consume and how much to save out of the initial endowment. Following Maćkowiak and Wiederholt (2009), we solve these two sub-problems backward.

First, for *any* attention allocation strategy, we solve the following consumption-saving problem:

$$U = \frac{(Y_0 - K_1)^{1-1/\psi}}{1-1/\psi} + \beta \frac{\left(\mathbb{E}\left[(A_1 K_1 + Y_1)^{1-\gamma}\,|S_0\right]\right)^{\frac{1-1/\psi}{1-\gamma}}}{1-1/\psi}.$$

The first order condition for K_1 is:

$$\frac{\partial U}{\partial K_1} = -\left(Y_0 - K_1\right)^{-1/\psi} + \beta\left(\mathbb{E}\left[(A_1 K_1 + Y_1)^{1-\gamma}|S_0\right]\right)^{\frac{\gamma-1/\psi}{1-\gamma}} \mathbb{E}\left[(A_1 K_1 + Y_1)^{-\gamma} A_1|S_0\right] = 0. \quad (21)$$

It is straightforward that the first order condition determines a unique solution to the consumption problem. Solving the condition yields the optimal choice of K_1 in period 0. Plugging $K_1^*(S_a, S_y, \hat{\sigma}_a^2, \hat{\sigma}_y^2)$ back to the utility function gives us the indirect utility. Taking the unconditional expectations by evaluating over S_a and S_y allows us to solve the first-stage attention allocation problem. The detailed procedure is provided in Appendix 9.2.

It is worthwhile to briefly discuss two special cases. First, if consumers have infinite amount of attention, they know future labor-income and capital-income shocks perfectly, and we have a perfect-foresight case, in which

$$C_0 = (\beta A_1)^{-\psi} \frac{A_1 Y_0 + Y_1}{1 + A_1(\beta A_1)^{-\psi}}, \text{ and } C_1 = \frac{A_1 Y_0 + Y_1}{1 + A_1(\beta A_1)^{-\psi}}.$$

Second, if consumers have zero attention devoted to monitoring their future income risks, then no signal is received and they just use the prior information to forecast future income risks:[29]

$$U = \frac{(Y_0 - K_1)^{1-1/\psi}}{1-1/\psi} + \beta \frac{\left(\mathbb{E}\left[(A_1 K_1 + Y_1)^{1-\gamma}\right]\right)^{\frac{1-1/\psi}{1-\gamma}}}{1-1/\psi}.$$

If there is no labor income in period 1 and the utility function is of a CRRA-type expected utility, as shown in Yin (2021), the problem with finite attention or no attention can be solved analytically. Especially, if attention capacity is zero, the optimal saving in period 0 is

$$K_1^{\kappa=0} = \frac{1}{\left[\beta \exp\left(\frac{-\gamma(1-\gamma)}{2}\sigma_a^2\right)\right]^{-1/\gamma} + 1} Y_0, \tag{22}$$

which increases with capital income risk measured by σ_a^2 if $0 < 1/\gamma < 1$, and decreases with σ_a^2 if $1/\gamma > 1$. Higher σ_a^2 has two effects: saving in the risky asset becomes less attractive (save less due to the substitution effect) and consumers face higher uncertainty in the future (save more due to the income effect). This implies that when the EIS$=1/\gamma < 1$, the more reluctant agents are to substitute consumption intertemporally, and are willing to save more to against consumption fluctuations (the income effect dominates the substitution effect as discussed in, e.g., Sandmo (1970)). This holds in an infinite-time-horizon framework.

However, once we have additive labor income risk, the model becomes difficult to solve in a life-cycle model. We find two papers (Skinner (1988) and Mason and Wright (2001)) that solve the consumption problem with two risks by approximating the Euler equation. However, this approximation method delivers analytical solution only in special cases with quadratic utility, or with only capital/labor income risk. Without linearizing the Euler equation, this model becomes very complicated in the life-cycle model under RI, even when people try to solve it numerically. The main reason can be found in Skinner (1988), in which consumption-attention decision today depends on attention-choice in the future, as it affects future consumption-saving decision and thus future resources. Due to the tractability reason, we consider a two-period model to shut down the effects of future attention allocation on future resources.

4.2 Parameterization

In this section, we provide details how we set parameter values in this section by going through each block of parameters. In the next section, we also illustrate how different key parameter

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[29] Our setup is different from the RI models in Sims (2003) and Luo (2008), in which agents have no perfect knowledge about the current state. However, our model is similar to Van Nieuwerburgh and Veldkamp (2010), in which agents have no perfect information about a shock that is “announced” or “introduced” now, but will be revealed in the future.

values impact the attention allocation and consumption-saving decisions.

Capital and labor income risks. Following Campbell (2003), we set the prior variance of capital income risk σ_a^2 to 0.03 and assume that the ratio of prior variance of the labor income risk to that of the capital income risk $\sigma_y^2/\sigma_a^2 \in [1, 3, 5, 7, 9, 11, 13, 15]$. The benchmark value of labor income risk is 0.42.[30] We set the unconditional mean of capital return to 1.03. According to HRS data, the average drop of financial asset compared to one-year ago is about 28%. Therefore, the true realization of capital return is 0.72. If households with one member become unemployed, the total households' income becomes 1.475.[31]

Initial endowment. For the baseline parameterization, we set the unconditional expectation of labor income to 1 and the endowment in the initial period to $Y_0 = 7$. This value is calculated by using HRS data as follows. We first normalize each household's net financial wealth by the mean of its income and calculate the average value, which is about 3.5. We set the unconditional mean of a representative agent's labor income in the second period to 1 if employed and 0.5 if unemployed with unemployment probability 5%.[32] We then obtain the individual's unconditional expected income is 0.975. From our sample, the average household size is two, thus the expected income of a household with two workers is 1.95. To obtain a wealth-to-income ratio of 3.5, we need initial wealth to be about 7. [33]

Attention capacity. Directly measuring the degree of attention capacity is difficult. Though some estimates in the literature exist for the amount of information that humans can process (Landauer (1986)), it is difficult to map these numbers into the amount of attention that is actually allocated to monitoring the economic shocks faced by a typical household. Luo (2008) shows that when $\kappa = 0.5$ nats, the otherwise standard permanent income model can generate the observed aggregate consumption smoothness.[34] For example, **?** show that when $\kappa = 0.6$ nats, the model with limited attention can significantly improve the model's prediction on the key moments of the joint dynamics of durable consumption, non-durable consumption, and labor income. In addition, Coibion and Gorodnichenko (2015) use the SPF forecast survey data to test the degree of information rigidities governed by the degree of inattention and find that their model can fit the data well when κ is close to 0.5 nat. Therefore, we set the baseline value of κ to be 0.5 nat. For the robustness check, we also consider the cases when $\kappa = 1$ nat and 2 nats.

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[30] This value is the average prior variance for individuals hit by the unemployment shock. The average prior variance for the whole sample is 0.36.

[31] We are not able to observe whether a spouse/partner was hit by the unemployment shock during the same period. Therefore, we calculate the household's total income when the respondent becomes unemployed as the sum of the respondent's unemployment benefit and the expected income of the partner: $0.5 + (1 \times 0.95 + 0.5 \times 0.05)$.

[32] The replacement ratio in the US was about 50% according to the information on the website of the US Department of Labor (Employment and Training Administration).

[33] The value of initial endowment (Y_0) may vary largely for different individuals, from 2 to 20 in the Survey of Consumer Finances (SCF) given that the mean income is 1.

[34] Here "nat" is the unit of information transmitted, and 1 nat is defined as the entropy of a discrete distribution with equal weight on two points is simply $\mathbb{E}[\ln(f(X))] = -0.5\ln(0.5) - 0.5\ln(0.5) = 1$ nat.

The Discount factor. We set $\beta = 0.97$ as the baseline value. We also check the robustness of our main results by setting $\beta = 0.7$ and 0.8.[35]

The CRRA. In macroeconomic studies, the value of γ is between 1 and 6. We calibrate this value to match the overall consumption decline of 21% in the sample period, which results in a value of 5.

The EIS. We set $\psi = 1/3$. However, there is no consensus on the magnitude of the EIS (ψ), and the evidence is still mixed as the literature has found a very wide range of values. For example, Visising-Jorgensen and Attanasio (2003) estimate the EIS to be well in excess of 1, while Campbell and Cocco (2007) estimate a value well below 1 (and possibly 0). Guvenen (2006) finds that stockholders have a higher EIS (around 1.0) than non-stockholders (around 0.1). Best, Cloyne, Ilzetzki, and Kleven (2020) use U.K. mortgage data and find the EIS is close to 0.1. Havránek (2015) surveys the vast literature and suggests that a range around 0.3-0.4 is appropriate after correcting for selective reporting bias.

Next, we will do comparative statics analysis for optimal attention-consumption choice by varying one parameter while holding other parameters fixed at their baseline values. As we will show below, our main results do not rely on the choice of these parameter values.

4.3 Optimal Joint Attention-Consumption/Savings Decisions

In this subsection, we study the effects on optimal attention-consumption/savings decisions of the following factors: the relative prior volatility of the labor income risk to the capital income risk, the endowments of wealth and attention, the risk and time preferences, the expected capital return, and expected labor income.

Before moving on to discussing the effects of limited attention on the joint attention-consumption/savings decisions, it is helpful to inspect the mechanism via which the capital return risk and the labor income risk affect the consumption and saving behavior in the standard RE case. It is worth noting that within our RU framework with two income risks, both of them may increase the amount of precautionary savings. Specifically, as shown in Weil (1990) and Angeletos (2007), in an RU model with only capital income risk, the responses of consumption and savings to the capital return risk is theoretically indeterminate. The signs of the responses are determined by the value of the EIS.[36] For example, when the EIS is small (i.e., the income effect is relatively small), a mean-preserving increase in the capital return risk leads to a lower certainty equivalent capital

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[35] Barsky, Juster, Kimball, and Shapiro (1997) use MSC data and show that the overall average slope of the desired consumption path at a zero interest rate is 0.78 percent per year.

[36] Note that in an expected utility (EU) framework, there are two competing influences of the capital return risk at work: (i) the riskiness of the capital return makes savings less attractive than saving at the risk-free rate with the same average return, and (ii) the capital income risk will induce a precautionary saving motive to the prudent consumer. When the degree of prudence is sufficiently high (i.e., is greater than the CRRA plus 1 in the CRRA utility case), the precautionary motive dominates.

return, a lower marginal propensity to consume (MPC), and higher savings.[37] In addition, the degree of risk aversion determines the *magnitudes* of the responses. In contrast, in a model with only labor income risk, the presence of uncertain labor income interacts with the convexity of the marginal utility (i.e., prudence) and leads to an additional demand for precautionary savings. The intuition behind this result is that consumers increase their saving in order to better prepare themselves to face future labor income risk, and is similar to that in Leland (1968), Caballero (1990), and Wang, Wang, and Yang (2016).

4.3.1 The Effects of Relative Prior Variance

To examine the importance of the relative prior variance of labor and capital income, we first fix other parameters at their benchmark values, and then vary the value of the relative prior variance (σ_y^2/σ_a^2). As shown in Figure 2, it is clear that agents allocate more attention to the labor income shock as the relative importance of the prior variance of the labor income to that of the capital income increases. This is in line with many previous studies on optimal attention allocation, such as Maćkowiak and Wiederholt (2009). The intuition for this result is straightforward. When labor income becomes more uncertain, monitoring the risk in the labor-income dimension becomes relatively more important; as a result, agents pay more attention to the labor income risk relative to the capital income risk.

However, different from most of the previous studies that usually assume symmetric risks, the two risks (i.e., labor income risk and capital income risk) enter our model asymmetrically meaning that the risk on labor income is *additive* while the risk on capital income is *multiplicative* (as it is on the capital return). This asymmetry generates new implications for optimal attention allocation. To be more specific, in previous studies with symmetric risks, we have often seen that when the prior variances of two risks are the same, the agent allocates equal amounts of attention to each risk. However, in our model and as shown in Figure 2, agents pay more attention to the capital income risk when the variance of the labor income shock is the same as that of the capital income shock. This is due to two opposite effects of increasing the prior variance of the labor income shocks: (i) the *direct* effect makes agents pay more attention to the labor income risk and (ii) the *indirect* effect makes agents pay more attention to the capital income risk because the increase in the prior variance of labor income increases the amount of savings and thus makes the capital return more attractive. As shown in Figure 2, when σ_y^2/σ_a^2 is below approximately 1.5, the indirect effect dominates, meaning that agents pay more attention to the capital income risk than the labor income risk.

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[37] This effect is due to the negative interest elasticity of savings.

4.3.2 The Effects of Initial Wealth

To examine how initial wealth (Y_0) affects optimal attention-wealth allocation, we first fix other parameters at their benchmark values and then vary the value of Y_0. We can see from Panel A (the first row) of Figure 3 that the optimal amount of attention devoted to monitoring the capital income risk is rising with the level of initial wealth, suggesting that rich agents pay more attention to the capital income risk compared to poor agents. One potential explanation is that rich people have more risky assets in absolute amount than poor people and therefore have a stronger incentive to pay more attention to the capital income risk. This attention allocation mechanism makes the rich households' posterior variance in capital income smaller and investing in this risky asset becomes more attractive. In addition, as shown in Figure 3, the attention amount devoted to the labor income risk, κ_y, is decreasing with initial wealth (Y_0), and rich people save at a higher rate partially due to the amount of the precautionary savings from the increased posterior variance of labor income. From the left-upper panel of Figure 4 we observe that wealthier people indeed save at higher rates on average. This is in line with many empirical studies that show heterogeneous saving behavior across different wealth groups (see Dynan, Skinner, and Zeldes (2004).). For example, increasing Y_0 from 6 to 8 leads to a rise of the expected saving rate by approximately 4%. Yin (2021) also considers the impact of wealth inequality on the attention choice and the consumption-saving behavior. However, Yin (2021) considers one income shock (a shock to capital income) and assumes the information-processing cost to be fixed, whereas the present paper investigates optimal attention allocation in a more general setting with both the labor and capital income shocks.

To show the effect of wealth status on attention allocation, we follow Kontny and Yin (2020) and use survey data of Michigan Survey of Consumers (MSC) to measure attention with news recall. In MSC, respondent were asked to report what news they heard in past few months. As in Kontny and Yin (2020), we use a dummy variable for attention to labor market if respondents reported that they heard news about unemployment, job vacancies etc. We use a dummy variable for attention to financial market if respondents reported that they heard news about stock returns, interest rates. We measure wealth as the sum of real income and real stock holdings. In the regression below, we control for respondents' demographic characteristic, macroeconomic condition (NBER recession index) and use wealth as the main explanatory variable. As shown in Table 3, wealthy agents pay less attention to information about labor income risk relative to information about capital income risk. This direct empirical evidence suggests that wealth level indeed plays a role in determining attention allocated to different income shocks.

4.3.3 The Effects of the CRRA and the EIS

We first fix the EIS at its benchmark value of 1/3 and then vary the value of γ from 4 to 6. Panel B of Figure 3 show that the optimal amount of attention devoted to monitoring the capital income risk rises with the degree of risk aversion, meaning that more risk-averse agents pay more attention to the capital income shock and less attention to the labor income shock. One possible explanation is when the CRRA increases, the agent has a stronger incentive to save due to the precautionary motive, which leads the agent to pay more attention to the capital income risk.

Next, we fix the CRRA at its benchmark value 5 and then vary the value of the EIS (ψ) from 1/3 to 0.75. From Panel C (the third row) of Figure 3, we can see that a reduction in the EIS leads to a smaller amount of attention devoted to monitoring the capital income shock, and a larger amount of attention to the labor income shock. When the EIS is small, the change in consumption is less sensitive to the change of the capital return.[38] As a result, consumers pay less attention to the capital income risk. However, as can be seen from the same panel of Figure 3, this relationship between the EIS and attention allocation to the capital income risk may reverse if labor income is very uncertain. This is because a larger prior variance of the labor income risk leads to higher savings, and the EIS also governs how reluctant consumers change consumption across periods. In this situation, consumers with a low EIS have a stronger distaste for intertemporal substitution and would like to pay more attention to the capital income risk due to the larger amount of savings. Note that under the EU specification, a lower EIS means a higher CRRA by a restriction, which means that a decrease in the EIS also leads to smaller attention to the capital income risk. However, with the RU specification, this result may be misleading. That is, a lower EIS and a higher CRRA have opposite effects on attention allocation. As we discussed above, the main reason for this result is that the CRRA and the EIS affect the optimal attention allocation via distinct mechanisms.

In the middle panels of Figure 4, it is worth noting that although increasing the CRRA and reducing the EIS have a similar effect on the expected saving rate, their economic intuitions are totally different. In our RU model with two income risks, both the CRRA and the EIS play roles in determining the demand of precautionary savings. Specifically, the EIS affects savings via the relative importance of the income and substitution effects (i.e., the sign of the interest elasticity of savings), while the CRRA determines the *magnitude* of the change in savings. The above analysis therefore shows the importance of introducing the recursive utility in our model.

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[38] The EIS governs the sensitivity of consumption change to the change of interest rate, i.e., the percent change in consumption in response to one percentage change of interest rate.

4.3.4 The Effects of Attention Capacity

The attention allocation also depends on the total attention capacity (κ). First, it is easy to see, from Panel D (the fourth row) of Figure 3, that the amounts of attention allocated to each of the two shocks increase with the total amount of attention capacity. Second, fixing the labor and capital income risks (as measured by the two prior variances), we investigate the way how agents allocate the attention in these two dimensions depends on the total attention capacity. In general, as shown in Panel E (the fifth row) of Figure 3, the less total capacity is, the larger share of total attention is allocated to the labor income dimension, especially when the labor income risk is large. This suggests that the labor income risk becomes relatively more important to agents when the total attention capacity is more limited.

The lower-right panel of Figure 4 shows that the average saving rate decreases with total attention capacity. To understand this, let us take an extreme case as an example. When total attention is zero, i.e., agents pay no attention to the income shocks, they face greater posterior uncertainty in future income than those who pay some attention, and consequently, they choose to save at a higher rate due to the precautionary saving motive. We also notice that the pattern of the expected saving rate becomes flatter and flatter when increasing the total attention capacity. This is intuitive because when agents have more capacity to process information, the difference in their posterior variance is not big no matter how large the prior variance is. In another extreme case, when $\kappa \to \infty$, our model becomes completely deterministic and the saving rate becomes flat.

5 Confronting the Model with Data

In this section, we show how the mechanisms we highlighted in the previous section can help explain the data in three aspects.

To fully explore the mechanism, we rewrite the expression for the change in consumption as follows:

$$\frac{\Delta C_1}{C_0} = \frac{A_1K_1 + Y_1 - (Y_0 - K_1)}{Y_0 - K_1} = \frac{Y_1/Y_0 - 1 + 2s}{1 - s} + \frac{s(A_1 - 1)}{1 - s}, \tag{23}$$

where $s = K_1/Y_0$ is the saving rate (or the marginal propensity to save, the MPS) and $1 - s$ is the marginal propensity to consume (the MPC).

In general, from the upper panel of Figure 5, we show that the decline in consumption due to labor and capital income shocks becomes smaller when the prior variance of the labor income shock increases. Next, we will split the total change into a part due to financial loss (the difference between capital return and 1) and unemployment (the reduction of labor income in period 1).

5.1 Consumption Response to the Prior of the Labor Income Risk

The first aspect we want to explore is how individual consumption responds to the prior labor income risk. Column 1 in Table 2 reports that, on average, individuals with 1 more unit of the prior variance of labor income, the decline in consumption after the unemployment shock becomes 11.6 percentage points smaller in absolute terms. Qualitatively, this finding is highly consistent with our model's prediction.

As shown in the lower-left panel of Figure 5, our calibrated model predicts that the reduction in consumption after unemployment is smaller for agents with larger prior variances of labor income. Using the results regarding the optimal saving behavior, this empirical fact can be explained as follows. An agent with a larger prior variance of the labor income risk will save more due to the precautionary saving motive. We call this channel the *direct* precautionary saving effect. But at the same time the agent also pays more attention to the labor income risk, leading to a reduction in the posterior variance about labor income and precautionary savings. Meanwhile, due to limited attention, the agent pays less attention to the capital income risk, making the risky asset riskier and less attractive and then reducing the saving rate. We call these two channels induced by limited attention the *indirect* precautionary saving effects. Figure 4 shows that the direct precautionary motive effect dominates the other two indirect effects, meaning a higher prior variance of the labor income risk causes a higher total saving rate, and thus the reduction of consumption is smaller.

5.2 Consumption Response to Financial Shocks

The second aspect we want to explore is how the consumption response to financial shocks depends on the prior labor income risk. As also shown in Column 1 of Table 2, when financial wealth drops, consumption declines more if the agent has a higher prior variance of the labor income risk. It is clear from equation (23) that the second term on the right hand side can be used to characterize the response of the consumption change to the loss of financial wealth. The empirical fact mentioned above can thus be explained as follows. When agents have a larger prior variance of the labor income risk, they save more, leading to an increase in $s/(1-s)$, i.e., the ratio of the MPS to the MPC. As a result, given the negative net capital return, $A_1 - 1$, the consumption declines more. In the lower-right panel of Figure 5, we plot the marginal effect of the loss in financial wealth for different values of the prior variance of labor income. It is clear that the curves in this figure are downward sloping and are thus consistent with the empirical facts shown in Table 2.

We make the following quantitative comparisons. In our empirical results, Fact 1 says that during the Great Recession period, households with a larger prior variance on their labor income experience a smaller consumption decline in percentage terms when they become unemployed and Fact 2 says that a higher prior variance on labor income leads to a larger consumption decline in response to a financial loss. In general, both finite-attention and zero-attention models can *qualitatively* capture the empirical evidence documented above. However, the finite-attention model can match the data much better. From HRS data, we find that the average change in consumption for those who experienced both labor and capital income shocks was about -21.1. In the finite-attention model, we find a consumption change of -19.4. However, in the corresponding zero-attention model, the consumption change becomes -14.7, which is well above the empirical counterpart. The main reason behind this difference is that the saving rate in the zero-attention model is too large. More precisely, if consumers pay no attention to reduce the uncertainty in the income shocks, their precautionary saving motives are very strong when increasing the prior variance of the labor income shock. However, if they can optimally allocate their finite attention to both income shocks, an increase in the variance of the labor income shock attracts more attention such that the precautionary saving motive becomes smaller. In addition, less attention devoted to the capital income risk makes the risky asset become a less attractive saving device.

Moreover, we can show that the finite-attention model does an even better job in explaining the data than the zero-attention model when the prior variance is large. We divide the prior variance of the labor income risk into four groups (quartiles). The second row of Table 4 shows the average prior variance in each quartile. The third row shows the average percent change in consumption. We then plug different prior variances into the model under assumption that labor income is 1.475 with unemployment shock and unemployment benefit, and other parameter values are calibrated as before. Comparing the results in the fourth and fifth rows, we can observe that the consumption behavior in the finite-attention model is closer to the empirical counterparts than in the zero-attention model, especially when the labor income risk is more volatile. To check the robustness of these results, we also consider the level of financial wealth loss in different quartiles of the prior variance in Table 5. It is also clear from the table that comparing different models' quantitative results with the empirical counterparts, our finite-attention model does a better job in explaining the empirical consumption behavior.

5.3 Heterogeneous Consumption Responses Due to Different Wealth Levels

The third aspect we want to examine is the heterogeneous consumption response driven by different wealth levels. As shown in Columns 3 and 4 of Table 2, the coefficient for the interaction term of financial asset and the unemployment shock is negative, meaning that wealthier individuals experience larger consumption decline (in percent) at unemployment.

We can better explain this empirical result by rewriting (23) as follows:

$$\frac{\Delta C_1}{C_0} = \frac{A_1 K_1 + Y_1 - (Y_0 - K_1)}{Y_0 - K_1} = \frac{s}{1-s} A_1 + \frac{Y_1}{C_0} - 1, \tag{24}$$

From (24), we can see that there are two opposite effects of an increase in initial wealth on the change in consumption. First, a higher level of initial wealth, Y_0, leads to a higher savings rate, as explained in the previous section and Figure 4. Holding everything else equal, a higher savings rate would increase the first two terms on the right side of (24) and thus lead to a smaller decline in consumption.[39] Second, a higher level of initial wealth lowers the gross growth rate of wealth, Y_1/Y_0, or the relative importance of unemployment benefits to initial wealth. This tends to reduce $\Delta C_1/C_0$ (i.e., leads to a larger percent decline in consumption). The upper panels of Figure 6 plots these two effects and shows that in our calibrated exercise, the second effect dominates the first, suggesting that wealthier individuals experience a larger consumption decline at unemployment. This model's prediction is also consistent with the empirical counterpart.

It is important to note that, quantitatively, these two effects are affected by the total attention capacity. As shown in the bottom-left panel of Figure 6, when attention capacity is increasing, the change in consumption becomes flatter and flatter. We can easily show that when attention capacity goes to infinity, the change in consumption in this deterministic scenario is $(\beta A_1)^{1/\gamma} - 1$. One potential explanation for this diminishing effect of initial wealth on the change in consumption is that increasing the attention capacity strengthens the effect of initial wealth on the saving behavior (the first effect mentioned above). Keeping all other parameters constant, the consumers' saving decisions depend on initial wealth and the precautionary saving motive due to income risks. When attention capacity is small, agents face higher perceived uncertainty and the effect of initial wealth on total savings is smaller. However, when increasing attention capacity, agents have less and less perceived uncertainty in their income; as a result, the effect of initial wealth on saving decisions becomes stronger. As shown in the bottom-right panel of Figure 6, the positive effects of increasing initial wealth on the expected saving rate becomes larger for larger attention capacity.

6 Welfare Implications

In this section, we compute the welfare gains if the inattentive agents are allowed to increase their channel capacity. Specifically, we follow Cochrane (1989), Luo (2008) and Maćkowiak and Wiederholt (2015), and also conduct a welfare analysis.[40] As shown in Table 6, we calculate the utility losses for three different values of κ and four different values of σ_y^2, Y_0, γ, and ψ. Here is the procedure to conduct the welfare analysis. Our main purpose for this exercise is to investigate how an increase in attention capacity affects the expected lifetime utility. For example, we first choose $\kappa = 1$ as the starting value, and calculate the corresponding unconditional expected lifetime utility. Then we increase each starting value of attention capacity κ by 100% and compute the

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[39] Notice that in the calibration, the consumption change in (24) is negative, and thus a smaller decline means consumption change is less negative.

[40] Different from our two-period consumption model with two income shocks, Luo (2008) studies an infinite horizon permanent income model with a single labor income shock. He examines the welfare effects of income shocks under rational inattention by calculating how much utility agents will lose if the actual consumption path under rational inattention deviates from the first-best consumption path under full information.

corresponding unconditional expected lifetime utility for each κ. Finally, we can compute the percentage increase of the expected lifetime utility using this formula:

$$\left| \frac{\mathbb{E}[U(\kappa_{\text{new}})] - \mathbb{E}[U(\kappa_{\text{baseline}})]}{\mathbb{E}[U(\kappa_{\text{baseline}})]} \right| . \tag{25}$$

Next, we will present two exercises for the welfare analysis. In the first exercise, we choose two different starting points of attention capacity, namely, 1 and 2. And then at each point, we increase attention capacity by 100%. First, we can see from all the three panels of Table 6 that the utility gains are increasing with the level of attention capacity. This result is intuitive and in line with the findings in Luo (2008): agents with higher attention capacity can better predict their future income shocks and in the extreme case when they have infinite capacity, it converges to the corresponding perfect-foresight scenario. Second, if we compare vertically for each panel, it clearly shows that the change in the expected utility is decreasing in κ. More precisely, as shown in the first column of Panel A where $Y_0 = 7$, increasing attention capacity from 1 to 2 leads to an increase of welfare by about 1.9%, while a rise of attention from 2 to 4 increases welfare by about 1%. Comparing Rows 1 and 2 of the table, these results suggest a heterogeneity in the welfare gain for agents with different levels of attention capacity.

Using the above welfare calculation formula, we can also examine the effects of changes in different factors on welfare gains. Panel A of Table 6 shows that for a given attention capacity, the expected lifetime utility is decreasing with initial wealth. As shown in the first row where we increase κ from 1 to 2, if Y_0 is increased from 6 to 8, we can see that the welfare gain decreases from about 2% to 1.8%. The intuition behind this result is as follows. Poor individuals also consume less, and have larger marginal utility; as a result, increasing an additional unit of attention is more beneficial for poor individuals as it can help them make more efficient consumption-saving plans. Panels B and C of Table 6 also show that the welfare gain is increasing with the discount factor and the CRRA. However, the intuitions for these results are different. For Panel B, more patient agents care more about their future utility and increasing attention capacity can reduce their uncertainty in future income and consumption. In contrast, the intuition of the results in Panel C is that, for more risk-averse agents, the larger the attention capacity, the higher the lifetime expected utility because with more attention, they would face less uncertainty from labor and capital incomes. Finally, the results in Panel D show that the welfare gain is decreasing with the EIS. When the EIS becomes smaller, agents prefer the consumption profile to be smoother across periods and increasing attention can help reduce the fluctuations.

In the second exercise, we investigate the effect of optimal attention allocation on the welfare gain. More specifically, as shown in the third column of Table 7, we first repeat the welfare analysis for different levels of initial wealth by increasing total attention capacity. These results are subject to the attention allocation mechanism. In the second column, we first solve the optimal attention allocation strategy when $\kappa = 1$ for each value of initial wealth. Then we fix this attention allocation strategy and compute the welfare gain by increasing total attention capacity to 2, 3, and ∞, respectively. When comparing these two columns, we can see that the welfare gains are consistently larger in the optimal/flexible attention allocation case. This implies that allowing flexible attention allocation makes the agent better off for different values of initial wealth, and the welfare gains from the flexible adjustment are significant. For example, when $Y_0 = 7$ and κ is increased from 1 to 2, the percentage change of the welfare gain from switching the fixed attention allocation mechanism to the flexible attention allocation mechanism is about 33.3%.

As a useful remark, as proposed in Sims (2010) and discussed in Paciello and Wiederholt (2014), we may model RI by assuming that agents choose the optimal degree of attention capacity to minimize the conditional variance of the state, given an endowed constant marginal cost of acquiring and processing information (i.e., a shadow price of attention capacity). This is called "elastic attention" in which case the agent aims to equalize the marginal benefit of reducing the conditional variance of the state (by increasing the channel capacity) and the marginal cost of acquiring and processing information. In such models with elastic attention, the relationship between the marginal cost of acquiring and processing information and the optimal channel capacity (and the Kalman gain in the filtering problem) is a one-to-one mapping, holding other parameters (income uncertainty, the discount factor, the CRRA, and the EIS) constant. Although we do not explicitly model elastic attention in the current paper, we may view the difference or change in attention capacity (κ) as a result of a change in the constant marginal cost of information-processing. For example, the two different starting points of attention capacity, $\kappa = 1$ and 2, may imply that the agent with $\kappa = 2$ can process relevant information more efficiently because he has a lower marginal cost of information-processing.

7 Further Discussions

7.1 Relative Consumption Dispersion to Income

The significant increase in household income inequality or dispersion for the U.S. in the 1980s and 1990s is a well-documented fact. Many studies have found that the dispersions of U.S. household earnings and incomes have a strong upward trend. In addition, the literature also documents that the recent increase in income inequality in the U.S. has not been accompanied by a corresponding rise in consumption inequality over the same period. In other words, over the sampling period, income and consumption inequality diverged and the relative dispersion/inequality of consumption to income has decreased, as discussed by Krueger and Perri (2006) and Blundell, Pistaferri, and Preston (2008). In the section, we show that our benchmark model proposed in Section 3 can also be used to study how attention allocation affects the relative dispersion of changes in consumption to income.[41] The left panel of Figure 7 shows the evolution of consumption and income dispersion as well as the relative dispersion of changes in consumption to income between 1980 and 2010.[42] It clearly shows that the relative dispersion declines while the volatility of labor income increases.

The right panel of Figure 7 shows that our model generates a similar pattern under different parameters. That is, as the prior variance of the labor income risk rises, the relative consumption dispersion declines. It is worth noting that in the literature, the most appropriate empirical measure of labor income uncertainty is not obvious. Some previous studies have proxied income uncertainty with either the variability of a household's income or the variability of its expenditures. However, as pointed out by Lusardi (1998) and Guiso, Jappelli, and Terlizzese (1992), the variability measures mentioned above may be poor proxies because they can contain large controllable elements. Here we follow Lusardi (1998) and Carroll, Dynan, and Krane (2003) and use the prior variance of labor income calculated using the probability of job loss to measure the uncertainty/volatility of labor income.

In addition, from the right panel of Figure 7, we can see that the relative dispersion is decreasing with κ. The intuition for this result is as follows. There are two factors that can affect the relative dispersion of consumption to income: one is attention capacity and the other is the saving behavior. When agents have a larger attention capacity, the relative dispersion of consumption becomes smaller because they have more precise signals (i.e., smaller variances of the noises).[43] Given the baseline calibration, the expected change in consumption is positive. When agents save at lower rates, their consumption becomes smoother and the relative dispersion becomes smaller. Therefore, a potential explanation for the positive correlation between the relative dispersion and the amount of attention is that when κ is large, agents save less and they have less perceived uncertainty in their future income and consumption; as a result, the change of consumption is less dispersed.

7.2 Implications on Tax Policies

In this subsection, we want to discuss how different types of taxation policy affect the consumption-saving allocation via the attention allocation channel. Specifically, we study a partial equilibrium case, where there exists a government who collect taxes on both capital income and labor income

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[41] The relative consumption dispersion/inequality is measured as the ratio of the standard deviation of the change in consumption to the standard deviation of the change in income. Luo, Nie, Wang, and Young (2017) study how information friction affects the consumption inequality in infinite-horizon settings.

[42] See Online Appendix A in Luo, Nie, and Young (2020) for more details on how the panel is constructed from Panel Study of Income Dynamics (PSID).

[43] We can show that consumption is a function of the noisy signal and thus the variance of consumption depends on the variance of the signals.

from the household sector.[44] The budget constraints can thus be written as:

$$C_0 + K_1 = Y_0, \tag{26}$$

$$C_1 = (1 - \tau_a) A_1 K_1 + (1 - \tau_y) Y_1, \tag{27}$$

where τ_a and τ_y are linear tax rates on capital income and labor income, respectively. The first order condition for K_1 in the second stage problem is then:

$$\beta \left(\mathbb{E} \left[(1 - \tau_a) A_1 K_1 + (1 - \tau_y) Y_1 \right)^{1-\gamma} | S_0] \right)^{\frac{\gamma - 1/\psi}{1-\gamma}} \mathbb{E} \left[((1 - \tau_a) A_1 K_1 + (1 - \tau_y) Y_1)^{-\gamma} (1 - \tau_a) A_1 | S_0 \right]$$
$$= (Y_0 - K_1)^{-1/\psi}.$$

Following the same two-step approach proposed in the previous section, we can solve the optimal attention-consumption attention under different tax schemes. Figure 8 illustrates how the saving rate and attention allocation vary with different types of income taxes.[45]

The upper two panels of Figure 8 presents how changing the tax rates affects the optimal attention allocation. We find that an increase in the marginal tax rate on labor (capital) income leads to a decrease in the amount of attention allocated to the labor (capital) income shock. This result is related to the effect of the change in the tax rate on the income uncertainty. For example, an increase in the marginal tax rate on labor income can cause a large reduction in the after-tax labor income risk, which leads to a lower amount of attention to the labor income shock. It is worth noting that Elmendorf and Kimball (2000) show that in an optimal saving-portfolio choice model, an increase in the labor income tax rate reduces the after-tax labor income risk, which leads to a significant increase in the optimal share of financial wealth invested in the risky asset. Similarly, it also holds for the change in the capital income tax.

We next consider the effects of different taxes on the consumption-saving decision because they are linked more directly. It is clear from the lower panels of Figure 8 that the saving rate increases with both the capital income tax rate (τ_a) and the labor income tax rate (τ_y). This is intuitive because a higher tax rate increases the household's saving motive. For example, given that $\tau_y = 21.4\%$, and keep all other parameters as in the baseline model, the expected saving rate increases with both τ_y and τ_a. Increasing taxes has a *direct* effect on savings through the smoothness motive channel, i.e., agents would like to save more to smooth consumption if tax rates are high. There is also *indirect* effects through the attention reallocation channel. More precisely, increasing τ_y leads to less attention to the labor income risk but more attention to the capital income risk and this will further increase the saving rate. On the other hand, increasing τ_a shifts more attention to the labor income risk but less attention to the capital income risk

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[44] Here we follow Luo (2017) and assume that government takes tax and consumes by itself. And thus there is no transfer from government to households.

[45] We again use the baseline parameter values for these exercises.

and this will reduce the saving rate. Therefore, we can observe that changing tax rates of labor income has larger effects on the household's consumption-saving behavior than changing that of capital income.

The discussions above provide us with useful policy implications on the effects of different types of income tax under RI. As increasing the labor income tax rate has larger effects on both attention allocation and consumption-saving allocation, we can argue that labor income taxes are more efficient for the government to achieve its goals of, for example, adjusting the household's savings than capital income taxes.

7.3 The Bequest Motive

In our benchmark model, we adopt a two-period specification so that we can fully solve the optimal consumption-attention allocation problem with both capital and income risks. One potential inconsistency of the model is that consumers in the second period will consume all their resources which looks different from the HRS data we use: people in the second period do not spend all their resources in reality. In this subsection, we introduce a bequest motive into the model to overcome this inconsistency and show that the key implications of the model remain unchanged. In other words, introducing bequest motives helps the model to generate a realistic consumption level in the second period but the implication regarding relative changes in consumption across individuals remain largely unchanged.

For convenience, we follow Carroll (1998) and Dynan, Skinner, and Zeldes (2002), and consider an EU model (a special case of our benchmark RU model) with constant relative risk aversion preferences:

$$\begin{aligned}
&\max U = \mathbb{E}\left[u(C_0) + \beta\mathbb{E}\left[u(C_1) + b(W)|S_0\right]\right], \\
&\text{subject to: } C_0 = Y_0 - K_1, \\
&C_1 = A_1K_1 + Y_1 - W, \\
&\kappa_a + \kappa_y \leq \kappa,
\end{aligned}$$

where W is the bequest left after period 1 and $b(\cdot)$ is the utility from leaving a bequest to their offsprings; signal structure and information-flow constraint are defined in Equations (9) and (14).

Carroll (1998) assumes that $u(C) = \frac{C^{1-\gamma}}{1-\gamma}$ and $b(W) = \frac{(W+\omega)^{1-\alpha}}{1-\alpha}$. For simplicity, we consider a special case with $\omega = 0$ and $\gamma = \alpha$. In this case, we first obtain that:

$$C_1 = W = \frac{A_1K_1 + Y_1}{2}, \tag{28}$$

and that:

$$U = \mathbb{E}\left[u(C_0) + \beta\mathbb{E}\left[u(C_1) + b(W)|S_0\right]\right]. \tag{29}$$

Plugging Equation (28) back into the above utility function, U, we obtain the first order condition for optimal savings K_1:

$$u'(Y_0 - K_1) = \beta \mathbb{E}\left[u'\left(\frac{A_1K_1 + Y_1}{2}\right)\frac{A_1}{2} + b'\left(\frac{A_1K_1 + Y_1}{2}\right)\frac{A_1}{2}|S_0\right] \tag{30}$$

Finally, using the result of optimal savings in U, we can find the optimal attention allocation that maximizes the expected utility.

We plot the optimal attention-consumption allocation and consumption responses in Figure 9. In these figures, we first notice that they are consistent with our main results obtained in our benchmark model: wealthier individuals pay more attention to the capital income risk than those with less initial wealth; people pay more attention to the labor income risk if the prior variance of the labor income shock increases; people with more initial wealth save at higher rate and have larger reduction in consumption when they experience unemployment shock and financial wealth loss. In addition, we also find that due to the bequest motive, consumers choose to save more than in the benchmark model (see the upper-left panel of Figure 4). This result is in line with the literature that studies the importance of the bequest motive in saving behavior, such as Carroll (1998) and Kopczuk and Lupton (2007). Moreover, the lower-right panel in Figure 9 shows that, after considering the bequest motive, the consumption response to income shocks is not quantitatively different from the main results of our benchmark model. This is because although people with the bequest motive save at higher rates, their consumption in period 1 is also only a part of the total resource.

8 Conclusion

In this paper, we construct and solve a consumption-saving model with recursive utility and two types of income risks under limited attention capacity to study how limited attention and its optimal allocation impact the optimal consumption and saving decisions. The key feature of this model is that agents with limited attention need to decide how to allocate their attention to monitor labor and capital income risks. We quantitatively evaluate how the optimal attention-consumption allocation is affected by the relative prior variance of the two exogenous income risks (capital income and labor income risks), differences in individuals' endowments of wealth and attention, and differences in their risk and time preferences. We find that our model can capture some key aspects of household consumption behavior observed in the U.S. micro-data. We also show that the welfare loss due to limited attention is significantly larger for households with lower wealth; allowing households to flexibly allocate their attention can significantly reduce this welfare loss.

9 Appendix

9.1 Data Appendix

This appendix describes the variables used in our empirical analysis as well as the sample treatment.

- Consumption change ($\frac{\Delta C_t}{C_{t-1}}$) is the total spending change compared to the previous year. These data are from HRS 2009 internet survey, in which respondents were directly asked "By how much has your household spending [increased/reduced] compared to a year ago?" In our analysis, we follow Christelis, Georgarakos, and Jappelli (2015) and exclude observations with consumption change larger than 80% or smaller than -80%.

- Financial wealth change ($\frac{\Delta WF_t}{WF_{t-1}}$) is the reduction of financial wealth since September of 2008. These data are from HRS 2009 internet survey, in which respondents were directly asked "How much has the total value of your retirement plans (IRA and Keogh plans, trust, mutual funds, stocks, and other investment) declined percentage-wise since September of 2008?"

- Likelihood of losing job (p) indicates the chance that the respondent believes that she/he will lose job during the next year. These data are from HRS 2006 core survey. We use this information as the subjective belief of being unemployed in the future.

- Labor income is the total amount of income from all jobs. We use both 2006 and 2008 core surveys of HRS to obtain individuals' labor income. Especially the labor income of 2006 is used to calculate the prior variance in labor income as shown below.

- Prior variance ($pvar$) is defined as $p(1-p)(1-\eta)^2\tilde{Y}_{2006}^2$. Here we follow the measured introduced by Lusardi (1998). $\tilde{Y}_{2006}$ is the labor income in 2006 normalized by the mean income in 2006. η is the replacement ratio of unemployment from the website of the US Department of Labor. In the empirical analysis, we exclude one outlier with prior variance larger than 10000.

- Unemployment shock (ΔU_t) shows whether a respondent became unemployed between the date of the survey and the same date a year ago. Here we want to construct an unemployment shock that affects respondent's current consumption, but not the consumption a year ago. These data are from HRS 2009 internet survey, in which respondents were asked to report their employment status, and the year and month of being unemployed.

- Net financial asset is the sum of respondent's financial asset (including retirement plans, IRA and Keogh plans, trust, mutual funds, stocks, and other investment) and debts. These data are from HRS 2008 core survey.

- Subjective belief of stock market is measured by individual's belief about the percentage chance that Dow Jones industrial average will increase next year. To keep the consistency with the prior belief about the labor income risk, we also use data from HRS 2006 core survey.

- Other personal characteristics (including, age, education, gender, marital status, retirement status, household size) are from HRS 2009 internet survey. Especially, education here is measured by years of education. Similar as the unemployment shock, *retirement* is a dummy variable indicates whether the respondent became retired between the data of interview and the same date a year ago.

9.2 Appendix: Solving the Recursive Utility Model

We adopt the outer optimization approach to solve the optimal attention allocation problem in the recursive utility case. For any given amount of attention to capital income shock κ_a, we can obtain the distribution of the signal on capital income shock, S_a. As the total amount of attention is fixed, we can also obtain the amount of attention to labor income shock, κ_y, and the distribution of the signal on capital income shock, S_y. Then, we can solve the optimal savings K_1^* for a combination of (s_a, s_y) according to Equation (21).

Here we maximize the unconditional expected utility (evaluating over possible signals) by choosing the optimal attention allocation:

$$\begin{aligned} U(K_1) &= \frac{(Y_0 - K_1)^{1-1/\psi}}{1-1/\psi} + \beta \frac{\left(\mathbb{E}\left[(A_1K_1 + Y_1)^{1-\gamma}|S_0\right]\right)^{\frac{1-1/\psi}{1-\gamma}}}{1-1/\psi} \\ &= U\left(S_a, S_y, \hat{\sigma}_a^2, \hat{\sigma}_y^2, K_1\right). \end{aligned}$$

Define $V(\hat{\sigma}_a^2, \hat{\sigma}_y^2) = \mathbb{E}_{\mathbb{I}}[U(S_a, S_y, \hat{\sigma}_a^2, \hat{\sigma}_y^2, K_1)]$, the attention allocation is to choose κ_a:

$$\max_{\kappa_a} V(\kappa_a), \tag{31}$$

subject to:

$$\frac{1}{2}\log\left(\frac{\sigma_a^2}{\hat{\sigma}_a^2}\right) = \kappa_a, \tag{32}$$

$$\frac{1}{2}\log\left(\frac{\sigma_y^2}{\hat{\sigma}_y^2}\right) = \kappa_y, \tag{33}$$

$$\kappa_a + \kappa_y = \kappa. \tag{34}$$

Here we can easily show the mean and variance of the signals can be written as:

$$\mathbb{E}[S_a] = \mu_a,\ \mathbb{V}(S_a) = \frac{(\sigma_a^2)^2}{\sigma_a^2 - \hat{\sigma}_a^2};\ \mathbb{E}[S_y] = \mu_y,\ \mathbb{V}(S_y) = \frac{(\sigma_y^2)^2}{\sigma_y^2 - \hat{\sigma}_y^2}.$$

Their corresponding density functions are :

$$f_{S_a} = \frac{1}{\sqrt{2\pi\mathbb{V}(S_a)}} \exp\left(-\frac{(s_a - \mu_a)^2}{2\mathbb{V}(S_a)}\right) \text{ and } f_{S_y} = \frac{1}{\sqrt{2\pi\mathbb{V}(S_y)}} \exp\left(-\frac{(s_y - \mu_y)^2}{2\mathbb{V}(S_y)}\right).$$

Define $t_a = \frac{s_a - \mu_a}{\sqrt{2\mathbb{V}(S_a)}}$ and $t_y = \frac{s_y - \mu_y}{\sqrt{2\mathbb{V}(S_y)}}$, we have

$$\begin{aligned} s_a &= \mu_a + \sqrt{2\mathbb{V}(S_a)}t_a = \mu_a + \frac{\sqrt{2}\sigma_a^2}{\sqrt{\sigma_a^2 - \hat{\sigma}_a^2}}t_a, \\ s_y &= \mu_y + \sqrt{2\mathbb{V}(S_y)}t_y = \mu_a + \frac{\sqrt{2}\sigma_y^2}{\sqrt{\sigma_y^2 - \hat{\sigma}_y^2}}t_y. \end{aligned}$$

We apply the Gaussian quadrature approach to approximate the unconditional expectation of the utility and obtain the value for some κ_a (κ_y). In the second step, we adopt the inner optimization approach to solve the corresponding optimal consumption-saving problem. Specifically, the RHS of the Euler equation (21) can be written as:

$$\beta\left(\mathbb{E}\left[(A_1K_1 + Y_1)^{1-\gamma}|S_0\right]\right)^{\frac{\gamma - 1/\psi}{1-\gamma}} \mathbb{E}\left[(A_1K_1 + Y_1)^{-\gamma}A_1|S_0\right].$$

The conditional distributions of $\epsilon_a|S_a = s_a$ and $\epsilon_y|S_y = s_y$ can be written as:

$$\begin{aligned} f_{\epsilon_a|S_a=s_a} &= \frac{1}{\sigma_a\sqrt{1-\rho_a^2}\sqrt{2\pi}} \exp\left(-\frac{\left(\epsilon_a - \left(\mu_a + \sigma_a\rho_a\frac{s_a - \mathbb{E}[S_a]}{\sqrt{\text{var}(S_a)}}\right)\right)^2}{2\sigma_a^2(1-\rho_a^2)}\right), \\ f_{\epsilon_y|S_y=s_y} &= \frac{1}{\sigma_y\sqrt{1-\rho_y^2}\sqrt{2\pi}} \exp\left(-\frac{\left(\epsilon_y - \left(\mu_y + \sigma_y\rho_y\frac{s_y - \mathbb{E}[S_y]}{\sqrt{\text{var}(S_y)}}\right)\right)^2}{2\sigma_y^2(1-\rho_y^2)}\right). \end{aligned}$$

Define

$$x_a = \frac{\epsilon_a - \left(\mu_a + \sigma_a\rho_a\frac{s_a - \mathbb{E}[S_a]}{\sqrt{\mathbb{V}(S_a)}}\right)}{\sigma_a\sqrt{1-\rho_a^2}\sqrt{2}} \text{ and } x_y = \frac{\epsilon_y - \left(\mu_y + \sigma_y\rho_y\frac{s_y - \mathbb{E}[S_y]}{\sqrt{\mathbb{V}(S_y)}}\right)}{\sigma_y\sqrt{1-\rho_y^2}\sqrt{2}},$$

we have

$$\epsilon_a = \sigma_a \sqrt{1-\rho_a^2}\sqrt{2}x_a + \mu_a + \sigma_a \rho_a \frac{s_a - \mathbb{E}[S_a]}{\sqrt{\mathbb{V}(S_a)}},$$
$$\epsilon_y = \sigma_y \sqrt{1-\rho_y^2}\sqrt{2}x_y + \mu_y + \sigma_y \rho_y \frac{s_y - \mathbb{E}[S_y]}{\sqrt{\mathbb{V}(S_y)}},$$

where $\rho_a^2 = 1 - \hat{\sigma}_a^2/\sigma_a^2$, $\sqrt{1-\rho_a^2} = \hat{\sigma}_a/\sigma_a$, $\mathbb{E}[S_a] = \mu_a$, $\mathbb{V}(S_a) = (\sigma_a^2)^2/\left(\sigma_a^2 - \hat{\sigma}_a^2\right)$, $\rho_y^2 = 1 - \hat{\sigma}_y^2/\sigma_y^2$, $\sqrt{1-\rho_y^2} = \hat{\sigma}_y/\sigma_y$, $\mathbb{E}[S_y] = \mu_y$, and $\mathbb{V}(S_y) = (\sigma_y^2)^2/\left(\sigma_y^2 - \hat{\sigma}_y^2\right)$. Finally, we have

$$\epsilon_a = \hat{\sigma}_a \sqrt{2}x_a + \frac{\hat{\sigma}_a^2}{\sigma_a^2}\mu_a + \left(1 - \frac{\hat{\sigma}_a^2}{\sigma_a^2}\right)s_a,$$
$$\epsilon_y = \hat{\sigma}_y \sqrt{2}x_y + \frac{\hat{\sigma}_y^2}{\sigma_y^2}\mu_y + \left(1 - \frac{\hat{\sigma}_y^2}{\sigma_y^2}\right)s_y.$$

Notice that when attention capacity is zero, noise variance is infinity and posterior variance equals the prior variance. In this case, signal is too noisy to be useful and agents make decisions based on their prior belief.

Then we rewrite $(A_1K_1 + Y_1)^{1-\gamma}$ and $(A_1K_1 + Y_1)^{-\gamma}$ in the RHS of the Euler equation as

$$f_1(\epsilon_a, \epsilon_y) = \left(\exp\left(\sqrt{2}\hat{\sigma}_a x_a + \frac{\hat{\sigma}_a^2}{\sigma_a^2}\mu_a + \left(1 - \frac{\hat{\sigma}_a^2}{\sigma_a^2}\right)s_a\right)K_1 + \exp\left(\sqrt{2}\hat{\sigma}_y x_y + \frac{\hat{\sigma}_y^2}{\sigma_y^2}\mu_y + \left(1 - \frac{\hat{\sigma}_y^2}{\sigma_y^2}\right)s_y\right)\right)^{1-\gamma},$$
$$f_2(\epsilon_a, \epsilon_y) = \begin{pmatrix} \exp\left(\sqrt{2}\hat{\sigma}_a x_a + \frac{\hat{\sigma}_a^2}{\sigma_a^2}\mu_a + \left(1 - \frac{\hat{\sigma}_a^2}{\sigma_a^2}\right)s_a\right)K_1 \\ + \exp\left(\sqrt{2}\hat{\sigma}_y x_y + \frac{\hat{\sigma}_y^2}{\sigma_y^2}\mu_y + \left(1 - \frac{\hat{\sigma}_y^2}{\sigma_y^2}\right)s_y\right) \end{pmatrix}^{-\gamma} \exp\left(\sqrt{2}\hat{\sigma}_y x_y + \frac{\hat{\sigma}_y^2}{\sigma_y^2}\mu_y + \left(1 - \frac{\hat{\sigma}_y^2}{\sigma_y^2}\right)s_y\right).$$

Applying the Gaussian quadrature approach, we can approximate the RHS as follows:

$$\mathbb{E}[f_1(\epsilon_a, \epsilon_y)|S_a, S_y] = \int\int f_1(x_a, x_y)e^{-x_a^2}e^{-x_y^2}dx_a dx_y \cong \sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{\pi}\omega_{a,i}^{GH}\omega_{y,j}^{GH}f_1^*(\xi_{a,i}^{GH}, \xi_{y,i}^{GH}),$$
$$\mathbb{E}[f_2(\epsilon_a, \epsilon_y)|S_a, S_y] = \int\int f_2(x_a, x_y)e^{-x_a^2}e^{-x_y^2}dx_a dx_y \cong \sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{\pi}\omega_{a,i}^{GH}\omega_{y,j}^{GH}f_2^*(\xi_{a,i}^{GH}, \xi_{y,i}^{GH}),$$

where ξ_a and ξ_y are nodes and ω_a and ω_y are weights.

Next, we solve for the optimal attention allocation:

$$\max_{\kappa_a, \kappa_y} V = \mathbb{E}[U(S_a, S_y, \hat{\sigma}_a^2, \hat{\sigma}_y^2)],$$

subject to (32)-(34). We then use the Gaussian-quadrature approach to approximate the indirect utility:

$$\mathbb{E}[U(S_a, S_y, \hat{\sigma}_a^2, \hat{\sigma}_y^2)] = \int\int U(S_a, S_y, \hat{\sigma}_a^2, \hat{\sigma}_y^2) dt_a dt_y \cong \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{1}{\pi} \omega_{sa,i}^{GH} \omega_{sy,j}^{GH} U(\xi_{sa,i}^{GH}, \xi_{sy,j}^{GH}),$$

where ξ_{sa} and ξ_{sy} are nodes and ω_a and ω_y are weights.

In summary, we solve the model backwards. First, Solving $F(K_1) = -(Y_0 - K_1)^{-1/\psi} + \beta \left(\mathbb{E}[f_1(\epsilon_a, \epsilon_y)|S_a, S_y]\right)^{\frac{\gamma - 1/\psi}{1-\gamma}} \mathbb{E}[f_2(\epsilon_a, \epsilon_y)|S_a, S_y] = 0$ yields the optimal savings, K_1^*. Then plugging this result back into the utility function yields the indirect utility, $U(S_a, S_y, \kappa_a, \kappa_y)$. We can then compute the unconditional expected utility evaluated over signal observations and solve for the optimal attention allocation, κ_a^* and κ_y^*, by maximizing the unconditional expected utility. The following is the detailed procedure of solving the model:

1. Set $\kappa_a^{min} = 0.0001$ and $\kappa_a^{max} = \kappa - 0.0001$, such that $\kappa_y^{max} = \kappa - 0.0001$ and $\kappa_y^{min} = 0.0001$.
2. For κ_a^{min}, use the Legendre-Gauss approach compute 7 nodes for S_a and their corresponding weights. Similarly S_y for κ_y^{max}. For each combination (s_a, s_y), compute the optimal savings K_1^*, and then compute the value of $V(\kappa_a^{min})$.
3. For κ_a^{max}, use the Legendre-Gauss approach compute 7 nodes for S_a and their corresponding weights. Similarly S_y for κ_y^{min}. For each combination (s_a, s_y), compute the optimal savings K_1^*, and then compute the value of $V(\kappa_a^{max})$.
4. Compute the slope $\left(V(\kappa_a^{max}) - V(\kappa_a^{min})\right) / \left(\kappa_a^{max} - \kappa_a^{min}\right)$. If the slope is positive, set $\kappa_a^{min} = \left(\kappa_a^{min} + \kappa_a^{max}\right)/2$; if the slope is negative, set $\kappa_a^{max} = \left(\kappa_a^{min} + \kappa_a^{max}\right)/2$.
5. Iterate the steps above till the slope is close to zero, and we have $\kappa_a = \kappa_a^{max} = \kappa_a^{min}$.

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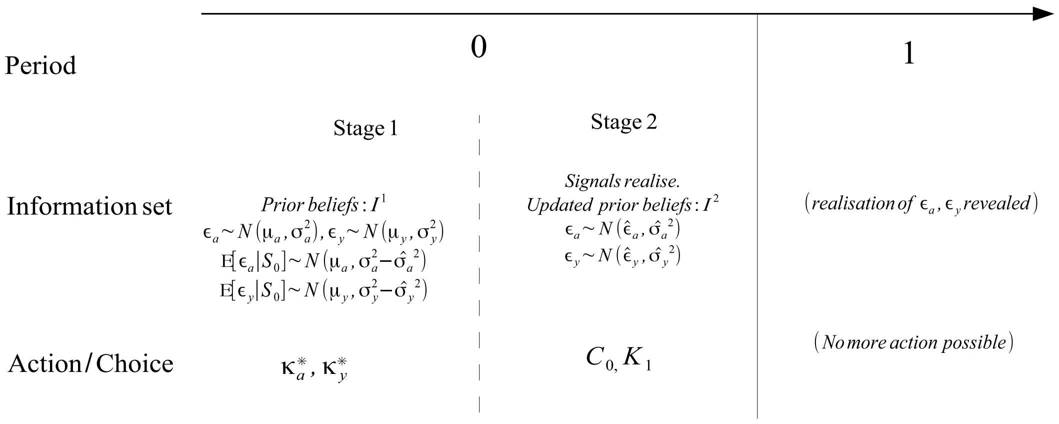
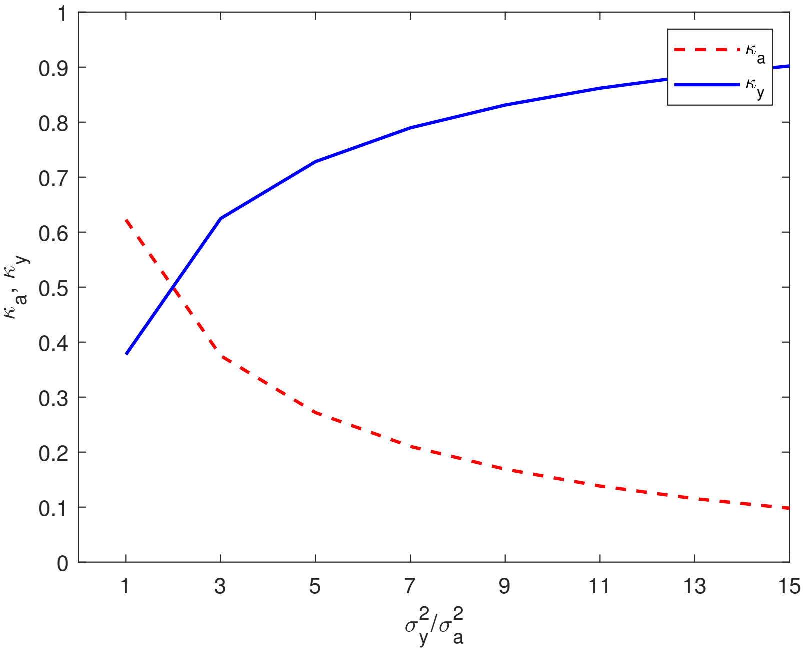
Figure 1: **Timeline**



Note: In this figure, we divide the optimization problem in period 0 into two stages to make the solution method clear; but in the model, the agent solves these two problems simultaneously.

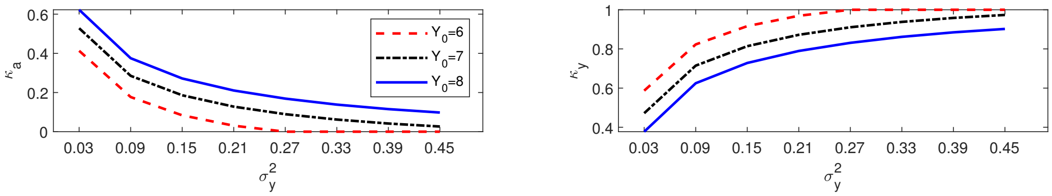
Figure 2: **Attention Allocation and Relative Prior Variance**



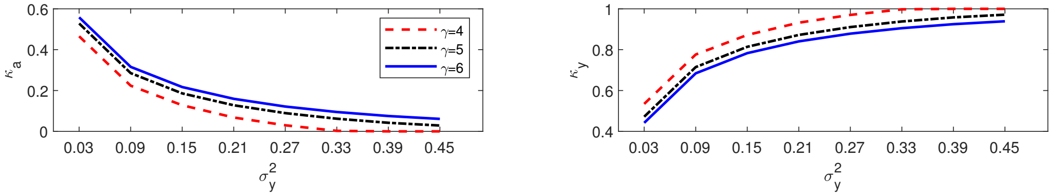
Note: This figure plots the optimal attention allocation to labor and capital income shocks (blue line and dashed red line respectively). The model is calibrated as follows: $\sigma_a^2 = 0.03$, $\kappa = 0.5$, $\beta = 0.97$, $\gamma = 5$, $\psi = 1/3$. Initial wealth Y_0 is set to 7, resulting in a wealth-to-income ratio 3.5, which is consistent with the empirical result in HRS data.

Figure 3: **Comparative Analysis: Attention Allocation**

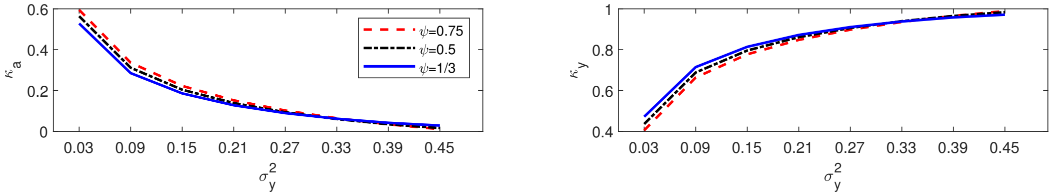
Panel A: Impact of Initial Wealth



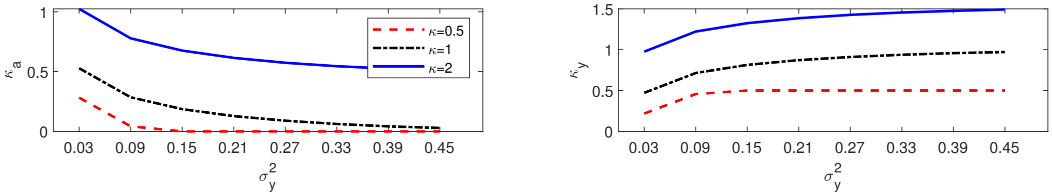
Panel B: Impact of CRRA



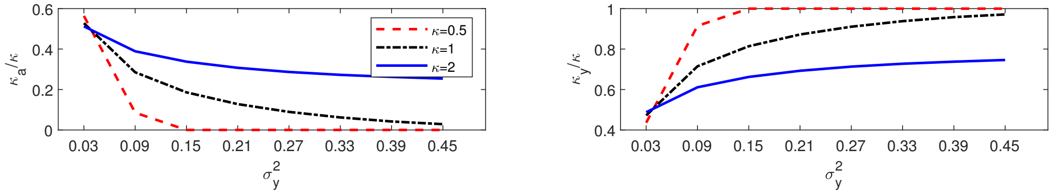
Panel C: Impact of EIS



Panel D: Impact of Attention Capacity

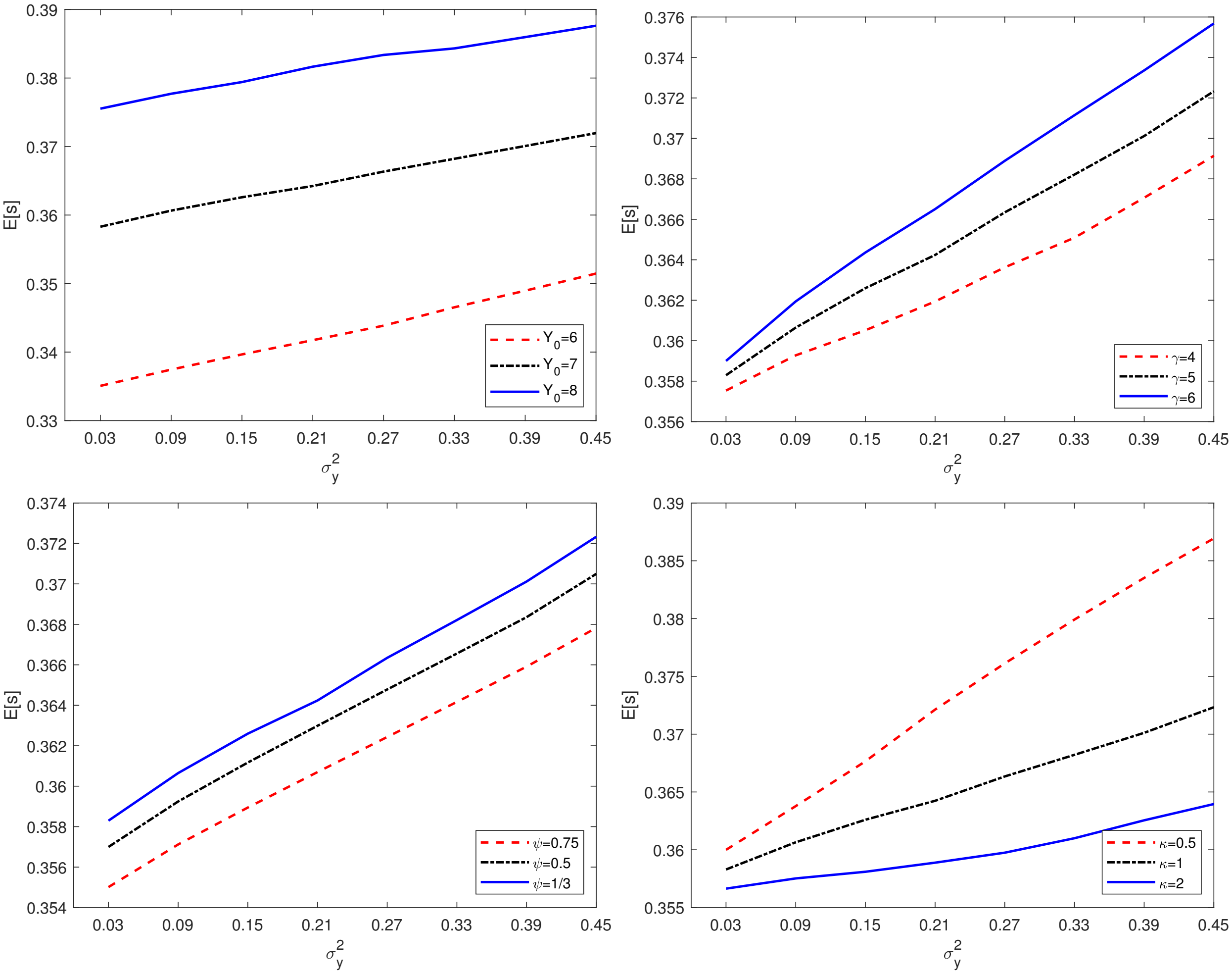


Panel E: Impact of Attention Capacity



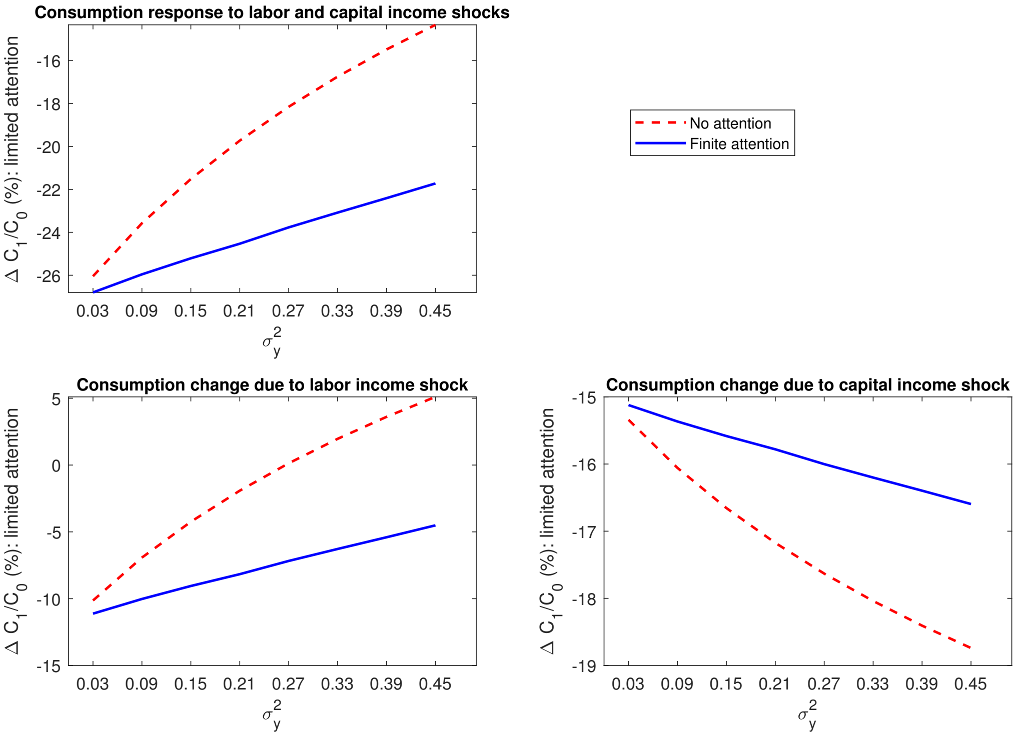
Note: This figure plots the optimal attention allocation to labor and capital income shocks (left panels and right panels respectively). The benchmark calibrations are as follows: $\sigma_a^2 = 0.03$, $\kappa = 0.5$, $\beta = 0.97$, $\gamma = 5$, $\psi = 1/3$. Initial wealth Y_0 is set to 7, resulting in a wealth-to-income ratio 3.5, which is consistent with the empirical result in HRS data.

Figure 4: **Comparative Analysis: Average Saving Rate**



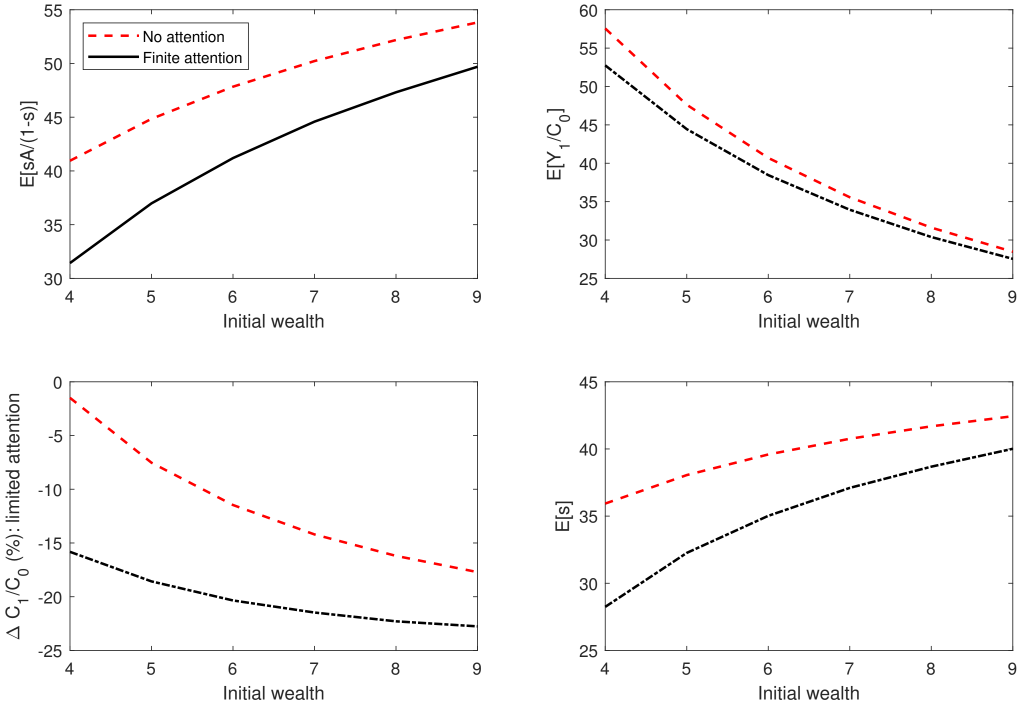
Note: This figure plots the relationship between expected saving rate and prior variance of the labor income risk under different calibrations of initial wealth, risk aversion coefficient, EIS and attention capacity. The benchmark calibrations are as follows: $\sigma_a^2 = 0.03$, $\kappa = 0.5$, $\beta = 0.97$, $\gamma = 5$, $\psi = 1/3$. Initial wealth Y_0 is set to 7, resulting in a wealth-to-income ratio 3.5, which is consistent with the empirical result in HRS data.

Figure 5: **Consumption Response to Labor and Capital Income Shocks**



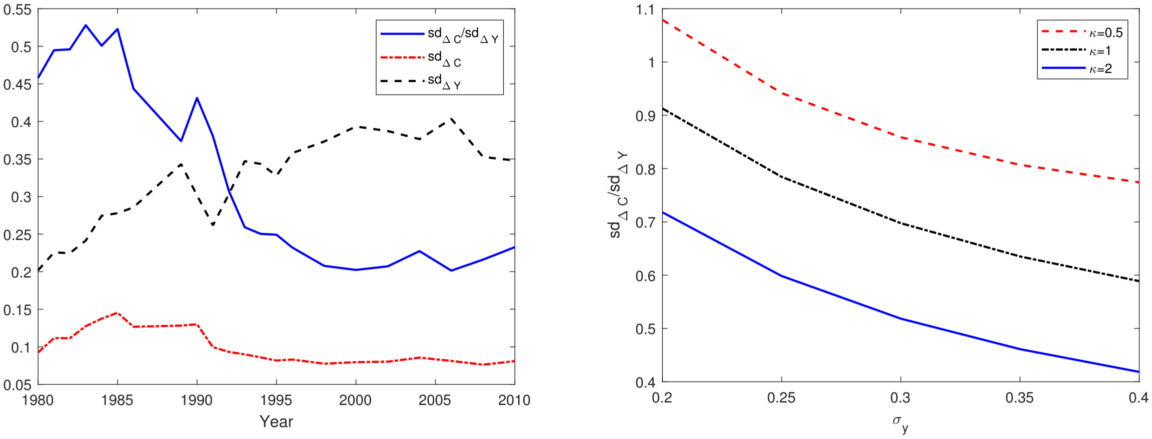
Note: The upper-left panel shows the response of consumption (in percentage change) to income shocks in the benchmark model; the lower-left panel shows the response of consumption (in percentage change) to labor income (unemployment) shock without financial loss; the lower-right figure shows the consumption response (in percentage change) to financial losses but without labor income (unemployment) shock. The benchmark calibrations are as follows: $\sigma_a^2 = 0.03$, $\kappa = 1$, $\beta = 0.97$, $\gamma = 5$, $\psi = 1/3$. Initial wealth Y_0 is set to 7, resulting in a wealth-to-income ratio 3.5, which is consistent with the empirical result in HRS data.

Figure 6: **Effects of Initial Wealth on Total Consumption Change and Its Components**



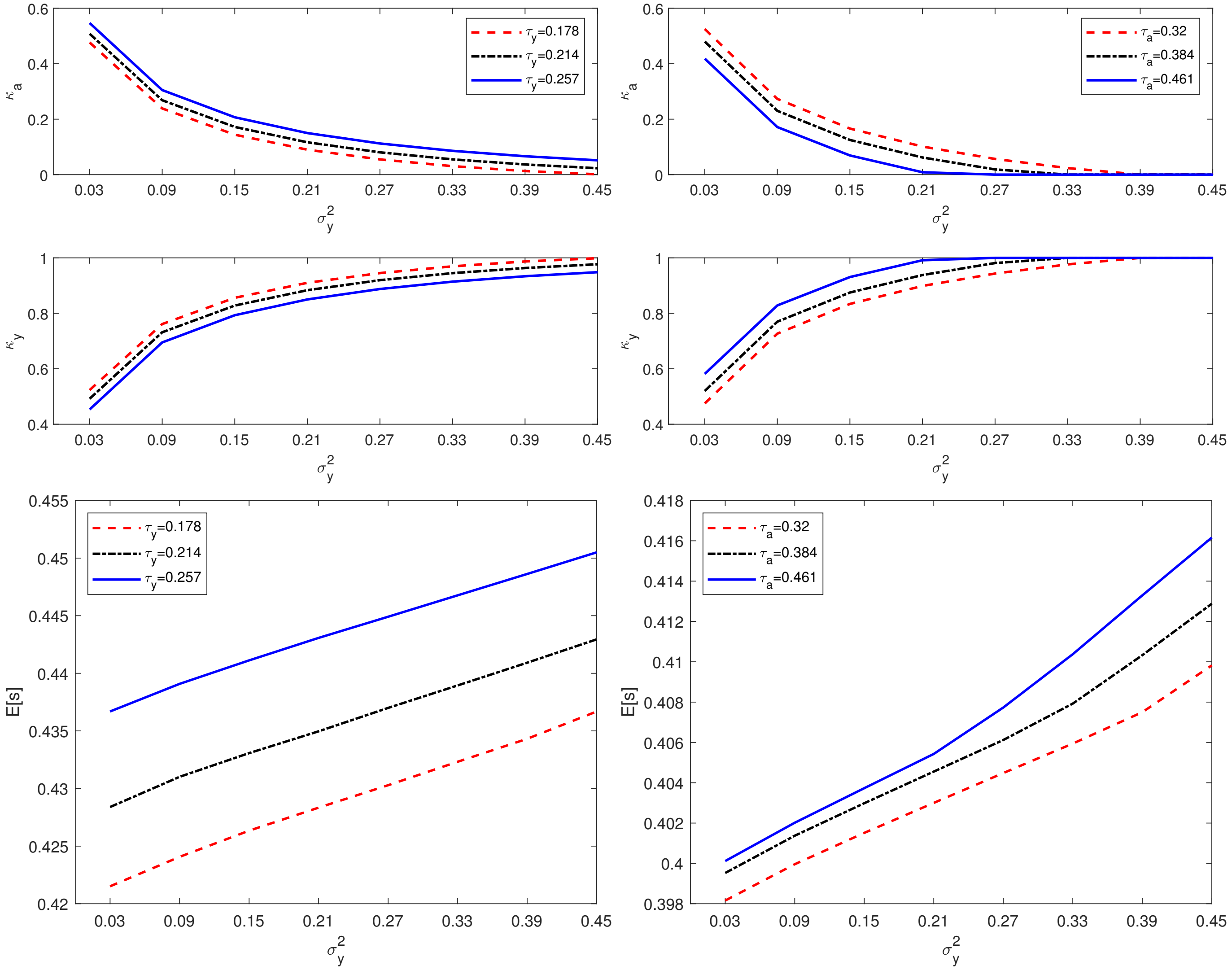
Note: This figure plots the effect of initial wealth (Y_0) on consumption change and its components under RI and zero capacity. The upper-left panel shows effects of initial wealth on consumption change via saving behavior; the upper-right panel shows effects via the relative importance of the unemployment benefit; the lower-left panel shows effects on total consumption change; the lower-right panel shows effects on saving rate (our first channel). The benchmark calibrations are as follows: $\sigma_a^2 = 0.03$, $\kappa = 0.5$, $\beta = 0.97$, $\gamma = 5$, $\psi = 1/3$. Initial wealth Y_0 is set to 7, resulting in a wealth-to-income ratio 3.5, which is consistent with the empirical result in HRS data.

Figure 7: **Relative Dispersion of Changes in Consumption and Income**



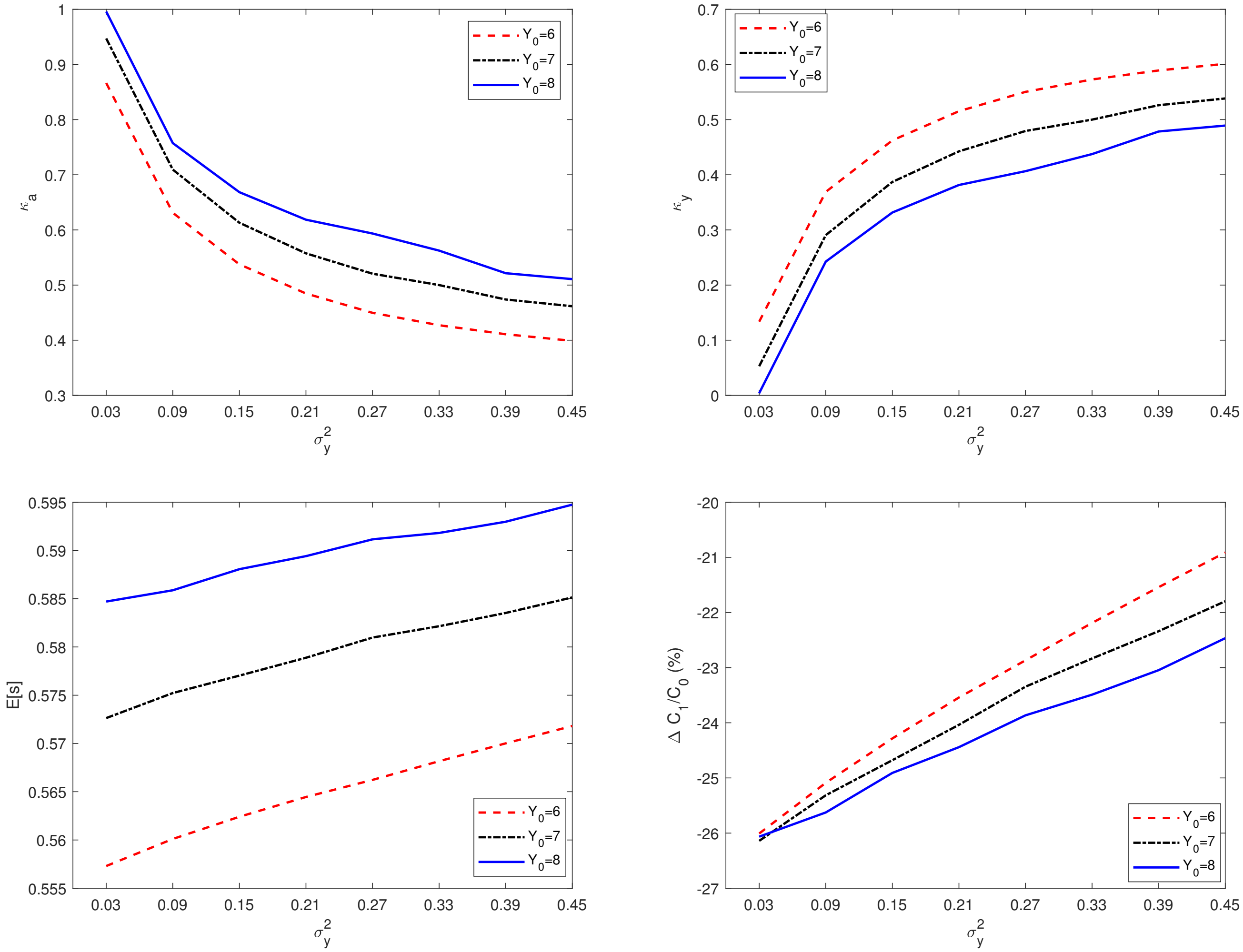
Note: The left panel plots the relative dispersion of changes in consumption and income using data from PSID. The right panel plots the relative dispersion in the model. The benchmark calibrations are as follows: $\sigma_a^2 = 0.03$, $\kappa = 1$, $\beta = 0.97$, $\gamma = 5$, $\psi = 1/3$. Initial wealth Y_0 is set to 7, resulting in a wealth-to-income ratio 3.5, which is consistent with the empirical result in HRS data.

Figure 8: **Attention Allocation, Saving Behavior with Different Taxes**



Note: This figure plots optimal attention allocation to income shocks and expected saving behavior with labor and capital income taxes. The benchmark calibrations are as follows: labor income tax $\tau_y = 0.214$, capital income tax $\tau_a = 0.384$ (according to), $\sigma_a^2 = 0.03$, $\kappa = 1$, $\beta = 0.97$, $\gamma = 5$, $\psi = 1/3$. Initial wealth Y_0 is set to 7, resulting in a wealth-to-income ratio 3.5, which is consistent with the empirical result in HRS data.

Figure 9: **Attention Allocation, Saving, and Consumption Change with Bequest Motivations**



Note: This figure plots optimal attention allocation to income shocks, expected saving behavior and consumption change with bequest motive. The benchmark calibrations are as follows: $\sigma_a^2 = 0.03$, $\kappa = 1$, $\beta = 0.97$, $\gamma = 5$, $\psi = 1/3$. Initial wealth Y_0 is set to 7, resulting in a wealth-to-income ratio 3.5, which is consistent with the empirical result in HRS data.

Table 1: Summary Table

	Mean
Consumption change (%)	-4.34
Financial wealth change (%)	-24.07
Prior variance	0.36
Unemployment shock	0.01
Age	61.74
Education	14.36
Sex	1.56
Married	0.81
Retirement	0.53
Chances that Dow Jones will increase next year	51.89
Net asset (thousand dollars)	160.75
Income (thousand dollars)	65.44
Household size	2.1
Number of observations	747

Table 2: Elasticities of Consumption w.r.t. Financial Losses and Unemployment Shocks

	(1)	(2)	(3)	(4)
Change in financial asset(%)	0.106**	0.107**	0.131***	0.133***
	(2.27)	(2.28)	(2.69)	(2.70)
Unemployment shock	-14.60***	-14.52***	-7.421**	-7.578**
	(-4.17)	(-4.16)	(-2.12)	(-2.17)
Prior variance	0.235	0.281	-0.329	-0.336
	(0.45)	(0.51)	(-0.96)	(-0.97)
Prior variance* change in financial asset(%)	0.0199*	0.0218*		
	(1.73)	(1.78)		
Prior variance*unemployment shock	11.68*	11.30*		
	(1.88)	(1.83)		
Financial asset (normalized by average income)	-0.00586	-0.00675	-0.0818	-0.0847
	(-0.23)	(-0.26)	(-1.09)	(-1.13)
Financial asset* change in financial asset(%)			-0.00355	-0.00363
			(-1.07)	(-1.09)
Financial asset*unemployment shock			-0.313***	-0.302***
			(-2.83)	(-2.71)
Observations	747	747	747	747

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Dependent variable is percentage change in consumption. Main explanatory variables are percentage change in financial assets, and whether respondent became unemployed in 2007 and 2008. Prior variance is the subjective belief about the volatility of income. Control variables include gender, age, marital status, retirement status, percentage change in value of the main residences, the prior mean of labor income, and individuals' expectation about the probability of an increase in Dow Jones Industrial Average. Columns 1 and 3 show the baseline estimation. Colunmns 2 and 4 include two extra control variables (household size and log of income). t-statistics are shown in parentheses.

Table 3: Wealth and attention to unemployment and capital return

	(1) Attention to unemployment	(2) Attention to financial market
log(Wealth)	-0.0312***	0.0451***
	(0.00192)	(0.00220)
Pseudo R^2	0.127	0.121
N	43881	43881

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Data are from Michigan Survey of Consumers. In column 1, the dependent variables is attention to unemployment, which is a dummy variable that equals 1 if respondents mentioned that they heard news about unemployment recently. In column 2, the dependent variables is attention to financial market, which is a dummy variable that equals 1 if respondents mentioned that they heard news about interest rate and stock return recently. Main explanatory variable is log of wealth that is the sum of real income and real stock holdings. t-statistics are shown in parentheses.

Table 4: Comparing data and model

	Quartile 1	Quartile 2	Quartile 3	Quartile 4
Prior variance	$\sigma_y^2 = 0.00001$	$\sigma_y^2 = 0.052$	$\sigma_y^2 = 0.42$	$\sigma_y^2 = 1.24$
Data: $\frac{\Delta C_{t+1}}{C_t}(\%)$	-26.25	-22.5	-21.67	-12.78
RI($\kappa = 0.5$): $\frac{\Delta C_{t+1}}{C_t}(\%)$	-27.46	-26.08	-20.47	-9.37
Zero Attention: $\frac{\Delta C_{t+1}}{C_t}(\%)$	-27.41	-25.21	-16.1	-4.21

We compare empirical results with theoretical counterparts. We divide the prior variance of labor income risk into 4 quartiles and compute the average consumption change in each quartile (third row) and the average prior variance (second row). We plug these average prior variances into our model to compute consumption change under RI (fourth row) and zero attention (fifth row). The benchmark calibrations are as follows: $\sigma_a^2 = 0.03$, $\kappa = 0.5$, $\beta = 0.97$, $\gamma = 5$, $\psi = 1/3$. Initial wealth Y_0 is set to 7, resulting in a wealth-to-income ratio 3.5, which is consistent with the empirical result in HRS data.

Table 5: Comparing data and model

	Quartile 1	Quartile 2	Quartile 3	Quartile 4
Prior variance	$\sigma_y^2 = 0.00001$	$\sigma_y^2 = 0.065$	$\sigma_y^2 = 0.53$	$\sigma_y^2 = 1.24$
Capital return	$A_1 = 0.675$	$A_1 = 0.89$	$A_1 = 0.74$	$A_1 = 0.67$
Data: $\frac{\Delta C_{t+1}}{C_t}(\%)$	-26.25	-22.5	-21.67	-12.78
RI ($\kappa = 0.5$)**:** $\frac{\Delta C_{t+1}}{C_t}(\%)$	-29.96	-16.44	-18.44	-14.05
Zero Attention: $\frac{\Delta C_{t+1}}{C_t}(\%)$	-29.9	-15.37	-13.73	-9.12

We compare empirical results with theoretical counterparts. We divide the prior variance of labor income risk into 4 quartiles and compute the average consumption change in each quartile (third row), the average prior variance (second row) and the average financial wealth loss (numbers in fourth row minus one). We plug these average prior variances into our model to compute consumption change under RI (fifth row) and zero attention (sixth row). The benchmark calibrations are as follows: $\sigma_a^2 = 0.03$, $\kappa = 0.5$, $\beta = 0.97$, $\gamma = 5$, $\psi = 1/3$. Initial wealth Y_0 is set to 7, resulting in a wealth-to-income ratio 3.5, which is consistent with the empirical result in HRS data.

Table 6: Welfare Analysis

Panel A	$Y_0 = 6$	$Y_0 = 7$	$Y_0 = 8$
κ from 1 to 2	2.0013%	1.9146%	1.8284%
κ from 2 to 4	1.0201%	0.9943%	0.9561%
Panel B	$\beta = 0.7$	$\beta = 0.8$	$\beta = 0.97$
κ from 1 to 2	1.8875%	1.9003 %	1.9146%
κ from 2 to 4	0.9699 %	0.9809%	0.9943%
Panel C	$\gamma = 4$	$\gamma = 5$	$\gamma = 6$
κ from 1 to 2	1.2537%	1.9146 %	2.5568%
κ from 2 to 4	0.6451%	0.9943%	1.3366%
Panel D	$\psi = 1/3$	$\psi = 0.5$	$\psi = 0.75$
κ from 1 to 2	1.9146%	1.1922%	0.4619%
κ from 2 to 4	0.9943%	0.6201%	0.2411%

Note: This table reports the welfare change (in percent) when increasing attention capacity by 100% under different parameter values. The benchmark parameter values are set as follows $Y_0 = 7$, $\sigma_y^2 = 0.03$, $\sigma_y^2 = 0.42$, $\beta = 0.97$, $\gamma = 5$, $\psi = 1/3$.

Table 7: Welfare Analysis (flexible attention allocation vs fixed attention allocation)

$Y_0 = 6$	fixed attention allocation $\kappa_a = 0.0001\bar{\kappa}$, $\kappa_y = 0.9999\bar{\kappa}$	flexible attention allocation
κ from 1 to 2	1.5325%	2.0013%
κ from 1 to 3	1.7534%	2.7320%
κ from 1 to ∞	1.8143%	3.1577%

$Y_0 = 7$	fixed attention allocation $\kappa_a = 0.0338\bar{\kappa}$, $\kappa_y = 0.9662\bar{\kappa}$	flexible attention allocation
κ from 1 to 2	1.4363%	1.9146%
κ from 1 to 3	1.7301%	2.6272%
κ from 1 to ∞	3.0185%	3.0427 %

$Y_0 = 8$	fixed attention allocation $\kappa_a = 0.1072\bar{\kappa}$, $\kappa_y = 0.8928\bar{\kappa}$	flexible attention allocation
κ from 1 to 2	1.5324%	1.8284%
κ from 1 to 3	1.9704%	2.5134%
κ from 1 to ∞	2.9043%	2.9149%

Note: This table reports the comparison of welfare change (in percent) when increasing attention capacity between optimal attention allocation and a fixed attention allocation strategy for people with different wealth. Other parameters are set at their benchmark values.