Robustness, Information-Processing Constraints, and the Current Account in Small Open Economies*

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Abstract

In this paper we examine the effects of two types of "*induced uncertainty*", model uncertainty due to robustness (RB) and state uncertainty due to finite information-processing capacity (called rational inattention or RI), on consumption and the current account. We show that the combination of RB and RI improves the model's predictions for (i) the contemporaneous correlation between the current account and income and (ii) the volatility and persistence of the current account in small open emerging and developed economies. In addition, we show that the two informational frictions improve the model's ability to match the impulse response of consumption to income and the relative volatility of consumption to income growth.

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1 Introduction

Current account models following the intertemporal approach feature a prominent role for the behavior of aggregate consumption. For given total income, consumption is the main determinant of national saving, and the balance of national saving in excess of investment is the major component of the current account. This important role for consumption has naturally led researchers to study current account dynamics using consumption models.[1] For example, the standard intertemporal current account (ICA) model is based on the standard linear-quadratic permanent income hypothesis (LQ-PIH) model proposed by Hall (1978) under the assumption of rational expectations (RE). Within the PIH framework, agents can borrow in the international capital market and optimal consumption is determined by permanent income rather than current income; consequently, permanent income also matters for the current account. For example, consumption only partly adjusts to temporary adverse income shocks, which makes the current account tend to be in deficit. In contrast, consumption fully adjusts to permanent income shocks, with little impact on the current account.

However, many empirical studies show that the standard RE-ICA models are often rejected in the post-war data.[2] In addition, the standard models also cannot explain the different behavior of the current account and consumption in emerging and developed countries.[3] It is not surprising that the standard RE-ICA models are rejected because the underlying standard permanent income models have encountered their own well-known empirical difficulties, particularly the well-known 'excess sensitivity' and 'excess smoothness' puzzles. Specifically, the main problems with the standard RE-ICA models are as follows. First, the models cannot generate low contemporaneous correlations between the current account and net income (net income is defined as output minus investment and government spending).[4] If net income is a persistent trend-stationary AR(1) process,[5] The model predicts that the current account is a linear function of net income and they

[1] See Obstfeld and Rogoff (1995) for a survey.

[2] See Sheffrin and Woo (1990), Ghosh (1995), Glick and Rogoff (1995), Obstfeld and Rogoff (1996), Ghosh and Ostry (1998), Bergin and Sheffrin (2000), Nason and Rogers (2003), and Gruber (2004).

[3] For example, see Neumeyer and Perri (2005), Aguiar and Gopinath (2007), Uribe (2009), among others.

[4] Note that here we follow Aguiar and Gopinath (2007) and Uribe (Chapter 1, 2009) and use the detrended data to compute the reported empirical second moments. Following Obstfeld and Rogoff (1995), Ghosh and Ostry (1998), Gruber (2004), Engel and Rogers (2006), among others, in this paper we net out investment and government spending because our model also suggests that consumption spending depends on income that is disposable for household consumption.

[5] It is well known that given the length and structure of the data on real GDP, it is difficult to distinguish persistent trend-stationary AR(1), unit root, and difference-stationary (DS) processes for real GDP. (See Chapter 4 of Deaton 1992 for a detailed discussion on this issue.) We focus on the AR(1) case in this paper; the results for the DS case are available from the authors upon request. In Section 3.2, we discuss the unit root case, in which the empirical second moments of the current account and net income are not finite. The RE model predicts that when net output follows a unit root process, the current account becomes constant.

are thus perfectly correlated, whereas in the data they are only weakly correlated.[6] Note that in the data the current account is countercyclical with real GDP and more countercyclical in the emerging economy. (For example, see Neumeyer and Perri 2005, Aguiar and Gopinath 2007, Uribe 2009). Second, they cannot generate low persistence of the current account.[7] The standard RE models predict that the current account and net income have the same degree of persistence, whereas in the data the persistence of the current account is much lower than that of net income in emerging countries and insignificantly lower than that of net income in developed countries (See Table 1).[8] Third, the models cannot generate observed volatility of the current account (Bergin and Sheffrin 2000; Gruber 2004). Fourth, they cannot generate volatile consumption growth and the observed hump-shaped impulse responses of consumption to income (Aguiar and Gopinath 2007). Finally, the assumption of certainty equivalence in these models ignore some important channels through which income shocks affect the current account. As shown in Ghosh and Ostry (1997) in post-war quarterly data for the US, Japan, and the UK, the current account is positively correlated with the amount of precautionary savings generated by uncertainty about future net income. Fogli and Perri (2008) also show that in OECD economies changes in country-specific macroeconomic volatility are strongly correlated with changes in net external asset position.

It is, therefore, natural to turn to new alternatives to the standard RE-ICA model and ask what implications they have for the joint dynamics of consumption, the current account, and income. In this paper, we show that two types of informational frictions, robustness (RB) and information-processing constraints (rational inattention or RI), can significantly improve the model's ability to fit the data discussed above. Specifically, these two types of information imperfections interact with the fundamental shock (the income shock in our model) and give rise to closely related "*induced uncertainty*": (i) model uncertainty and (ii) state uncertainty. These two types of induced uncertainty can affect the model's dynamics even within the linear-quadratic (LQ) framework.[9] We adopt Hall's LQ-PIH setting in this paper because the main purpose of this paper is to inspect the mechanisms through which the induced uncertainty affects the joint dynamics of consumption, the current account, and income, and it is much more difficult to study these informational

[6] See Tables 2-3 and Tables 4-5 for the statistics for emerging and developed countries, respectively. Here we follow Aguiar and Gopinath (2007) by dividing the small economies into emerging and developed economies and use annual data from World Development Indicators.

[7] Boz, Durdu, and Li (2010) also report the empirical autocorrelation of the current account and the correlation between the current account and real GDP in emerging countries, and examine how labor market frictions can improve the model's predictions on these dimensions.

[8] In this paper, we assume that there is only one shock to net income. If there are multiple structural shocks, the persistence of the detrended current account and that of detrended net income might be generated by the responses to the different shocks. See Kano (2008) for a detailed discussion.

[9] Note that in the traditional linear-quadratic, linearized, or log-linearized models, uncertainty measured by the variance of the fundamental shock does not affect the model dynamics.

frictions in non-LQ frameworks.[10] After solving the models explicitly, we then examine how the induced uncertainty due to RB and RI can improve the model's predictions on these important dimensions of the joint dynamics of the current account, consumption, and net income in emerging and developed countries we discussed above. In particular, we are interested in two key features of emerging market: consumption volatility exceeds income volatility and less procyclical current accounts with net income found in the data. Note that in the literature, it is widely documented that in emerging market consumption volatility exceeds income volatility and current accounts are strongly countercyclical.[11]

Hansen and Sargent (1995, 2007a) first introduced robustness (a concern for model misspecification) into economic models. In robust control problems, agents are concerned about the possibility that their model is misspecified in a manner that is difficult to detect statistically; consequently, they choose their decisions as if the subjective distribution over shocks was chosen by a malevolent nature in order to minimize their expected utility (that is, the solution to a robust decision-maker's problem is the equilibrium of a max-min game between the decision-maker and nature). Robustness models produce precautionary savings but remain within the class of LQ-Gaussian models, which leads to analytical simplicity.[12] A second class of models that produces precautionary savings but remains within the class of LQ-Gaussian models is the risk-sensitive model of Hansen, Sargent, and Tallarini (1999).[13] We show that even if the parameter value of robustness is the same for all small open countries, the RB model has the potential to lead to the observed different joint behavior of consumption and current accounts across the developed and emerging economies. The reason is that the amount of model uncertainty that affects the model's dynamics is determined by the interaction of the preference for robustness and income uncertainty; consequently, the model with the same parameter value of robustness can still lead to different

[10] See Hansen and Sargent (2007a) and Sims (2003, 2006) for detailed discussions on the difficulties in solving the non-LQ models with information imperfections. The primary alternative model is based on Mendoza (1991), a small open economy version of a real business cycle model. That model would be significantly less tractable than the one we use, because it involves multiple state variables.

[11] See Neumeyer and Perri (2005), Aguiar and Gopinath (2007), Boz, Durdu, and Li (2010) among others.

[12] It is worth noting that although both robustness (RB) and CARA preference (i.e., Caballero 1990 and Wang 2003) increase the precautionary savings premium via the intercept terms in the consumption functions, they have distinct implications for the marginal propensity to consume (MPC). Specifically, CARA has no impact on the MPC, whereas RB increases the MPC. That is, under RB, in response to a negative wealth shock, the consumer would choose to reduce consumption more than that predicted in the standard LQ or CARA model (i.e., save more to protect themselves against the negative shock). We think that it is a way to distinguish CARA preference and RB.

[13] See Hansen and Sargent (2007a) and Luo and Young (2010) for detailed comparisons of the two models. In our ICA model, it seems more plausible to have different degrees of robustness (ϑ) across countries than to assume different degrees of risk sensitivity (i.e., enhanced risk aversion) across countries to explain the observed different joint behavior of consumption and current accounts in emerging and developed economies. Backus, Routledge, and Zin (2004) also discuss this issue.

behavior of consumption and the current account because income uncertainty is different across countries.[14] Furthermore, we find that incorporating robustness can improve the model by along the following three dimensions in all small open countries: generating lower contemporaneous correlation between the current account and net income, lower persistence of the current account, and higher relative volatility of consumption growth to income growth. In addition, after calibrating the RB parameter using the detection error probability, we find that RB can help generate the different stochastic properties of the emerging and developed economies. Specifically, the current account in the emerging economy is (1) less correlated with net income, (2) less persistent, and (3) less volatile than that in the developed economy. However, quantitatively, we find that RB by itself cannot fully explain the joint behavior of consumption and the current account in the two small-open economies.

We therefore consider the model with imperfect state observation (*state uncertainty*) due to finite information-processing capacity (rational inattention or RI). Sims (2003) first introduced RI into economics and argued that it is a plausible method for introducing sluggishness, randomness, and delay into economic models. In his formulation agents have finite Shannon channel capacity, limiting their ability to process signals about the true state of the world. As a result, an impulse to the economy induces only gradual responses by individuals, as their limited capacity requires many periods to discover just how much the state has moved; one key change relative to the RE case is that consumption has a hump-shaped impulse response to changes in income.[15] Using the results in Luo (2008), it is straightforward to show that RI by itself still leads to counterfactual strongly-procyclical current accounts and cannot generate precautionary savings in the LQG setting.[16] However, the combination of RB and RI produces a model that captures many of the facts that are seen as anomalous through the lens of an RE model, while producing consumption dynamics that are consistent with the data. The intuition is that RI introduces (i) slow adjustment to the income shock and (ii) an endogenous noise into the model, which amplifies the importance of model uncertainty in determining the model's dynamics and further improves the model's predictions on the joint behavior of consumption and the current account.

We briefly list the results of the RB-RI model. First, we can produce a low correlation between the current account and net income, and in fact can even produce negative correlations for some parameter settings; the key requirement to get low correlations is that the agent have a strong fear of model misspecification. Second, we can produce low persistence in the current account, a consequence of the slow movements in consumption that RI produces. Third, if information-

[14] As is well known in the literature, income uncertainty is much larger in emerging countries than that in developed countries.

[15] See Sims (2003) and Luo (2008).

[16] Habit formation also worsens the model's predictions on the current account dynamics; consumption adjusts slowly with respect to income shocks under habit formation, as shown in Gruber (2004), generating procyclical current accounts. Luo (2008) compares the consumption predictions of habit formation and RI.

processing is sufficiently restricted, current account volatility can match that observed in the data for emerging markets, although not for developed economies. Fourth, the model produces a hump-shaped consumption response to income, a consequence of RI, and can produce highly volatile consumption growth in emerging economies. Fifth, the precautionary savings effect generated by RB is consistent with the positive correlation between income volatility and average current accounts. We detail in the main body of the paper the intuition for all of these results.

The remainder of the paper is organized as follows. Section 2 presents key facts of small open economy business cycles. Section 3 reviews the standard RE-ICA model and discuss the puzzling implications of the model. Section 4 presents the RB ICA model and discusses some results regarding the joint dynamics of consumption, the current account, and income. Section 5 solves the RB-RI ICA model and presents the implications for the same variables. Section 6 concludes.

2 Facts

In this section we document key aspects of small open economy business cycles. We follow Aguiar and Gopinath (2007) by dividing these small economies into two groups, labeled emerging economies and developed economies.[17] We use annual data from World Development Indicators. Net income (y) is constructed as real GDP$-i-g$, where i is Gross Fixed Capital Formation and g is General Government Final Consumption Expenditure. Consumption (c) in defined as Household Final Consumption Expenditure, ca refers to the Current Account, and holdings of bonds (b) corresponds to Net Foreign Assets.

Tables 2 and 3 report key statistics of emerging economies, and Tables 4 and 5 report the same statistics for developed countries. To provide a comparison for the reader, we report the average values of these moments of both emerging countries and developed countries in Table 1; we report both the results using a linear filter and the Hodrick-Prescott (HP) filter (with a smoothing parameter of 100) in the same table. For the variable growth (with a symbol Δ) the unfiltered series are used. The numbers in the parentheses are the GMM-corrected standard errors of the statistics across countries.[18] Since our permanent income model is stationary, we need to remove the low frequency component from the data. Thus in this paper we focus primarily on the linear filter when we calibrate the parameters and compare models with data.

`[Insert Tables 1-5 Here]`

[17] Israel and the Slovak Republic are not in our list because some variables from these two countries are missing from our data set.

[18] The standard errors are computed under the assumption of independence across the countries. The standard error of $\sigma(y)/\mu(y)$ in the tables refers to the standard error of $\sigma(y)$ as the ratio of $\mu(y)$. $\mu(y)$ is the average level of net income.

We briefly list the facts we focus on. First, the correlation between the current account and net income is positive but small (and insignificant when detrended with the HP filter). Second, the relative volatility of the current account to net income is smaller in emerging countries than in developed economies, although the difference is not statistically significant when the series are detrended with the HP filter. Third, the persistence of the current account is smaller than that of net income, and less persistent in emerging economies. And fourth, the volatility of consumption growth relative to income growth is larger in emerging economies than in developed economies.

3 A Stylized Intertemporal Model of the Current Account

In this section we present a standard RE version of the ICA model and will discuss how to incorporate RB and RI into this stylized model in the next sections. Following common practice in the literature, we assume that the model economy is populated by a continuum of identical infinitely-lived consumers, and the only asset that is traded internationally is a risk-free bond.

3.1 Model Setup

The RE ICA model, the small-open economy version of Hall's permanent income model, can be formulated as

$$\max_{\{c_t\}} E_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right] \tag{1}$$

subject to the flow budget constraint

$$b_{t+1} = Rb_t + y_t - c_t, \tag{2}$$

where $u(c_t) = -\frac{1}{2}(\overline{c} - c_t)^2$ is the utility function, $\overline{c}$ is the bliss point, c_t is consumption, R is the exogenous and constant gross world interest rate, b_t is the amount of the risk-free foreign bond held at the beginning of period t, and y_t is net income in period t and is defined as output less than investment and government spending. Let $\beta R = 1$; then this specification implies that optimal consumption is determined by permanent income:

$$c_t = (R-1)\, s_t \tag{3}$$

where

$$s_t = b_t + \frac{1}{R} \sum_{j=0}^{\infty} R^{-j} E_t \left[y_{t+j} \right] \tag{4}$$

is the expected present value of lifetime resources, consisting of financial wealth (the risk-free foreign bond) plus human wealth. As shown in Luo (2008) and Luo and Young (2010), in order to facilitate the introduction of robustness and rational inattention we reduce the above multivariate model with a general income process to a univariate model with iid innovations to permanent income s_t that can be solved in closed-form. Specifically, if s_t is defined as a new state variable, we can reformulate the above PIH model as

$$v(s_0) = \max_{\{c_t, s_{t+1}\}_{t=0}^{\infty}} \left\{ E_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right] \right\} \tag{5}$$

subject to

$$s_{t+1} = Rs_t - c_t + \zeta_{t+1}, \tag{6}$$

where the time $(t+1)$ innovation to permanent income can be written as

$$\zeta_{t+1} = \frac{1}{R} \sum_{j=t+1}^{\infty} \left(\frac{1}{R} \right)^{j-(t+1)} (E_{t+1} - E_t) [y_j] ; \tag{7}$$

$v(s_0)$ is the consumer's value function under RE.[19] Under the RE hypothesis, this model with quadratic utility leads to the well-known random walk result of Hall (1978),

$$\begin{aligned} \Delta c_t &= \frac{R-1}{R} (E_t - E_{t-1}) \left[\sum_{j=0}^{\infty} \left(\frac{1}{R} \right)^j y_{t+j} \right] \\ &= (R-1) \zeta_t, \end{aligned} \tag{8}$$

which relates the innovations in consumption to income shocks.[20] In this case, the change in consumption depends neither on the past history of labor income nor on anticipated changes in labor income. In addition, the model specification also implies the certainty equivalence property holds, and thus uncertainty has no impact on optimal consumption.

Substituting (2) and (3) into the current account identity,

$$ca_t = b_{t+1} - b_t = (R-1) b_t + y_t - c_t, \tag{9}$$

gives

$$ca_t = - \sum_{j=t+1}^{\infty} \left(\frac{1}{R} \right)^{j-t} E_t [\Delta y_j] , \tag{10}$$

which means that the current account equals minus the present discounted value of future expected net income changes. This expression also reflects the fact that consumers smooth income shocks by borrowing or lending in international financial markets. If income is expected to decline in the future, then the current account rises immediately as current consumption determined by permanent income is less than current income; the opposite occurs if income is expected to rise in the future.

[19] In the next section, we will introduce robustness directly into this 'reduced' permanent income model, and show that this univariate RB model and the corresponding multivariate RB model lead to the same consumption function. We may also imagine that consumers form the reduced model after many years' experience.

[20] Note that under RE the expression of the change in individual consumption is the same as that of the change in aggregate consumption.

3.2 Model Predictions for Consumption and the Current Account

We close the model by specifying the stochastic process for net output. Specifically, we assume that the deviation of net output from its mean follows an AR(1) process

$$y_{t+1} - \overline{y} = \rho(y_t - \overline{y}) + \varepsilon_{t+1}, \tag{11}$$

where $\rho \in (0, 1]$ is the persistence coefficient of output and ε_{t+1} is an iid normal shock with mean 0 and variance ω^2.[21] In this case, (7) implies that $\zeta_{t+1} = \frac{1}{R-\rho}\varepsilon_{t+1}$ and $s_t = b_t + \frac{1}{R-\rho}y_t$. In the RE version of the ICA model, substituting (3) into the current account identity, (9), gives

$$ca_t = \frac{1-\rho}{R-\rho}y_t, \tag{12}$$

which means that given ρ and R, the current account inherits the properties of the stochastic process for net output (in particular, the persistence of net output). (12) also clearly shows that the value of ρ affects how output determines the behavior of the current account. Here we discuss two possibilities for the exogenous process of net output.

Case 1 ($0 < \rho < 1$).

When $\rho < 1$, the shock is temporary and consumers adjust their optimal plans by only consuming the annuity value of the increase in total income. In this case, the current account works as a shock absorber, and consumers borrow to finance negative income shocks and save in response to positive shocks. In other words, the current account in this case is *procyclical*: $\frac{\partial ca_t}{\partial \varepsilon_t} > 0$, which means that the current account improves during expansions and deteriorates during recessions. The solid line in Figure 1 illustrates the impulse response of the current account to the income shock when $R = 1.04$ and $\rho = 0.7$. (We set R to be 1.04 throughout the paper; we treat it as a compromise of different asset returns in the economy.) Equation (12) also means that the contemporaneous correlation between the current account and income, $\mathrm{corr}\,(ca_t, y_t)$, is 1. This model prediction contradicts the empirical evidence: in small open economies the correlation between the current account and net output is positive but close to 0. As reported in Panel A (HP filter) of Table 1, $\mathrm{corr}\,(ca_t, y_t) = 0.04$ (*s.e.* 0.04) in emerging countries and 0.06 (*s.e.* 0.05) in developed economies. Similarly, in Panel B (linear filter) of Table 1, $\mathrm{corr}\,(ca_t, y_t) = 0.13$ (*s.e.* 0.05) in emerging countries and 0.17 (*s.e.* 0.05) in developed economies. In other words, the model predicts too high a correlation between the current account and net output.

Equation (12) clearly shows that the volatility of the current account is less than that of income:

$$\mu = \frac{\mathrm{sd}\,(ca_t)}{\mathrm{sd}\,(y_t)} = \frac{1-\rho}{R-\rho} < 1,$$

where sd denotes standard deviation. Note that $\frac{\partial \mu}{\partial \rho} < 0$. Using the estimated ρ reported in Panel A (HP filter) of Table 1 and assume that $R = 1.04$, the RE model predicts that $\mu = 0.926$

[21] See Table 1 for our estimates of the output process.

in emerging countries and $\mu = 0.933$ in developed countries. However, in the data (using HP filter) reported in Table 1, $\mu = 1.53$ (*s.e.* 0.09) in emerging countries and $\mu = 1.60$ (*s.e.* 0.08) in developed countries.[22] In other words, given the estimated income processes, the model cannot correctly predict the relative volatility of the current account to net output in emerging and developed economies.[23]

Equation (12) also implies that the persistence of the current account is the same as that of net output. However, in the data the current account is significantly less persistent than net output, and is less persistent in emerging economies than in developed economies. As shown in Panel B (linear filter) of Table 1, $\rho(y_t, y_{t-1}) = 0.8$ (*s.e.* 0.02) and 0.79 (*s.e.* 0.02) in emerging and developed countries, respectively, while the corresponding $\rho(ca_t, ca_{t-1}) = 0.53$ (*s.e.* 0.04) and 0.71 (*s.e.* 0.02).[24]

Furthermore, given the AR(1) income specification, the change in aggregate consumption is

$$\Delta c_t = \frac{R-1}{R-\rho}\varepsilon_t,$$

which means that consumption growth is white noise and the impulse response of consumption to the income shock is *flat* with an immediate upward jump in the initial period that persists indefinitely (see the solid line in Figure 2). However, as well documented in the consumption literature (such as Reis 2006), the impulse response of aggregate consumption to aggregate income takes a *hump-shaped* form, which means that aggregate consumption growth reacts to income shocks gradually.

The relative volatility of consumption growth and income growth can be written as

$$\mu_c = \frac{\mathrm{sd}[\Delta c_t]}{\mathrm{sd}[\Delta y_t]} = \frac{R-1}{R-\rho}\sqrt{\frac{1+\rho}{2}},$$

where we use the facts that $\zeta_t = \frac{\varepsilon_t}{R-\rho}$, $\Delta c_t = (R-1)\zeta_t$, and $\Delta y_t = \varepsilon_t + (\rho-1)\frac{\varepsilon_{t-1}}{1-\rho\cdot L}$, where L is the lag operator. This expression is strictly increasing in ρ, implying that consumption growth should be relatively more volatile in emerging economies (which is consistent with the data). However, given the values of ρ from Table 1, the volatility of consumption growth is much too low relative to net output. For example, if $R = 1.04$, the RE model predicts that the relative volatility of consumption growth to income growth in emerging and developed economies would be 0.28 and 0.24, respectively. In contrast, in the data, the corresponding μ values are 1.35 and 0.98, respectively.[25]

[22] Given the estimated ρ using the linear filter reported in Panel B of Table 1, the RE model predicts that $\Lambda = 0.83$ in emerging countries and $\Lambda = 0.84$ in developed countries. However, in the data reported in Table 1, $\Lambda = 0.8$ (*s.e.* 0.06) in emerging countries and $\Lambda = 1.35$ (*s.e.* 0.06) in developed countries.

[23] Given the standard errors reported in parentheses in Panel B in Table 1, the result is significant.

[24] As shown in Panel A of Table 1, using HP filter shows the same pattern.

[25] Here we use the linear filter to obtain these results; using the HP filter leads to similar results.

Case 2 ($\rho = 1$).

When $\rho = 1$, net output follows a unit root process and the current account becomes constant because consumers allocate all of the increase in net income to current consumption. Intuitively, when the income shocks are permanent, the best response is to adjust consumption plan permanently. (Note that when $\rho = 1$ the empirical second moments of the current account and net income are not finite.) This principle is called "finance temporary shocks, adjust to permanent shocks" in the literature. As a result, $\text{var}\left[ca_t\right] = 0$, which strongly contradicts the evidence that the current account is highly volatile in all small open economies.

In sum, comparing with the stylized facts reported in Table 1, it is clear that the stylized RE-ICA model with AR(1) income processes cannot account for the following key business cycle features in small open countries:

1. The contemporaneous correlation between the current account and net output is close to 0 in small open economies, and is slightly smaller in emerging markets.

2. The excess relative volatility of the current account to net output in emerging and developed economies.

3. The persistence of the current account is smaller than that of net output, and it is smaller in emerging economies than in developed economies.

4. The hump-shaped impulse responses of consumption to income shocks.

5. The relative volatility of consumption growth to income growth is larger in emerging economies than in developed economies.

Finally, in the standard ICA model the current account is independent of the uncertainty in output ω^2; that is, the amount of precautionary savings does not affect the current account surplus. The reason is that the LQ setup satisfies the certainty equivalence property, ruling out any response of saving to uncertainty. However, as shown in Ghosh and Ostry (1997), in the post-war quarterly data for the US, Japan, and the UK, the greater the uncertainty in income, the greater will be the incentive for precautionary saving and, ceteris paribus, the larger the current account surplus.[26]

4 Intertemporal Models of Current Account with Robustness

In this section, we introduce a concern for model uncertainty (robustness, RB) into the stylized intertemporal current account model (ICA) proposed in Section 3, and explore how this informa-

[26] Recent work examines the importance of precautionary savings for current account dynamics, including Mendoza, Quadrini, and Ríos-Rull (2009) and Carroll and Jeanne (2009); such models are not analytically tractable (with the exception of Carroll and Jeanne 2009) and the analysis is therefore somewhat less transparent.

tion imperfection affects the dynamics of consumption and the current account in the presence of income shocks.

4.1 Optimal Consumption and the Current Account under Robustness

A robust optimal control problem considers the question of how to make decisions when the agent does not know the probability model that generates the data. In the ICA model present in Section 3, an agent with a preference for robustness considers a range of models surrounding the given approximating model, (6), and makes decisions that maximize expected utility given the worst possible model. Following Hansen and Sargent (2007a), a robustness version of the ICA model proposed in Section 3 can be written as

$$v\left(s_t\right) = \max_{c_t} \min_{\nu_t} \left\{ -\frac{1}{2}\left(\overline{c} - c_t\right)^2 + \beta\left[\vartheta \nu_t^2 + E_t\left[v\left(s_{t+1}\right)\right]\right] \right\} \tag{13}$$

subject to the distorted transition equation (i.e., the worst-case model):

$$s_{t+1} = Rs_t - c_t + \zeta_{t+1} + \omega_\zeta \nu_t, \tag{14}$$

where ν_t distorts the mean of the innovation and $\vartheta > 0$ controls how bad the error can be.[27] As shown in Hansen, Sargent, and Tallarini (1999, henceforth, HST) and Hansen and Sargent (2007a), this class of models can produce precautionary behavior while maintaining tractability within the LQ-Gaussian framework.

When output follows an AR(1) process, (11), solving this robust control problem and using the current account identity yields the following proposition:

Proposition 1 *Under RB, the consumption function is*

$$c_t = \frac{R-1}{1-\Sigma} s_t - \frac{\Sigma \overline{c}}{1-\Sigma}, \tag{15}$$

the mean of the worst-case shock is

$$\omega_\zeta \nu_t = \frac{(R-1)\Sigma}{1-\Sigma} s_t - \frac{\Sigma}{1-\Sigma}\overline{c}, \tag{16}$$

the current account is

$$ca_t = \frac{1-\rho}{R-\rho} y_t + \Gamma s_t + \frac{\Sigma \overline{c}}{1-\Sigma}, \tag{17}$$

and $s_t \left(= b_t + \frac{1}{R-\rho} y_t\right)$ *is governed by*

$$s_{t+1} = \rho_s s_t + \zeta_{t+1}, \tag{18}$$

[27] Formally, this setup is a game between the decision-maker and a malevolent nature that chooses the distortion process ν_t. $\vartheta \geq 0$ is a penalty parameter that restricts attention to a limited class of distortion processes; it can be mapped into an entropy condition that implies agents choose rules that are robust against processes which are close to the trusted one. In a later section we will apply an error detection approach to calibrate ϑ.

where $\zeta_{t+1} = \varepsilon_{t+1}/(R-\rho)$, $\Sigma = R\omega_\zeta^2/(2\vartheta) \in (0,1)$ *measures the effect of the preference for robustness,* $\Gamma = -\frac{\Sigma(R-1)}{1-\Sigma} < 0$, *and* $\rho_s = \frac{1-R\Sigma}{1-\Sigma} \in (0,1)$.

Proof. See Appendix 7.1. ■

Our univariate RB model leads to the same consumption function as the corresponding multivariate RB model (i.e., the simplified HST model without habit and adjustment costs) in which the state variables are b_t and y_t.[28] Theoretically, the preference of robustness, ϑ, affects both the coefficients attached to b_t and y_t in the consumption function of the multivariate model. That is, in our model the evil agent distorts the transition equation of permanent income s_t, whereas in the multivariate HST model the evil agent distorts the income process y. In other words, the key difference between the multivariate HST model and our univariate model is that in the former RB may affect the relative importance of the two state variables on the consumption function, whereas in the latter the relative importance of the two effects are *fixed* by reducing the state space. However, after solving the two-state model numerically using the standard procedure proposed in Hansen and Sargent (2007a), we can see that the two models lead to the same decision rule (see Appendix 7.2 for the detailed derivation).

The effect of the preference for robustness, Σ, is jointly determined by the RB parameter, ϑ, and the volatility of the permanent income, ω_ζ. This interaction provides a novel channel that the income shock can affect the consumption and the current account for different countries. That is, when there is a preference for robustness (i.e., $\vartheta < \infty$), the different volatilities for the income processes in two countries can lead to different consumption and current account dynamics. This effect will disappear (i.e., $\Sigma = 0$) if there is no preference for robustness (i.e., $\vartheta \to \infty$).

Note that $\Sigma < 1$ comes from the requirement of the second-order condition of the optimization problem. The second-order condition for a minimization by nature can be rearranged into

$$\vartheta > \frac{1}{2}R^2\omega_\zeta^2.$$

Using the definition of $\Sigma = R\omega_\zeta^2/(2\vartheta)$, we obtain $1 > R\Sigma$. Since $R > 1$, we must have $\Sigma < 1$.

The consumption function under RB, (15), shows that the RB parameter, ϑ, affects the precautionary savings increment, $-\frac{\Sigma}{1-\Sigma}\overline{c}$. The smaller the value of ϑ the larger the precautionary saving increment. The consumption function also implies that the stronger the preference for robustness, the more consumption responds initially to changes in permanent income; that is, under RB consumption is more sensitive to unanticipated income shocks. This response is referred to as "making hay while the sun shines" in van der Ploeg (1993).

For the special case $\rho = 1$,

$$ca_t = \Gamma s_t + \frac{\Sigma\overline{c}}{1-\Sigma}, \tag{19}$$

[28] Note that the equivalence between the two models can be extended to the case with more state variables. We are grateful to an anonymous referee for suggesting us to check the possibility that the univariate and multivariate models are identical in the sense that they lead to the same solution.

which clearly shows that the current account is countercyclical even if output follows a random walk.

4.1.1 Impulse Responses of the Current Account

When $\rho \in (0,1)$, the effect of a change in net output on the current account is determined by the first two terms in (17), and the current account includes a unit root. Specifically,

$$\frac{\partial ca_t}{\partial \varepsilon_t} = \frac{\Gamma + 1 - \rho}{R - \rho}, \tag{20}$$

which means that the current account will be procyclical if the effect of the robust preference is not sufficiently strong:

$$\Sigma < \Sigma_1 = \frac{1-\rho}{R-\rho}\left(= 1 - \frac{R-1}{R-\rho}\right). \tag{21}$$

For the special case that $\rho = 1$, introducing robustness generates countercyclical behavior of the current account as $\Sigma > 0$.[29]

Figure 1 shows the impulse response functions (IRF) of the current account to income shock under different values of Σ. As they show, the current account can respond very differently to income shocks as the effect of the preference for robustness varies. For example, when Σ is zero (the RE model) or small, the current account responds positively to an income shock and slowly declines to zero. However, when Σ becomes large enough (such as when $\Sigma = 0.95$ as shown in Figure 1), the current account initially responds *negatively* to a (positive) income shock. As we will discuss more in section 5.2, these different shapes are supported by the VAR evidence from the studied emerging and developed countries. (See Figures 10 and 11.)

It is worth noting that the trade balance $(y_t - c_t)$ is also countercyclical if the same condition for the preference for robustness as specified in (21) holds, namely that $\Sigma > (1-\rho)/(R-\rho)$. The intuition for this result is very simple: the only difference between the trade balance and the current account is the net return on holding foreign bonds ($(R-1)\,b_t$), and this term is not affected by the income innovation at time $t+1$.

4.1.2 Volatility of the Current Account

We now examine how RB affects the relative volatility of the current account to net income. Using (17), the relative volatility of the current account to net income can be written as

$$\mu = \frac{\mathrm{sd}\,(ca_t)}{\mathrm{sd}\,(y_t)} = \sqrt{\left\{(1-\rho^2)\left[\frac{1-\rho}{1+\rho} + \frac{\Gamma^2}{1-\rho_s^2} + \frac{2\,(1-\rho)\,\Gamma}{1-\rho\rho_s}\right]\right\}/\,(R-\rho)^2} < 1, \tag{22}$$

[29] While the current account is not countercyclical with respect to net income, it is countercyclical with respect to GDP in many countries. Standard models attribute this countercyclicality to investment flows (Backus, Kehoe, and Kydland 1994). Our model offers an alternative interpretation.

where we use the facts that

$$\operatorname{var}(ca_t) = \left[\frac{1-\rho}{1+\rho} + \frac{\Gamma^2}{1-\rho_s^2} + \frac{2(1-\rho)\Gamma}{1-\rho\rho_s}\right]\frac{\omega^2}{(R-\rho)^2}, \tag{23}$$

$\operatorname{var}(y_t) = \frac{\omega^2}{1-\rho^2}$, $\operatorname{var}(s_t) = \frac{\omega^2}{(R-\rho)^2(1-\rho_s^2)}$, and $\operatorname{cov}(y_t, s_t) = \frac{\omega^2}{(R-\rho)(1-\rho\rho_s)}$.

Given R and ρ, (22) shows that μ is affected by the amount of robustness (Σ). Note that μ is not a monotonic function of Σ, as $\frac{\Gamma^2}{1-\rho_s^2}$ in (22) is increasing with Σ and $\frac{2(1-\rho)\Gamma}{1-\rho\rho_s}$ in (22) is decreasing with Σ. Given the complexity of this expression, we cannot obtain an explicit result about how RB affects μ. Figure 3 illustrates that how RB affects the relative volatility for different values of ρ. It is clear that μ is decreasing with Σ when Σ is relatively small and is increasing with Σ when Σ is large. The reason is that when Σ is large, the second term (the volatility term about s_t) in the bracket of (23) dominates the third term (the negative covariance term about s_t and y_t) there. (Note that $\Gamma < 0$.) RB thus has a potential to make the model fit the data better along this dimension when Σ in small open economies is large enough and is larger in emerging economies than in developed economies. Note that we have shown in Section 3.2 that the stylized model cannot generate sufficiently-volatile current accounts, and the relative volatility of the current account to income is smaller in emerging economies than in developed economies.

4.1.3 Persistence of the Current Account

The persistence of the current account is measured by its first autocorrelation. Using (17), the first autocorrelation of the current account, $\rho(ca_t, ca_{t-1})$, can be written as

$$\begin{aligned}\rho(ca_t, ca_{t+1}) &= \frac{\operatorname{cov}(ca_t, ca_{t+1})}{\sqrt{\operatorname{var}(ca_t)}\sqrt{\operatorname{var}(ca_{t+1})}} \\ &= \left[\frac{\rho(1-\rho)}{1+\rho} + \frac{\rho_s\Gamma^2}{1-\rho_s^2} + \frac{(\rho+\rho_s)(1-\rho)\Gamma}{1-\rho\rho_s}\right] / \left[\frac{1-\rho}{1+\rho} + \frac{\Gamma^2}{1-\rho_s^2} + \frac{2(1-\rho)\Gamma}{1-\rho\rho_s}\right],\end{aligned} \tag{24}$$

which converges to ρ (the persistence of net income) as Σ goes to 0. Given the complexity of this expression, we cannot obtain an explicit result about how RB affects $\rho(ca_t, ca_{t+1})$. Figure 4 illustrates how RB affects the persistence of the current account for different values of ρ. It is clear that $\rho(ca_t, ca_{t+1})$ is decreasing with Σ. RB thus has a potential to make the model fit the data better along this dimension. In addition, introducing RB can also explain that $\rho(ca_t, ca_{t+1})$ is smaller in emerging countries than in developed countries if Σ is larger in emerging countries.[30] The standard RE-ICA model predicts that the current account and income have the same degree of persistence, which contradicts the evidence that the current account is significantly less persistent

[30] If net income is a pure random walk, the current account under RB can be written as

$$ca_t = \Gamma s_t + \frac{\Sigma\overline{c}}{1-\Sigma},$$

than income in small open economies and the persistence of net income is larger in emerging counties than in developed countries.

4.1.4 Correlation between the Current Account and Income

An alternative description of the comovement of the current account and income is the contemporaneous correlation between the current account and income, $\operatorname{corr}(ca_t, y_t)$. Under RB, the correlation can be written as:

$$\operatorname{corr}(ca_t, y_t) = \left(\frac{\Gamma}{1-\rho\rho_s} + \frac{1}{1+\rho}\right) / \sqrt{\frac{1}{(1+\rho)^2} + \frac{\Gamma^2}{(1-\rho^2)(1-\rho_s^2)} + \frac{2\Gamma}{(1+\rho)(1-\rho\rho_s)}}, \tag{26}$$

which converges to 1 as Σ converges to 0. Figure 5 illustrates that how RB affects the correlation between the current account and net income for different values of ρ. It is clear that $\operatorname{corr}(ca_t, y_t)$ is decreasing with Σ (note that in the figure we restrict the values of Σ to be less than 0.83 such that $\operatorname{corr}(ca_t, y_t)$ is positive as generated in the data). RB thus aligns the model and the data more closely along this dimension. In addition, introducing RB can also account for the fact that $\operatorname{corr}(ca_t, y_t)$ is smaller in emerging countries than in developed countries, provided Σ is larger in emerging countries.

4.1.5 Implications of Macroeconomic Uncertainty for the Current Account under RB

Finally, the last term in (17) determines the effect of precautionary savings on the current account. It is clear that with the preference for robustness, the greater the uncertainty in net income, the greater the amount of precautionary saving, and the larger the current account surplus, as

$$\frac{\partial ca_t}{\partial \omega_\zeta^2} > 0. \tag{27}$$

This result is consistent with the empirical evidence that the current account and macroeconomic volatility are positively correlated (Ghosh and Ostry 1997, Fogli and Perri 2008). This result is also related to Mendoza, Quadrini, and Ríos-Rull (2009) and Carroll and Jeanne (2009) in which they solve the models with CRRA utility numerically and examine the importance of precautionary savings for current account dynamics. Our model therefore also contributes to this literature by providing a new mechanism through which precautionary saving due to induced uncertainty affects the current account. Note that the precautionary savings induced by a concern about

which clearly shows that the current account is countercyclical because $\Gamma < 0$. Given (18), the current account can be written as

$$ca_{t+1} = \rho_s ca_t + \Gamma\zeta_{t+1} + (1-\rho_s)\frac{\Sigma\bar{c}}{1-\Sigma}, \tag{25}$$

which means that RB reduces the persistence of the current account because $\partial\rho_s/\partial\Sigma < 0$.

robustness differs from the usual precautionary savings motive that emerges when labor income uncertainty interacts with the convexity of the marginal utility of consumption. This type of precautionary savings emerges because consumers facing more model uncertainty want to save more as protection against model misspecification and thus occurs even in models with quadratic utility.

4.1.6 Implication for Consumption Volatility

Although introducing robustness has a potential to improve the model's predictions on the dynamics of the current account and precautionary savings, it worsens the model's prediction for the joint dynamics of consumption and income. Given (15) and (18), the change in aggregate consumption can be written as

$$c_{t+1} = \rho_s c_t - \frac{(1-R)\,\Sigma\overline{c}}{1-\Sigma} + \frac{R-1}{(1-\Sigma)\,(R-\rho)}\varepsilon_{t+1}, \tag{28}$$

where $\rho_s = \frac{1-R\Sigma}{1-\Sigma}$ and we use the fact that $\zeta_{t+1} = \varepsilon_{t+1}/\,(R-\rho)$. Therefore, aggregate consumption under RB follows an AR(1) process, which contradicts the evidence that in the data consumption reacts to income gradually and with delay. In other words, RB does not produce any propagation in consumption after an income shock. As emphasized in Sims (1998, 2003), VAR studies show that most cross-variable relationships among macroeconomic time series are smooth and delayed. Figure 2 illustrates the response of aggregate consumption growth to an aggregate income shock ε_{t+1}; comparing the solid line (RE) with the dash-dotted line, it is clear that RB raises the sensitivity of consumption growth to unanticipated changes in aggregate income.

Furthermore, the relative volatility of consumption growth to income growth, μ, can be written as

$$\mu_c = \frac{\mathrm{sd}\,[\Delta c_t]}{\mathrm{sd}\,[\Delta y_t]} = \frac{R-1}{(1-\Sigma)\,(R-\rho)}\sqrt{\frac{1+\rho}{1+\rho_s}}, \tag{29}$$

where we also use the fact that $\Delta y_t = \varepsilon_t + (\rho-1)\,\frac{\varepsilon_{t-1}}{1-\rho L}$.[31] It is clear from (29) that RB increases the relative volatility via two channels: first, it strengthens the marginal propensity to consume out of permanent income $\left(\frac{R-1}{1-\Sigma}\right)$; and second, it increases consumption volatility by reducing the persistence of permanent income measured by ρ_s: $\frac{\partial\rho_s}{\partial\Sigma} < 0$. Furthermore, if Σ is larger in emerging economies, the RB-ICA model will predict that the relative volatility of consumption to income is greater in emerging economies than in developed economies.

[31] We use the relative volatility of consumption growth to income growth instead of that of consumption to income to compare the implications of RE and RB models, as consumption follows a random walk under RE and the volatility of consumption is not well defined in this model.

4.2 Investment and the Current Account under RB

In this paper we focus on examining how the two types of "induced uncertainty" affect the joint behavior of consumption, the current account and income, and abstract from production and investment decisions. Since investment is an important force in determining the current account, in this subsection we briefly examine how the presence of investment decision affects the behavior of consumption and the current account. To maintain our analysis within the LQG setting, we follow Glick and Rogoff (1995), assume that output is determined by the following production function:

$$y_t = a_t k_t^{\alpha} \left[1 - \frac{g}{2} \left(\frac{i_t^2}{k_t} \right) \right],$$

where a_t is aggregate productivity, k_t is capital stock, and the second term in the bracket captures the adjustment costs in capital. Taking a linear approximation to the first-order conditions of the firm's optimizing problem yields

$$y_t \simeq \alpha_i i_t + \alpha_k k_t + \alpha_a a_t, \tag{30}$$

$$k_t \simeq \lambda_1 k_{t-1} + \lambda_1 \sum_{j=0}^{\infty} (\beta \lambda_1)^j E_t [a_{t+j}] \triangleq \lambda_1 k_{t-1} + \lambda_{ka} a_t \tag{31}$$

$$i_t \simeq \lambda_1 i_{t-1} + \eta \sum_{j=1}^{\infty} \lambda_2^2 (E_t [a_{t+j}] - E_{t-1} [a_{t+j-1}]) \triangleq \lambda_1 k_{t-1} + \lambda_{ia} a_t \tag{32}$$

where $\alpha_i < 0$, $\alpha_k > 0$, and $\alpha_a > 0$ are the linearization coefficients, $\lambda_1 \in (0, 1)$, $\lambda_2 > 0$, and $\eta > 0$.[32] Using Expressions (7), (30), (31), and (32), it is straightforward to show that the innovation to permanent income can be written as a linear function of the change in aggregate productivity:

$$\zeta_{t+1} = \frac{1}{R-1} \left\{ \alpha_a + \frac{\phi \left[(\alpha_i - 1)(R-1) + \alpha_k \right]}{R - \lambda_1} \right\} \Delta a_{t+1}, \tag{33}$$

where $\phi = \eta \left[\lambda_2 / (1 - \lambda_2) \right]$.

After substituting the consumption function and net output into the current account identity,

$$\Delta ca_t = (R-1)\, ca_{t-1} + \Delta y_t - \Delta i_t - \Delta c_t, \tag{34}$$

we have

$$\begin{aligned} \Delta ca_t &= (R-1)\, ca_{t-1} + \Psi \Delta a_t + \gamma i_{t-1}, \text{ or} \\ ca_t &= R ca_{t-1} + \Psi \Delta a_t + \gamma i_{t-1}, \end{aligned} \tag{35}$$

where $R > 1$, $\phi = \eta \left[\lambda_2 / (1 - \lambda_2) \right]$, $\gamma = (\lambda_1 - 1)(\alpha_i - 1) + \alpha_k$, $\Psi = \frac{\phi[\alpha_k + (\alpha_i - 1)(1 - \lambda_1)]}{R - \lambda_1}$, and investment follows an AR(1) process: $i_t = \lambda_1 i_{t-1} + \phi \Delta a_t$. We can see from (35) that endogenizing

[32] See Glick and Rogoff (1995) for the detailed derivation.

investment affects the current account dynamics by introducing a lagged investment term and the term of the change in aggregate productivity. It is also clear from (35) that RB affects the current account by affecting the value of Ψ. Given the structure of the current account specified in (35), it is impossible to obtain the explicit expression for the stochastic properties of the current account.[33] However, we can still examine how RB affects the current account by inspecting (35).

As shown in Hansen and Sargent (2007), introducing RB into the decision problem will strengthen the responses of the control variables to both endogenous and exogenous state variables. In other words, in the consumer problem, consumption is more sensitive to the income shock (i.e., a linear function of productivity shocks), and in the firm problem capital stock and investment are more sensitive to the productivity shock (i.e., the values of λ_1, λ_{ka}, and λ_{ia} are larger under RB). Given (17), (34), and $\Delta y_t = \alpha_i \Delta i_t + \alpha_k \Delta k_t + \alpha_a \Delta a_t$ ($\alpha_i < 0$), it is straightforward to show that introducing RB will make the current account be more negatively correlated with the aggregate productivity by making Ψ more negative. In other words, the stronger the preference for RB, the more countercyclical the current account is.

4.3 Calibrating the RB Parameter

Having examined the implications of RB for the relative volatility and persistence of the current account, and the correlation between the current account and income, it is clear that RB has a potential to improve the model's predictions on the joint dynamics of the current account and net income. A requirement for matching these facts is that the fear of misspecification is stronger in emerging economies. This requirement is obviously subject to empirical testing, the task we turn to now.

Specifically, we use the procedure outlined in Hansen and Sargent (2007a) to calibrate the RB parameter (ϑ or Σ). We calibrate ϑ by using the notion of a model detection error probability that is based on a statistical theory of model selection (the approach will be precisely defined below). We can then infer what values of the RB parameter ϑ imply reasonable fears of model misspecification for empirically-plausible approximating models. The model detection error probability is a measure of how far the distorted model can deviate from the approximating model without being discarded; low values for this probability mean that agents are unwilling to discard very many models (as they want errors to be rare), implying that the cloud of models surrounding the approximating model is large.

[33] Glick and Rogoff (1995) examined the responses of the current account and investment to the productivity shock, and did not explore the other stochastic properties of the current account (e.g., the volatility and persistence of the current account).

4.3.1 The Definition of the Model Detection Error Probability

Let model A denote the approximating model and model B be the distorted model. Define p_A as

$$p_A = \text{Prob}\left(\log\left(\frac{L_A}{L_B}\right) < 0 \middle| A \right), \tag{36}$$

where $\log\left(\frac{L_A}{L_B}\right)$ is the log-likelihood ratio. When model A generates the data, p_A measures the probability that a likelihood ratio test selects model B. In this case, we call p_A the probability of the model detection error. Similarly, when model B generates the data, we can define p_B as

$$p_B = \text{Prob}\left(\log\left(\frac{L_A}{L_B}\right) > 0 \middle| B \right). \tag{37}$$

Following Hansen, Sargent, and Wang (2002) and Hansen and Sargent (2007b), the detection error probability, p, is defined as the average of p_A and p_B:

$$p(\vartheta) = \frac{1}{2}(p_A + p_B), \tag{38}$$

where ϑ is the robustness parameter used to generate model B. Given this definition, we can see that $1-p$ measures the probability that econometricians can distinguish the approximating model from the distorted model. Now we show how to compute the model detection error probability in the Robustness model.

4.3.2 Calibrating the RB Parameter in the ICA Model

Under RB, assuming that the approximating model generates the data, the state, s_t, evolves according to the transition law

$$\begin{aligned} s_{t+1} &= Rs_t - c_t + \zeta_{t+1}, \\ &= \frac{1 - R\Sigma}{1-\Sigma} s_t + \frac{\Sigma}{1-\Sigma}\bar{c} + \zeta_{t+1}. \end{aligned} \tag{39}$$

In contrast, assuming that the distorted model generates the data, s_t evolves according to

$$\begin{aligned} s_{t+1} &= Rs_t - c_t + \zeta_{t+1} + \omega_\zeta \nu_t, \\ &= s_t + \zeta_{t+1}. \end{aligned} \tag{40}$$

In order to compute p_A and p_B, we use the following procedure:

1. Simulate $\{s_t\}_{t=0}^T$ using (39) and (40) a finite number of times. The number of periods used in the simulation, T, is set to be the actual length of the data for each individual country.
2. Count the number of times that $\log\left(\frac{L_A}{L_B}\right) < 0 \Big| A$ and $\log\left(\frac{L_A}{L_B}\right) > 0 \Big| B$ are each satisfied.

3. Determine p_A and p_B as the fractions of realizations for which $\log\left(\frac{L_A}{L_B}\right) < 0\Big| A$ and $\log\left(\frac{L_A}{L_B}\right) > 0\Big| B$, respectively.

In practice, given Σ, to simulate the $\{s_t\}_{t=0}^T$ we need to know a) the volatility of ζ_t in (39) and (40), and b) the value of $\overline{c}$. For a), we can compute it from $\mathrm{sd}(\zeta) = \frac{\sqrt{1-\rho^2}}{R-\rho}\,\mathrm{sd}(y)$ where $\mathrm{sd}(y)$ is the standard deviation of net income. For b), we use the local coefficient of relative risk aversion $\gamma = -\frac{u''(c)c}{u'(c)} = \frac{c}{\overline{c}-c}$ to recover the value of $\overline{c}$: $\overline{c} = \left(1+\frac{1}{\gamma}\right)E[c]$ where $E[c]$ is mean consumption. We choose $\gamma = 2$. Finally, we assume that consumers in our model economy are impatient enough such that they cannot resolve their model misspecification fears during the actual length of the data for each individual country.

4.4 Calibration Results and Main Findings

After simulating the models and obtaining the detection error probability that circumscribes a neighborhood of models against which consumers want to assure robustness, we can find the values of ϑ and Σ associated with that probability. Having shown how the RB parameter is related to the model detection error probability, in this section we report the calibrated values of the RB parameters by setting the model detection error probability to different targeted values. As a benchmark, we choose the RB parameter to match the model detection error probability of $p = 0.1$. That is, the probability that the agent can distinguish the approximating model from the distorted model is 0.9.

Tables 7 and 8 report the calibrated values of RB parameter, $\Sigma \equiv R\omega_\zeta^2/(2\vartheta)$, as well as the associated model detection error, p, the autocorrelation coefficient of GDP, ρ, and the ratios of the standard deviation of real income and permanent income to the mean of real income (undetrended), $\sigma(y)/\mu(y)$ and $\sigma(\zeta)/\mu(y)$, respectively.[34] For simplicity here we only report the results using the linear filter; using the HP filter generates similar patterns from the model. We use $\sigma(y)/\mu(y)$ to measure the relative volatility of fundamental uncertainty. Table 6 reports the averages over the emerging countries and that for the developed countries, respectively, and shows that on average:

1. Emerging countries face more volatile income processes than do developed countries. That is, macroeconomic uncertainty is higher in emerging countries.

2. After setting the detection error probability $p(\vartheta, \Sigma)$ to be the same in the two economies, the recovered Σ is larger in emerging countries.

Therefore, the effect of the preference for robustness (measured by Σ) in emerging countries is stronger than in developed countries. The intuition is simple: agents in the emerging economy are

[34] All tables in this paper are generated using the estimated parameters in the exogenous income processes. See Table 1 for the estimated parameters.

more concerned about model misspecification because they face larger macroeconomic uncertainty and instability than those in developed countries. It is worth noting that a larger Σ does not necessarily imply a smaller value of ϑ since ω_ζ (i.e., $\sigma(\zeta)$) can be different. As we have shown in Section 4.1, RB influences the countercyclical behavior of the current account and the relative volatility of consumption to income in the model through the interaction of ϑ and ω_ζ in Σ instead of ϑ.

We first consider a comparison between the standard RE model and the RB model. In Tables 9-10, p is set to 0.1 such that $\Sigma = 0.524$ in emerging countries and 0.205 in developed countries. In this case the first three columns of the tables clearly show that RB can improve the model's predictions along the following three dimensions: the contemporaneous correlation between the current account and net income, the persistence of the current account, and the relative volatility of consumption growth to income growth, but worsens the model prediction on the relative volatility of the current account to net income. Specifically, for emerging countries, given the calibrated Σs RB reduces the correlation between the current account and net income from 1 to 0.62; reduces the first-order autocorrelation from 0.8 to 0.74; increases the relative volatility of consumption growth to income growth from 0.28 to 0.9; and reduces the relative volatility of the current account to income from 0.71 to 0.49. The intuition that RB reduces the volatility of the current account is that RB increases the response of consumption to income shock, and thus reduces the response of the current account.

In Tables 11-12, we reduce the detection error probability to 0.01 and find that in this case RB can improve the model's predictions along all the four dimensions including the relative volatility of the current account to net income. When the RB parameter is large enough, the second term in the bracket of (23) dominates the third term, and thus the volatility of the current account increases. However, $p = 0.01$ is an extremely low value and means that agents rarely make mistakes and thus can distinguish the models quite well.[35] As shown in Tables 11-12, even for this extremely low detection error probability, the RB model still cannot generate the observed volatility of the current account. In the next section, we will show that introducing another informational friction, rational inattention, helps resolve this anomaly.

`[Insert Tables 6-8 Here]`

[35] Alternatively, low p means that we impose weak limits on the evil nature who distorts the model.

5 RB-RI Model

5.1 Optimal Consumption and the Current Account under RB and RI

5.1.1 Information-Processing Constraints

Under RI, consumers in the economy face both the usual flow budget constraint and information-processing constraint due to finite Shannon capacity first introduced by Sims (2003). As argued by Sims (2003, 2006), individuals with finite channel capacity cannot observe the state variables perfectly; consequently, they react to exogenous shocks incompletely and gradually. They need to choose the posterior distribution of the true state after observing the corresponding signal. This choice is in addition to the usual consumption choice that agents make in their utility maximization problem.[36]

Following Sims (2003), the consumer's information-processing constraint can be characterized by the following inequality:

$$\mathcal{H}\left(s_{t+1}|\mathcal{I}_t\right)-\mathcal{H}\left(s_{t+1}|\mathcal{I}_{t+1}\right)\leq\kappa, \tag{41}$$

where κ is the consumer's channel capacity, $\mathcal{H}\left(s_{t+1}|\mathcal{I}_t\right)$ denotes the entropy of the state prior to observing the new signal at $t+1$, and $\mathcal{H}\left(s_{t+1}|\mathcal{I}_{t+1}\right)$ is the entropy after observing the new signal.[37] The concept of *entropy* comes from information theory, and it characterizes the uncertainty in a random variable. The right-hand side of (41), being the reduction in entropy, measures the amount of information in the new signal received at $t+1$. Hence, as a whole, (41) means that the reduction in the uncertainty about the state variable gained from observing a new signal is bounded from above by κ. Since the *ex post* distribution of s_t is a normal distribution, $N\left(\widehat{s}_t,\sigma_t^2\right)$, (41) can be reduced to

$$\log|\psi_t^2|-\log|\sigma_{t+1}^2|\leq 2\kappa \tag{42}$$

where $\widehat{s}_t$ is the conditional mean of the true state, and $\sigma_{t+1}^2=\mathrm{var}\left[s_{t+1}|\mathcal{I}_{t+1}\right]$ and $\psi_t^2=\mathrm{var}\left[s_{t+1}|\mathcal{I}_t\right]$ are the posterior variance and prior variance of the state variable, respectively. To obtain (42), we use the fact that the entropy of a Gaussian random variable is equal to half of its logarithm variance plus a constant term.

It is straightforward to show that in the univariate case (42) has a unique steady state σ^2.[38] In that steady state the consumer behaves as if observing a noisy measurement which is $s_{t+1}^*=$

[36] More generally, agents choose the joint distribution of consumption and current permanent income subject to restrictions about the transition from prior (the distribution before the current signal) to posterior (the distribution after the current signal), and nature then draws current consumption from this distribution. The budget constraint implies a link between the distribution of consumption and the distribution of next period permanent income.

[37] We regard κ as a technological parameter. If the base for logarithms is 2, the unit used to measure information flow is a 'bit', and for the natural logarithm e the unit is a 'nat'. 1 nat is equal to $\log_2 e\approx 1.433$ bits.

[38] Convergence requires that $\kappa>\log(R)\approx R-1$; see Luo and Young (2010) for a discussion.

$s_{t+1}+\xi_{t+1}$, where ξ_{t+1} is the endogenous noise and its variance $\alpha_t^2 = \text{var}\left[\xi_{t+1}|\mathcal{I}_t\right]$ is determined by the usual updating formula of the variance of a Gaussian distribution based on a linear observation:

$$\sigma_{t+1}^2 = \psi_t^2 - \psi_t^2\left(\psi_t^2+\alpha_t^2\right)^{-1}\psi_t^2. \tag{43}$$

Note that in the steady state $\sigma^2 = \psi^2 - \psi^2\left(\psi^2+\alpha^2\right)^{-1}\psi^2$, which can be solved as $\alpha^2 = \left[\left(\sigma^2\right)^{-1} - \left(\psi^2\right)^{-1}\right]^{-1}$. Note that (43) implies that in the steady state $\sigma^2 = \left(\frac{1}{R-\rho}\right)^2\frac{\omega^2}{\exp(2\kappa)-R^2}$.and $\alpha^2 = \text{var}\left[\xi_{t+1}\right] = \frac{\left[\omega^2/(R-\rho)^2+R^2\sigma^2\right]\sigma^2}{\omega^2/(R-\rho)^2+(R^2-1)\sigma^2}$.

5.1.2 Considering RB in the RI Model

We now incorporate RI into the RB model and examine how the combination of the two types of information imperfections affect the joint dynamics of consumption, the current account, and income.[39] A key assumption in the RB-RI model is that we assume that the consumer not only has doubts about the process for the shock to permanent income ζ_{t+1}, but also distrusts his regular Kalman filter hitting the endogenous noise (ξ_{t+1}) and updating the estimated state. As a result, our agents have an additional dimension along which they desire robustness.

Specifically, the regular RI-induced Kalman filter equation updating the estimated state ($\widehat{s}_t$),

$$\widehat{s}_{t+1} = (1-\theta)\left(R\widehat{s}_t - c_t\right) + \theta\left(s_{t+1}+\xi_{t+1}\right), \tag{44}$$

where $\widehat{s}_t = E\left[s_t|\mathcal{I}_t\right]$ is the conditional mean of s_t, ξ_{t+1} is the iid endogenous noise with $\alpha^2 = \text{var}\left[\xi_{t+1}\right] = \frac{\left[\omega^2/(R-\rho)^2+R^2\sigma^2\right]\sigma^2}{\omega^2/(R-\rho)^2+(R^2-1)\sigma^2}$, $\theta = \sigma^2/\alpha^2 = 1 - 1/\exp(2\kappa) \in [0,1]$ is the constant optimal weight on any new observation, and $s_0 \sim N\left(\widehat{s}_0, \sigma^2\right)$ is fixed.[40] Combining (44) with the budget constraint, $s_{t+1} = Rs_t - c_t + \zeta_{t+1}$, yields the following equation governing the dynamics of the perceived state $\widehat{s}_t$ that matters in agents' decision problems:

$$\widehat{s}_{t+1} = R\widehat{s}_t - c_t + \eta_{t+1}, \tag{45}$$

where

$$\eta_{t+1} = \theta R\left(s_t - \widehat{s}_t\right) + \theta\left(\zeta_{t+1}+\xi_{t+1}\right) \tag{46}$$

is the innovation to the mean of the distribution of perceived permanent income,

$$s_t - \widehat{s}_t = \frac{(1-\theta)\zeta_t}{1-(1-\theta)R\cdot L} - \frac{\theta\xi_t}{1-(1-\theta)R\cdot L} \tag{47}$$

[39] The RB-RI model proposed in this paper encompasses the hidden state model discussed in Hansen, Sargent, and Wang (2002) and Hansen and Sargent (2007b); the main difference is that none of the states in the RB-RI model are perfectly observable (or controllable).

[40] θ measures how much new information is transmitted each period or, equivalently, how much uncertainty is removed upon the receipt of a new signal.

and $E_t[\eta_{t+1}] = 0$. To introduce robustness into the RI model, we assume that the agent thinks that (45) is the approximating model for the true model that governs the data but that he cannot specify. Following Hansen and Sargent (2007a), we surround (45) with a set of alternative models to represent his preference for robustness:

$$\widehat{s}_{t+1} = R\widehat{s}_t - c_t + \omega_\eta \nu_t + \eta_{t+1}. \tag{48}$$

Under RI the innovation η_{t+1} that the agent distrusts is composed of two MA(∞) processes and includes the entire history of the exogenous income shock and the endogenous noise, $\{\zeta_{t+1}, \zeta_t, \cdots, \zeta_0; \xi_{t+1}, \xi_t, \cdots, \xi_0\}$.

The optimizing problem for this RB-RI model is formulated as

$$\widehat{v}(\widehat{s}_t) = \max_{c_t} \min_{\nu_t} \left\{ -\frac{1}{2}(c_t - \overline{c})^2 + \beta E_t\left[\vartheta \nu_t^2 + \widehat{v}(\widehat{s}_{t+1})\right] \right\}, \tag{49}$$

subject to (48)-(47). (49) is a standard dynamic programming problem. The following proposition summarizes the solution to the RB-RI model.

Proposition 2 *Given ϑ and θ, the consumption function under RB and RI is*

$$c_t = \frac{R-1}{1-\Sigma}\widehat{s}_t - \frac{\Sigma \overline{c}}{1-\Sigma}, \tag{50}$$

the mean of the worst-case shock is

$$\omega_\eta \nu_t = \frac{(R-1)\Sigma}{1-\Sigma}\widehat{s}_t - \frac{\Sigma}{1-\Sigma}\overline{c}, \tag{51}$$

and $\widehat{s}_t$ is governed by

$$\widehat{s}_{t+1} = \rho_s \widehat{s}_t + \eta_{t+1}. \tag{52}$$

where $\rho_s = \frac{1-R\Sigma}{1-\Sigma} \in (0, 1)$,

$$\Sigma = R\omega_\eta^2 / (2\vartheta) > 0, \tag{53}$$

$$\omega_\eta^2 = \mathrm{var}[\eta_{t+1}] = \frac{\theta}{1-(1-\theta)R^2}\omega_\zeta^2. \tag{54}$$

It is clear from (50)-(54) that RB and RI affect the consumption function via two channels in the model: (1) the marginal propensity to consume (MPC) out of the perceived state $\left(\frac{R-1}{1-\Sigma}\right)$ and (2) the dynamics of the perceived state ($\widehat{s}_t$). Given $\widehat{s}_t$, stronger degrees of RI and RB increase the value of Σ, which increases the MPC. Furthermore, from (53) and (54), we can see that imperfect state observation due to RI can amplify the importance of model uncertainty measured by Σ in determining consumption and precautionary savings.

Before proceeding, we want to draw a distinction between the model proposed above and similar ones used in Luo and Young (2010) and Luo, Nie, and Young (2011). In those other papers, agents were assumed to trust the Kalman filter they use to process information, meaning that decisions were only robust to misspecification of the income process. An implicit assumption in the two papers is that the evil agent (the minimizing agent) has the same information set as the consumer (the maximizing agent). In that model Σ was independent of θ, and for the questions at hand here the resulting values were too small.[41] By adding the additional concern for robustness developed here, we are able to strengthen the effects of robustness on decisions. In addition, our setup here is arguably more consistent with the underlying primitive structure of ambiguity that gives rise to robust decision-making (Gilboa and Schmeidler 1989).

5.1.3 The Joint Dynamics of Consumption, the Current Account, and Net Income under RB-RI

Furthermore, in the RB-RI model individual dynamics are not identical to aggregate dynamics. Combining (50) with (48) yields the change in individual consumption in the RI-RB economy:

$$\Delta c_t = \frac{(1-R)\,\Sigma}{1-\Sigma}\left(c_{t-1}-\overline{c}\right) + \frac{R-1}{1-\Sigma}\left(\frac{\theta\zeta_t}{1-(1-\theta)R\cdot L} + \theta\left(\xi_t - \frac{\theta R\xi_{t-1}}{1-(1-\theta)R\cdot L}\right)\right),$$

where L is the lag operator and we assume that $(1-\theta)R < 1$.[42] This expression shows that consumption growth is a weighted average of all past permanent income and noise shocks. Since this expression permits exact aggregation, we can obtain the change in aggregate consumption as

$$\Delta c_t = \frac{(1-R)\,\Sigma}{1-\Sigma}\left(c_{t-1}-\overline{c}\right) + \frac{R-1}{1-\Sigma}\left(\frac{\theta\zeta_t}{1-(1-\theta)R\cdot L} + \theta\left(\overline{\xi}_t E^i\left[\xi_t\right] - \frac{\theta R\overline{\xi}_{t-1}}{1-(1-\theta)R\cdot L}\right)\right), \tag{55}$$

where i denotes a particular individual, $E^i\left[\cdot\right]$ is the population average, and $\overline{\xi}_t = E^i\left[\xi_t\right]$ is the common noise.[43] This expression shows that even if every consumer only faces the common shock ζ, the RI economy still has heterogeneity since each consumer faces the idiosyncratic noise induced by finite channel capacity. As argued in Sims (2003), although the randomness in an individual's response to aggregate shocks will be idiosyncratic because it arises from the individual's own information-processing constraint, there is likely a significant common component. The intuition is that people's needs for coding macroeconomic information efficiently are similar, so they rely on common sources of coded information. Therefore, the common term of the idiosyncratic error, $\overline{\xi}_t$, lies between 0 and the part of the idiosyncratic error, ξ_t, caused by the common shock to permanent income, ζ_t. Formally, assume that ξ_t consists of two independent noises: $\xi_t = \overline{\xi}_t + \xi_t^i$,

[41] Due to limited space, we do not report the results of this RB-RI model; they are available from the authors by request.

[42] This assumption requires $\kappa > \frac{1}{2}\log(R) \approx \frac{R-1}{2}$, which is weaker than the condition needed for convergence of the filter.

[43] For simplicity, here we use the same notation c for aggregate consumption.

where $\overline{\xi}_t = E^i[\xi_t]$ and ξ_t^i are the common and idiosyncratic components of the error generated by ζ_t, respectively. A single parameter,

$$\lambda = \frac{\operatorname{var}\left[\overline{\xi}_t\right]}{\operatorname{var}\left[\xi_t\right]} \in [0,1],$$

can be used to measure the common source of coded information on the aggregate component (or the relative importance of $\overline{\xi}_t$ vs. ξ_t).[44] Figure 2 also shows how RI can help generate the smooth and hump-shaped impulse response of consumption to the income shock, which, as argued in Sims (1998, 2003), fits the VAR evidence better.

In a recent paper, Angeletos and La'O (2009) show how dispersed information about the underlying aggregate productivity shock contributes to significant noise in the business cycle and helps explain cyclical variations in observed Solow residuals and labor wedges in the RBC setting. In contrast, Lorenzoni (2009) examines how demand shocks, defined as noisy news about future aggregate productivity, contribute to business cycles fluctuations in a new Keynesian model. In the next section, after calibrating the RB parameter we also show that the common noise due to finite capacity simultaneously increases the relative volatility of consumption growth and income growth and reduces the contemporaneous correlation between the current account and income, which makes the RB-RI model fit the data better.

Substituting (50) into the current account identity, the current account in the RB-RI model economy can be written as

$$ca_t = \frac{1-\rho}{R-\rho}y_t - \frac{\Sigma(R-1)}{1-\Sigma}s_t + \frac{R-1}{1-\Sigma}(s_t - \widehat{s}_t) + \frac{\Sigma}{1-\Sigma}\overline{c}, \tag{56}$$

where

$$s_t - \widehat{s}_t = \frac{(1-\theta)\zeta_t}{1-(1-\theta)R\cdot L} - \frac{\theta E^i[\xi_t]}{1-(1-\theta)R\cdot L} \tag{57}$$

is the error in estimating s_t. It is clear that when $\theta = 1$, (56) reduces to (17) in Section 4.1. The expression for the current account, (56), clearly shows that in the RB-RI model the current account is determined by four factors:

1. The income process, $-\frac{\rho}{R-\rho}\Delta y_t$. Holding other factors constant, the current account deteriorates in response to a positive shock to income.

2. The overreaction in consumption due to the preference for robustness, $-\frac{\Sigma(R-1)}{1-\Sigma}s_t$. This expression means that the stronger the preference for robustness, the more countercyclical the current account is. Under robustness, consumption is more sensitive to the unanticipated income shock, and thus the increase in consumption is larger than that of income itself; consequently, the current account deteriorates.

[44] It is worth noting that the special case that $\lambda = 1$ can be viewed as a representative-agent model in which we do not need to discuss the aggregation issue.

3. The forecast error term due to rational inattention, $\frac{R-1}{1-\Sigma}(s_t - \widehat{s}_t)$. Consumers with finite capacity cannot observe the state perfectly, and thus adjust optimal consumption gradually and with delay. For a positive income shock, a gradual adjustment in consumption improves the current account.

4. The precautionary savings term, $\frac{\Sigma \overline{c}}{1-\Sigma}$. The precautionary saving premium due to the fear of model misspecification induces a bias toward current account surplus.

Figure 1 also plots the impulse response of the current account to the income shock when $\Sigma = 0.95$ and $\theta = 80\%$. It clearly shows that the current account also responds to the income shock smoothly and gradually, which can better fit the VAR evidence that most cross-variable relationship among macroeconomic time series are smooth and delayed. Using (56) it is straightforward to show that the current account is procyclical if the following inequality is satisfied:

$$\Sigma < 1 - \theta \frac{R-1}{R-\rho}.$$

As will be shown in section 5.2 that the RB-RI model also has the potential to generate the different shapes of the IRFs in different countries.

5.1.4 Volatility of the Current Account

Under RB-RI, using (56), the relative volatility of the current account and net income can be written as

$$\mu = \frac{\mathrm{sd}\,(ca_t)}{\mathrm{sd}\,(y_t)} = \frac{\sqrt{1-\rho^2}}{R-\rho}\sqrt{\begin{array}{c} \frac{1-\rho}{1+\rho} + \frac{\Gamma^2}{1-\rho_s^2} + \left(\frac{R-1}{1-\Sigma}\right)^2 \left[\frac{(1-\theta)^2}{1-\rho_\theta^2} + \frac{\theta\lambda^2}{1-\rho_\theta^2}\frac{1}{(1/(1-\theta)-R^2)}\right] \\ + \frac{2(1-\rho)\Gamma}{1-\rho\rho_s} + \left(\frac{R-1}{1-\Sigma}\right)\frac{2(1-\rho)(1-\theta)}{1-\rho\rho_\theta} + \left(\frac{R-1}{1-\Sigma}\right)\frac{2\Gamma(1-\theta)}{1-\rho_s\rho_\theta} \end{array}}. \tag{58}$$

Given the complexity of this expression, we cannot obtain an explicit result about how the interactions of RI and RB affect the relative volatility. As in the RB case, we thus use a figure to illustrate how RB and RI affect the relative volatility. Figure 6 illustrates the effects of RI on the relative volatility when $R\omega_\zeta^2/(2\vartheta) = 0.5$ and $\rho = 0.8$. Note that in the RB-RI case, $\Sigma = R\omega_\eta^2/(2\vartheta) = \frac{\theta}{1-(1-\theta)R^2}R\omega_\zeta^2/(2\vartheta)$ as $\omega_\eta^2 = \frac{\theta}{1-(1-\theta)R^2}\omega_\zeta^2$. It is clear from the figure that given the aggregation factor (λ), the relative volatility is decreasing with the degree of attention (θ); given θ, the relative volatility is increasing with λ. The intuition for the first result is that holding the aggregation factor fixed (i.e., given the impact of the common noise), reducing θ increases the smoothness of aggregate consumption, and thus increases the volatility of the current account. The intuition for the second result is that holding θ fixed, increasing λ strengthens the importance of the common noise, which leads to more volatile consumption and current accounts. Therefore, RI measured by θ and λ has the potential to make the model fit the data better along this dimension. In the next section, we will examine how RI and RB improve the model's quantitative predictions.

5.1.5 Persistence of the Current Account

Under RB-RI, using (56), the first-order autocorrelation of the current account can be written as:

$$\rho\left(ca_t, ca_{t+1}\right)$$
$$= \frac{\frac{\rho(1-\rho)}{1+\rho} + \frac{\rho_s\Gamma^2}{1-\rho_s^2} + \left(\frac{R-1}{1-\Sigma}\right)^2 \frac{\rho_\theta(1-\theta)^2}{1-\rho_\theta^2} + \frac{(\rho+\rho_s)(1-\rho)\Gamma}{1-\rho\rho_s} + \left(\frac{R-1}{1-\Sigma}\right)\frac{(\rho+\rho_\theta)(1-\rho)(1-\theta)}{1-\rho\rho_\theta} + \left(\frac{R-1}{1-\Sigma}\right)\frac{(\rho_s+\rho_\theta)\Gamma(1-\theta)}{1-\rho_s\rho_\theta}}{\left\{\frac{1-\rho}{1+\rho} + \frac{\Gamma^2}{1-\rho_s^2} + \left(\frac{R-1}{1-\Sigma}\right)^2\left[\frac{(1-\theta)^2}{1-\rho_\theta^2} + \frac{\theta\lambda^2}{1-\rho_\theta^2}\frac{1}{(1/(1-\theta)-R^2)}\right] + \frac{2(1-\rho)\Gamma}{1-\rho\rho_s} + \left(\frac{R-1}{1-\Sigma}\right)\frac{2(1-\rho)(1-\theta)}{1-\rho\rho_\theta} + \left(\frac{R-1}{1-\Sigma}\right)\frac{2\Gamma(1-\theta)}{1-\rho_s\rho_\theta}\right\}}. \tag{59}$$

Using this explicit expression, Figure 7 illustrates the effects of RI on $\rho\left(ca_t, ca_{t+1}\right)$ when $R\omega_\zeta^2/\left(2\vartheta\right) = 0.5$ and $\rho = 0.8$. It clearly shows that given θ, the persistence of the current account is decreasing with λ. In contrast, the effects of θ on the persistence depends on the values of the aggregation factor (λ). When λ is large, (e.g., $\lambda = 1$,) the persistence is decreasing with the degree of RI; when λ is small, (e.g., $\lambda = 0.1$,) the persistence is increasing with the degree of RI. The intuition behind these results is as follows. Given the degree of attention (θ), λ has *no* impact on the covariance between ca_t and ca_{t+1} but increases the variance of the current account, which in turn reduces $\rho\left(ca_t, ca_{t+1}\right)$. It is obvious that RI and RB have the most significant impact on $\rho\left(ca_t, ca_{t+1}\right)$ in the representative agent case ($\lambda = 1$) because the impact of the noise due to RI on the variances of ca_t that appear in the denominator of (59) is largest in this case. In the next section using the calibrated model we show that the aggregate noise can quantitatively improve the model's predictions for the first-order autocorrelation of the current account.

5.1.6 Correlation between the Current Account and Income

Similarly, under RB-RI, the correlation between the current account and net income can be written as:

$$\operatorname{corr}\left(ca_t, y_t\right) = \frac{\frac{(1-\rho)(R-\rho)}{1-\rho^2} + \frac{(R-\rho)\Gamma}{1-\rho\rho_s} + \left(\frac{R-1}{1-\Sigma}\right)\frac{(1-\theta)(R-\rho)}{1-\rho\rho_\theta}}{\sqrt{\frac{(R-\rho)^2}{1-\rho^2}}\sqrt{\begin{array}{c}\frac{1-\rho}{1+\rho} + \frac{\Gamma^2}{1-\rho_s^2} + \left(\frac{R-1}{1-\Sigma}\right)^2\left[\frac{(1-\theta)^2}{1-\rho_\theta^2} + \frac{\theta\lambda^2}{1-\rho_\theta^2}\frac{1}{(1/(1-\theta)-R^2)}\right] \\ + \frac{2(1-\rho)\Gamma}{1-\rho\rho_s} + \left(\frac{R-1}{1-\Sigma}\right)\frac{2(1-\rho)(1-\theta)}{1-\rho\rho_\theta} + \left(\frac{R-1}{1-\Sigma}\right)\frac{2\Gamma(1-\theta)}{1-\rho_s\rho_\theta}\end{array}}}. \tag{60}$$

Using this expression, Figure 8 illustrates the effects of RI on the correlation when $R\omega_\zeta^2/\left(2\vartheta\right) = 0.5$ and $\rho = 0.8$. The figure also shows that given θ, the correlation is increasing with λ. In contrast, the effects of θ on the correlation are complicated and depend on the value of λ. Specifically, when λ is large ($\lambda = 1$), the persistence is decreasing with the degree of RI; when λ is small ($\lambda = 0.1$) the correlation could be increasing with the degree of RI. The intuition behind these results is similar as that for $\rho\left(ca_t, ca_{t+1}\right)$: given θ, λ has *no* impact on the covariance between ca_t and y_t but increases the volatility of the current account, which in turn reduces $\operatorname{corr}\left(ca_t, y_t\right)$.

5.1.7 Implication for Consumption Volatility

Using (55), the relative volatility of aggregate consumption growth relative to income growth can be written as

$$\mu_c = \frac{\mathrm{sd}\left[\Delta c_t\right]}{\mathrm{sd}\left[\Delta y_t\right]} = \frac{\theta\left(R-1\right)}{\left(1-\Sigma\right)\left(R-\rho\right)}\sqrt{\left(\frac{1+\rho}{2}\right)\left(\sum_{j=0}^{\infty}\Gamma_j^2 + \frac{\lambda^2\left(1-\theta\right)}{\theta\left(1-\left(1-\theta\right)R^2\right)}\sum_{j=0}^{\infty}\left(\Gamma_j - R\Gamma_{j-1}\right)^2\right)}, \tag{61}$$

where we use the fact that $\omega_\xi^2 = \mathrm{var}\left[\xi_t\right] = \frac{1-\theta}{\theta(1-(1-\theta)R^2)}\omega_\varepsilon^2$, $\rho_1 = \rho_s = \frac{1-R\Sigma}{1-\Sigma} \in (0,1)$, $\rho_2 = (1-\theta)R \in (0,1)$, and $\Gamma_j = \sum_{k=0}^{j}\left(\rho_1^{j-k}\rho_2^k\right) - \sum_{k=0}^{j-1}\left(\rho_1^{j-1-k}\rho_2^k\right)$, for $j \geq 1$,and $\Gamma_0 = 1$. Figure 9 illustrates how the combination of θ and λ affects the relative volatility of consumption growth to income growth when $R\omega_\zeta^2/(2\vartheta) = 0.5$, $\rho = 0.8$, and $R = 1.04$. It is clear that given θ, the relative volatility μ_c is increasing with λ. The effect of θ on μ_c is not monotonic, and depends on the values of λ. Specifically, When λ is large ($\lambda = 1$), the relative volatility is decreasing with the degree of attention (θ); when λ is small ($\lambda = 0.1$), the relative volatility is decreasing with θ first and then increasing with θ. The intuition behind this result is as follows. Given λ is small, when θ is low, the presence of the common noise, $\lambda\xi_t$, dominates the smoothness of consumption caused by the gradual responses to fundamental shocks; in contrast, when θ is large, the gradual response effect dominates the common noise effect, which reduces the relative volatility.

5.2 Comparing the Implications of Different Models

To illustrate the quantitative implications of the RB-RI model on the stochastic properties of the joint dynamics of consumption, the current account, and net income, we fix the RB parameter at the same levels we obtain in Section 4.4 and vary the two RI parameters, λ and θ.[45] As in Section 4.4, we first set the detection error probability, p, to be a plausible value, 10%. Tables 9-10 compare the model performance under different assumptions (RE, RB, and RB-RI) on matching four important dimensions of the data we documented in Section 2: (1) the contemporaneous correlation between the current account and net income, (2) the volatility of the current account, (3) persistence of the current account, and (4) the relative volatility of consumption growth to income growth. The tables clearly show that RI could help further improve the RB model's predictions along all these four dimensions. Specifically, for emerging countries, in the representative agent case ($\lambda = 1$), when $\theta = 0.5$ (i.e., 50% of the uncertainty regarding permanent income can

[45] The reason why we use the calibrated RB parameter values and vary the two RI parameters is that we want to distinguish the different effects of RB and RI on the model's dynamics. If we use the DEP to calibrate RB in the RB-RI model, it is difficult to separate the different effects of RB and RI within the model. We recalibrated the value of ϑ using the DEP in the RB-RI model and found that it does not change our main conclusions about the effects of RB and RI on the joint dynamics of consumption, the current account, and income. The calibration procedure and results are available from the authors by request.

be removed upon receiving a new signal), the interaction of RB and RI reduces the correlation between the current account and net income from 1 to 0.58; reduces the first-order autocorrelation from 0.8 to 0.36; increases the relative volatility of the current account to income from 0.71 to 0.79, and increases the relative volatility of consumption growth to income growth from 0.28 to 1.36, bringing all of them closer to the data.

We make three comments about this result. First, we have seen that in this case ($\lambda = 1$ and $\theta = 0.5$) the interaction of RB and RI make the model fit the data quite well along dimensions (3) and (4), while also quantitatively improving the model's predictions along dimensions (1) and (2). Second, this improvement does not preclude the model from matching the first two dimensions as well (i.e., the contemporaneous correlation between the current account and net income and the volatility of the current account). For example, holding λ equal to 1 and further reducing θ generates a smaller contemporaneous correlation between the current account and net income which is closer to the data. And holding $\theta = 0.5$ and reducing λ to 0.1 makes the relative volatility of the current account to net income very close to the data. Third, and mostly importantly, all these quantitative results are consistent with the theoretical results we obtained in Section 5.1.

As being mentioned earlier, Figure 1 shows the impulse response functions from the RB and RB-RI model under different parameters values. To have a comparison, Figures 10 and 11 report the empirical IRFs of the current account to the income shocks for all small open economies studied in this paper.[46] As these figures show, the shape of the IRFs are very different among different countries. For example, in some countries (such as those shown in Panel A of Figure 10), the current account responds positively to the income shock, while in some other countries (such as those shown in Panel D of Figure 11), the current account responds negatively to income shocks. And there are also some countries (such as those shown in Panel C of Figure 11) whose current accounts initially respond negatively and then increase to positively before the effects diminish to zero. These (empirical) shapes of IRFs are consistent with those generated by the RB and RB-RI models in Figure 1. Actually, as shown in Figure 1, without RB and RI, the RE model can only generate a positive response of the current account to income shock.[47] But the RB-RI model can help generate more flexible shapes of the IRFs consistent with the data. These results further show that introducing RB and RI into the standard model can help better explain

[46] To get the empirical IRFs, we run the following bivariate VAR:

$$\begin{bmatrix} y_{t+1} \\ ca_{t+1} \end{bmatrix} = A \begin{bmatrix} y_t \\ ca_t \end{bmatrix} + \begin{bmatrix} e_{1,t+1} \\ e_{2,t+1} \end{bmatrix}, \tag{62}$$

where A is a 2×2 coefficient matrix, ca_t and y_t are the detrended current account and net income, and $e_{1,t+1}$ and $e_{2,t+1}$ are the VAR innovations to net income and the current account, respectively. We use a triangular rotation matrix with net income ordered first.

[47] This response is consistent with the empirical evidence reported in Kano (2008) in which he found that the current account in Canada and UK responds positively to a positive transitory country-specific shock to net output initially and monotonically converges to zero in subsequent periods.

the data.

To check how robust these results are, we set the detection error probability to be 0.01 and report the results in Tables `11-12`. From these tables, it is clear that in this case RI can improve the model's predictions on the correlation of the current account and the first-order autocorrelation of the current account. For example, for emerging countries, in the representative agent case ($\lambda = 1$), when $\theta = 95\%$ (the agent can process almost all available information about the state), the combination of RB and RI reduces the correlation between the current account and income to 0.09 and reduces the first-order autocorrelation of the current account to 0.52. It is worth noting that given the high calibrated Σ when $p = 0.01$, the model generates very volatile processes of consumption growth (the relative volatility of consumption growth to income growth increases to 2.09 in this case).

`[Insert Tables 9-12 Here]`

6 Conclusion

We have examined how introducing two types of information imperfections, robustness and rational inattention, into an otherwise standard intertemporal current account model changes the dynamic effects of income shocks on the joint dynamics of consumption and the current account. We have shown that a model with agents who have both a preference for robustness and limited information processing capacity has the potential to better account for the data along a number of dimensions.

The model proposed in this paper can also be used to address the international diversification and consumption correlations puzzles (Backus, Kehoe, and Kydland, 1992). In Luo, Nie, and Young (2011) we show that the RB-RI model reduces the correlation of consumption across countries, and can in fact produce consumption correlations lower than income correlations. RB will lower the international consumption correlations by generating heterogenous responses of consumption to income shocks across countries, provided countries differ in terms of their preference for robustness. The effect of RI on consumption correlations depends on the relative importance of the gradual responses to income shocks versus the effect of the common noise due to limited capacity; the first effect increases the correlations, whereas the second reduces them. Whether the model predicts a low correlation turns out to hinge critically on the extent to which noise shocks are correlated across agents.

In addition, in contrast to the intertemporal consumption approach we consider here, the 'new rule' approach to the current account assigns the preeminent role to portfolio choice (for conflicting views on the relevance of the new rule, see Kraay and Ventura 2000,2003 and Tille and van Wincoop 2010). An interesting extension to our study would be to permit portfolio choice and study the dynamics of the current account in the RB-RI model.

Finally, to explore the mechanisms through which the two informational frictions interact and work, in this paper we have set up the model in a parsimonious way so that we can obtain a closed-form solution. We think that the mechanisms and insights we have explored in this simple framework can be carried over to more general cases. In particular, extending the model to incorporate the global interest rate shock emphasized by Nason and Rogers (2006) and Kano (2009) will be critical for demonstrating conclusively the utility of the RB-RI framework; given that such an extension is nontrivially difficult, we leave it for future work.

7 Appendix

7.1 Solving the Robust ICA Model Analytically

To solve the Bellman equation (13), we conjecture that

$$v(s_t) = -As_t^2 - Bs_t - C,$$

where A, B, and C are undetermined coefficients. Substituting this guessed value function into the Bellman equation gives

$$-As_t^2 - Bs_t - C = \max_{c_t}\min_{\nu_t}\left\{-\frac{1}{2}(\overline{c}-c_t)^2 + \beta E_t\left[\vartheta\nu_t^2 - As_{t+1}^2 - Bs_{t+1} - C\right]\right\}. \tag{63}$$

We can do the min and max operations in any order, so we choose to do the minimization first. The first-order condition for ν_t is

$$2\vartheta\nu_t - 2AE_t\left[\omega_\zeta\nu_t + Rs_t - c_t\right]\omega_\zeta - B\omega_\zeta = 0,$$

which means that

$$\nu_t = \frac{B + 2A(Rs_t - c_t)}{2\left(\vartheta - A\omega_\zeta^2\right)}\omega_\zeta. \tag{64}$$

Substituting (64) back into (63) gives

$$-As_t^2 - Bs_t - C = \max_{c_t}\left\{-\frac{1}{2}(\overline{c}-c_t)^2 + \beta E_t\left[\vartheta\left[\frac{B + 2A(Rs_t - c_t)}{2\left(\vartheta - A\omega_\zeta^2\right)}\omega_\zeta\right]^2 - As_{t+1}^2 - Bs_{t+1} - C\right]\right\},$$

where

$$s_{t+1} = Rs_t - c_t + \zeta_{t+1} + \omega_\zeta\nu_t.$$

The first-order condition for c_t is

$$(\overline{c}-c_t) - 2\beta\vartheta\frac{A\omega_\zeta}{\vartheta - A\omega_\zeta^2}\nu_t + 2\beta A\left(1 + \frac{A\omega_\zeta^2}{\vartheta - A\omega_\zeta^2}\right)(Rs_t - c_t + \omega_\zeta\nu_t) + \beta B\left(1 + \frac{A\omega_\zeta^2}{\vartheta - A\omega_\zeta^2}\right) = 0.$$

Using the solution for ν_t the solution for consumption is

$$c_t = \frac{2A\beta R}{1 - A\omega_\zeta^2/\vartheta + 2\beta A} s_t + \frac{\overline{c}\left(1 - A\omega_\zeta^2/\vartheta\right) + \beta B}{1 - A\omega_\zeta^2/\vartheta + 2\beta A}. \tag{65}$$

Substituting the above expressions into the Bellman equation gives

$$\begin{aligned}
& - As_t^2 - Bs_t - C \\
& = -\frac{1}{2}\left(\frac{2A\beta R}{1 - A\omega_\zeta^2/\vartheta + 2\beta A} s_t + \frac{-2\beta A\overline{c} + \beta B}{1 - A\omega_\zeta^2/\vartheta + 2\beta A}\right)^2 \\
& + \frac{\beta\vartheta\omega_\zeta^2}{\left(2\left(\vartheta - A\omega_\zeta^2\right)\right)^2}\left(\frac{2AR\left(1 - A\omega_\zeta^2/\vartheta\right)}{1 - A\omega_\zeta^2/\vartheta + 2\beta A} s_t + B - \frac{2\overline{c}\left(1 - A\omega_\zeta^2/\vartheta\right)A + 2\beta AB}{1 - A\omega_\zeta^2/\vartheta + 2\beta A}\right)^2 \\
& - \beta A\left(\left(\frac{R}{1 - A\omega_\zeta^2/\vartheta + 2\beta A} s_t - \frac{-B\omega_\zeta^2/\vartheta + 2c + 2B\beta}{2\left(1 - A\omega_\zeta^2/\vartheta + 2\beta A\right)}\right)^2 + \omega_\zeta^2\right) \\
& - \beta B\left(\frac{R}{1 - A\omega_\zeta^2/\vartheta + 2\beta A} s_t - \frac{-B\omega_\zeta^2/\vartheta + 2c + 2B\beta}{2\left(1 - A\omega_\zeta^2/\vartheta + 2\beta A\right)}\right) - \beta C.
\end{aligned}$$

Given $\beta R = 1$, collecting and matching terms, the constant coefficients turn out to be

$$A = \frac{R(R-1)}{2 - R\omega_\zeta^2/\vartheta}, \tag{66}$$

$$B = -\frac{R\overline{c}}{1 - R\omega_\zeta^2/(2\vartheta)}, \tag{67}$$

$$C = \frac{R}{2\left(1 - R\omega_\zeta^2/2\vartheta\right)}\omega_\zeta^2 + \frac{R}{2\left(1 - R\omega_\zeta^2/2\vartheta\right)(R-1)}\overline{c}^2. \tag{68}$$

Substituting (66) and (67) into (65) yields the consumption function (15) in the text. Substituting (15) into the current account identity and using the expression for s_t yields the expression for the current account (17).

7.2 Solving the Robust iCA Model Numerically

To directly compare the two-state HST model and our univariate model, we solve the following two-state HST robust control problem:

$$v(b_t, y_t) = \max_{c_t}\min_{\nu_t}\left\{-\frac{1}{2}(\overline{c} - c_t)^2 + \beta\left[\vartheta\nu_t^2 + E_t\left[v(b_{t+1}, y_{t+1})\right]\right]\right\},$$

subject to the approximating state transition equation:

$$\begin{bmatrix} b_{t+1} \\ y_{t+1} \end{bmatrix} = \begin{bmatrix} R & 1 \\ 0 & \rho \end{bmatrix} \begin{bmatrix} b_t \\ y_t \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} c_t + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \varepsilon_{t+1},$$

which means that the distortion model is

$$\begin{bmatrix} b_{t+1} \\ y_{t+1} \end{bmatrix} = \begin{bmatrix} R & 1 \\ 0 & \rho \end{bmatrix} \begin{bmatrix} b_t \\ y_t \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} c_t + \begin{bmatrix} 0 \\ 1 \end{bmatrix} (\varepsilon_{t+1} + \omega_\varepsilon \nu_t). \tag{69}$$

Following the standard procedure proposed in Hansen and Sargent (Chapters 2 and 10, 2007), we can solve this robust LQ regulator numerically and obtain a linear function of c_t in terms of (b_t, y_t):

$$c_t = \mathrm{MPC}\,(b_t + \lambda y_t)\,,$$

where λ measures the relative importance of net income (y) and financial wealth (b) in determining optimal consumption. In contrast, our model predicts that

$$c_t = \mathrm{MPC}\, s_t,$$

where we ignore the constant term, $\mathrm{MPC} = \frac{R-1}{1-\Sigma}$, $s_t = b_t + \frac{1}{R-\rho} y_t$, and $\Sigma = R\omega_\zeta^2 / (2\vartheta)$. It is easy to check that the MPCs are identical in the two cases and λ in the HST model is just $\frac{1}{R-\rho}$. For example, when $R = 1.04$, $\rho = 0.65$, and $\omega^2 = 1$, the HST model predicts that

$$c_t = 0.0821 b_t + 0.2105 y_t$$

when $1/\vartheta = 0.15$, whereas our model predicts that

$$c_t = 0.0821\,(b_t + 2.5641 y_t) = 0.0821 b_t + 0.2105 y_t$$

for the same value of ϑ (i.e., $\Sigma = 0.5128$).

7.3 Deriving the Joint Dynamics between the Current Account and Income under RB-RI

7.3.1 Deriving the Volatility of the Current Account under RB-RI

Given (56), $ca_t = \frac{1-\rho}{R-\rho} y_t - \frac{\Sigma(R-1)}{1-\Sigma} s_t + \frac{R-1}{1-\Sigma}(s_t - \widehat{s}_t) + \frac{\Sigma}{1-\Sigma}\overline{c}$, we first derive the variance of the current account as follows:

$$
\begin{aligned}
\operatorname{var}(ca_t) &= \operatorname{var}\left(\frac{(1-\rho)\,\zeta_t}{1-\rho\cdot L}+\frac{\Gamma\zeta_t}{1-\rho_s\cdot L}+\frac{R-1}{1-\Sigma}\left(\frac{(1-\theta)\,\zeta_t}{1-\rho_\theta\cdot L}-\frac{\theta E^i[\xi_t]}{1-\rho_\theta\cdot L}\right)\right)\\
&= (1-\rho)^2\frac{\omega_\zeta^2}{1-\rho^2}+\Gamma^2\frac{\omega_\zeta^2}{1-\rho_s^2}+\left(\frac{R-1}{1-\Sigma}\right)^2\operatorname{var}\left(\frac{(1-\theta)\,\zeta_t}{1-\rho_\theta\cdot L}-\frac{\theta E^i[\xi_t]}{1-\rho_\theta\cdot L}\right)\\
&+2\operatorname{cov}\left(\frac{(1-\rho)\,\zeta_t}{1-\rho\cdot L},\frac{\Gamma\zeta_t}{1-\rho_s\cdot L}\right)+2\operatorname{cov}\left(\frac{(1-\rho)\,\zeta_t}{1-\rho\cdot L},\frac{R-1}{1-\Sigma}\left(\frac{(1-\theta)\,\zeta_t}{1-\rho_\theta\cdot L}-\frac{\theta E^i[\xi_t]}{1-\rho_\theta\cdot L}\right)\right)\\
&+2\operatorname{cov}\left(\frac{\Gamma\zeta_t}{1-\rho_s\cdot L},\frac{R-1}{1-\Sigma}\left(\frac{(1-\theta)\,\zeta_t}{1-\rho_\theta\cdot L}-\frac{\theta E^i[\xi_t]}{1-\rho_\theta\cdot L}\right)\right)\\
&= (1-\rho)^2\frac{\omega_\zeta^2}{1-\rho^2}+\Gamma^2\frac{\omega_\zeta^2}{1-\rho_s^2}+\left(\frac{R-1}{1-\Sigma}\right)^2\left[\frac{(1-\theta)^2}{1-\rho_\theta^2}+\frac{\theta\lambda^2}{1-\rho_\theta^2}\frac{1}{(1/(1-\theta)-R^2)}\right]\omega_\zeta^2\\
&+2\frac{(1-\rho)\,\Gamma\omega_\zeta^2}{1-\rho\rho_s}+2(1-\rho)(1-\theta)\left(\frac{R-1}{1-\Sigma}\right)\frac{\omega_\zeta^2}{1-\rho\rho_\theta}+2\Gamma(1-\theta)\left(\frac{R-1}{1-\Sigma}\right)\frac{\omega_\zeta^2}{1-\rho_s\rho_\theta}\\
&= \left\{\begin{array}{c}\frac{1-\rho}{1+\rho}+\frac{\Gamma^2}{1-\rho_s^2}+\left(\frac{R-1}{1-\Sigma}\right)^2\left[\frac{(1-\theta)^2}{1-\rho_\theta^2}+\frac{\theta\lambda^2}{1-\rho_\theta^2}\frac{1}{(1/(1-\theta)-R^2)}\right]\\+\frac{2(1-\rho)\Gamma}{1-\rho\rho_s}+\left(\frac{R-1}{1-\Sigma}\right)\frac{2(1-\rho)(1-\theta)}{1-\rho\rho_\theta}+\left(\frac{R-1}{1-\Sigma}\right)\frac{2\Gamma(1-\theta)}{1-\rho_s\rho_\theta}\end{array}\right\}\omega_\zeta^2. \qquad (70)
\end{aligned}
$$

Using the definition of the relative volatility of the current account and net income, we can obtain (58) in the text.

7.3.2 Deriving the Persistence of the Current Account under RB-RI

Given (56), the first-order autocovariance of the current account can be derived as follows:

$$
\begin{aligned}
&\operatorname{cov}(ca_t, ca_{t+1}) = \operatorname{cov}\left(\begin{array}{c}\frac{(1-\rho)\zeta_t}{1-\rho\cdot L}+\frac{\Gamma\zeta_t}{1-\rho_s\cdot L}+\frac{R-1}{1-\Sigma}\frac{(1-\theta)\zeta_t}{1-\rho_\theta\cdot L},\\ \frac{\rho(1-\rho)\zeta_t}{1-\rho\cdot L}+\frac{\rho_s\Gamma\zeta_t}{1-\rho_s\cdot L}+\frac{R-1}{1-\Sigma}\frac{(1-\theta)\rho_\theta\zeta_t}{1-\rho_\theta\cdot L}\end{array}\right)\\
&= \rho\operatorname{var}\left(\frac{(1-\rho)\,\zeta_t}{1-\rho\cdot L}\right)+\rho_s\operatorname{var}\left(\frac{\Gamma\zeta_t}{1-\rho_s\cdot L}\right)+\rho_\theta\operatorname{var}\left(\frac{R-1}{1-\Sigma}\frac{(1-\theta)\,\zeta_t}{1-\rho_\theta\cdot L}\right)\\
&+(\rho+\rho_s)\operatorname{cov}\left(\frac{(1-\rho)\,\zeta_t}{1-\rho\cdot L},\frac{\Gamma\zeta_t}{1-\rho_s\cdot L}\right)+(\rho+\rho_\theta)\operatorname{cov}\left(\frac{(1-\rho)\,\zeta_t}{1-\rho\cdot L},\frac{R-1}{1-\Sigma}\frac{(1-\theta)\,\zeta_t}{1-\rho_\theta\cdot L}\right)\\
&+(\rho_s+\rho_\theta)\operatorname{cov}\left(\frac{\Gamma\zeta_t}{1-\rho_s\cdot L},\frac{R-1}{1-\Sigma}\frac{(1-\theta)\,\zeta_t}{1-\rho_\theta\cdot L}\right)\\
&= \left[\begin{array}{c}\frac{\rho(1-\rho)}{1+\rho}+\frac{\rho_s\Gamma^2}{1-\rho_s^2}+\rho_\theta\left(\frac{R-1}{1-\Sigma}\right)^2\frac{(1-\theta)^2}{1-\rho_\theta^2}+(\rho+\rho_s)(1-\rho)\,\Gamma\frac{1}{1-\rho\rho_s}\\+(\rho+\rho_\theta)(1-\rho)(1-\theta)\left(\frac{R-1}{1-\Sigma}\right)\frac{1}{1-\rho\rho_\theta}+(\rho_s+\rho_\theta)\,\Gamma(1-\theta)\left(\frac{R-1}{1-\Sigma}\right)\frac{1}{1-\rho_s\rho_\theta}\end{array}\right]\omega_\zeta^2. \qquad (71)
\end{aligned}
$$

Using (70) and (71), we can obtain (59) in the text.

7.3.3 Deriving the Correlation between the Current Account and Net Income under RB-RI

Given (56), the covariance between the current account and net income is

$$\begin{aligned}
\operatorname{cov}(ca_t, y_t) &= \operatorname{cov}\left(\frac{1-\rho}{R-\rho}y_t + \Gamma s_t + \frac{R-1}{1-\Sigma}(s_t - \widehat{s}_t), y_t\right) \\
&= \frac{1-\rho}{R-\rho}\operatorname{var}(y_t) + \Gamma \operatorname{cov}\left(\frac{(R-\rho)\zeta_t}{1-\rho \cdot L}, \frac{\zeta_t}{1-\rho_s \cdot L}\right) + \frac{R-1}{1-\Sigma}\operatorname{cov}\left(\begin{array}{c}\frac{(R-\rho)\zeta_t}{1-\rho \cdot L}, \\ \frac{(1-\theta)\zeta_t}{1-\rho_\theta \cdot L} - \frac{\theta E^i[\xi_t]}{1-\rho_\theta \cdot L}\end{array}\right) \\
&= \left[\frac{(1-\rho)(R-\rho)}{1-\rho^2} + \frac{(R-\rho)\Gamma}{1-\rho\rho_s} + \left(\frac{R-1}{1-\Sigma}\right)\frac{(1-\theta)(R-\rho)}{1-\rho\rho_\theta}\right]\omega_\zeta^2. \qquad (72)
\end{aligned}$$

Using (70) and (72), the correlation between the current account and net income can be written as (60) in the text.

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Figure 1: Responses of Current Account to Income Shock ε

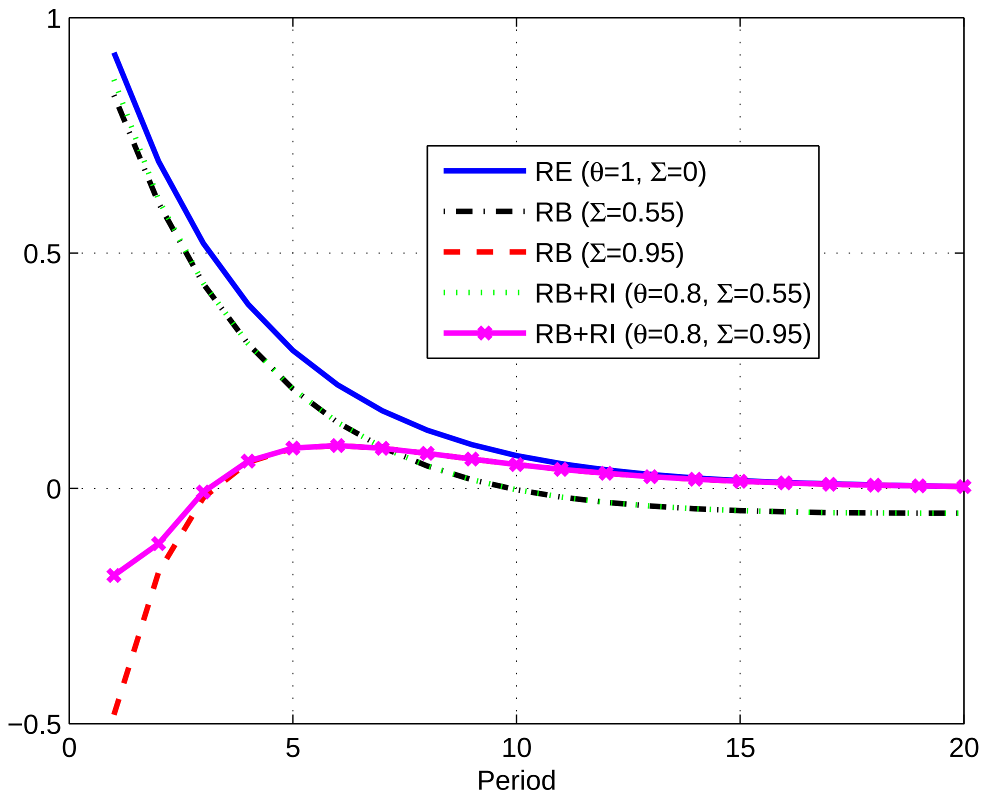


Figure 2: Responses of Consumption to Income Shock ε

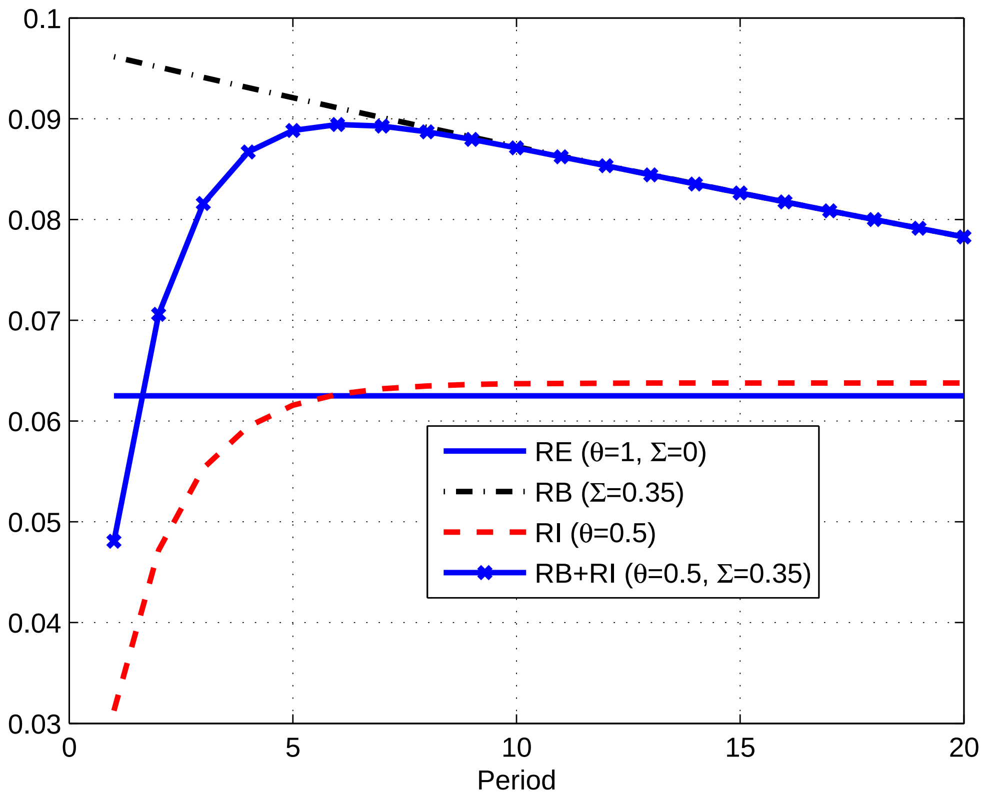


Figure 3: The Relative Volatility of Current Accounts to Net Income under RB

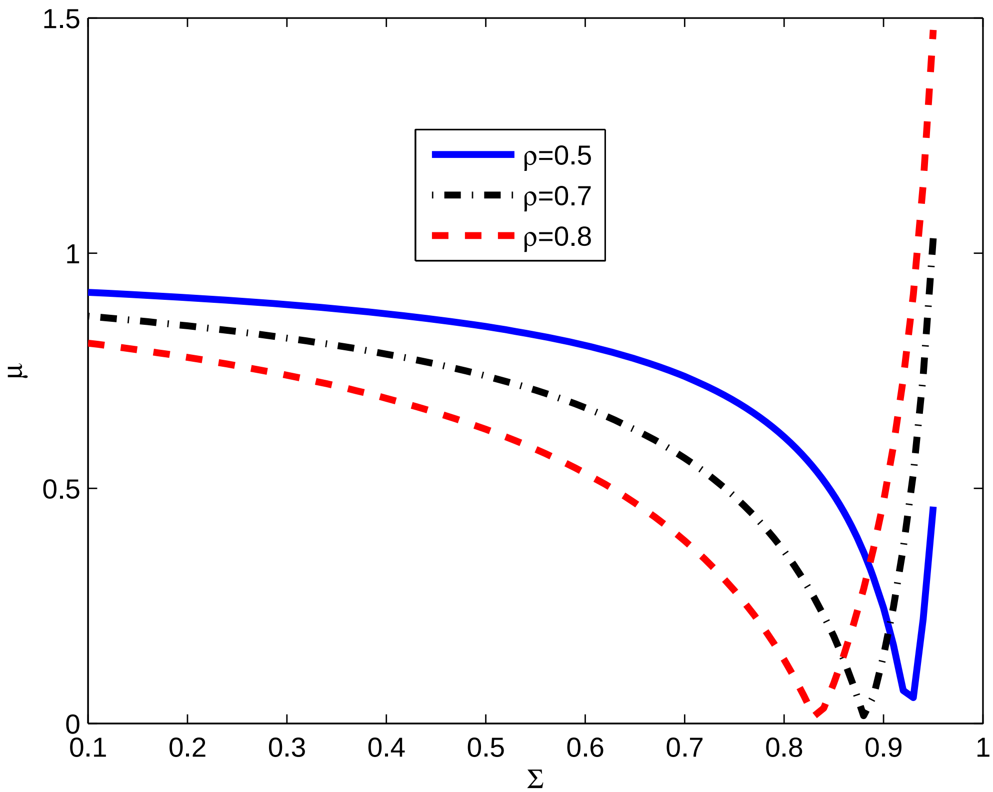


Figure 4: The Persistence of Current Accounts under RB

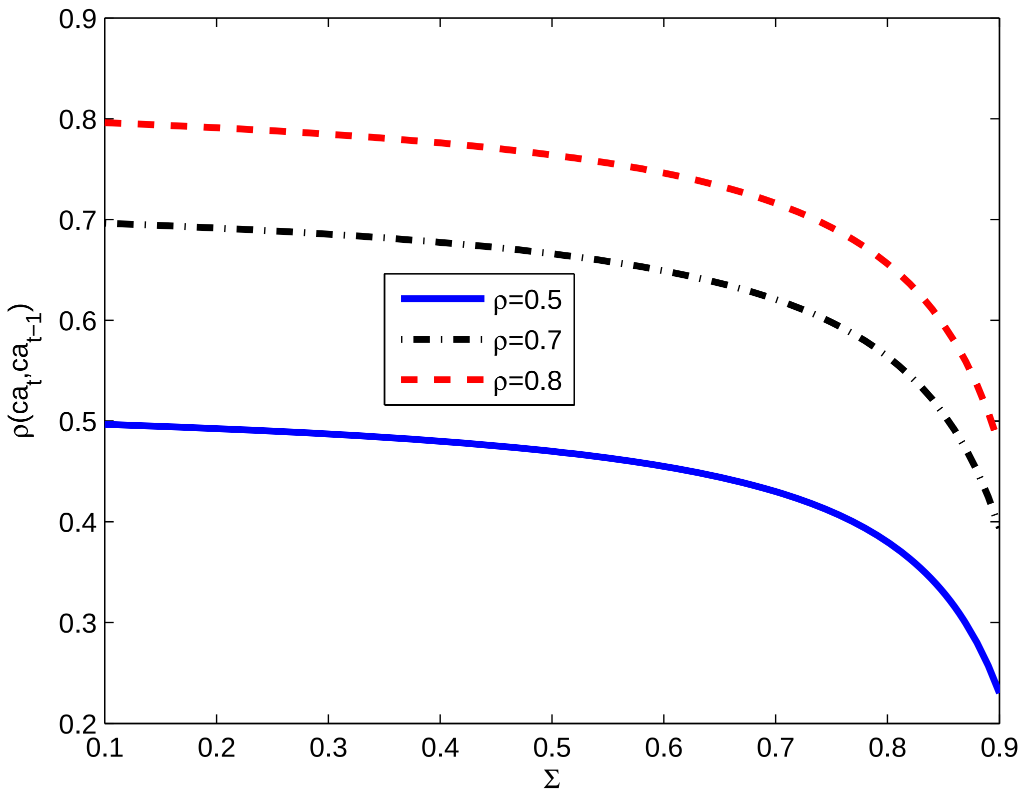


Figure 5: The Correlation between the Current Accounts and Net Income under RB

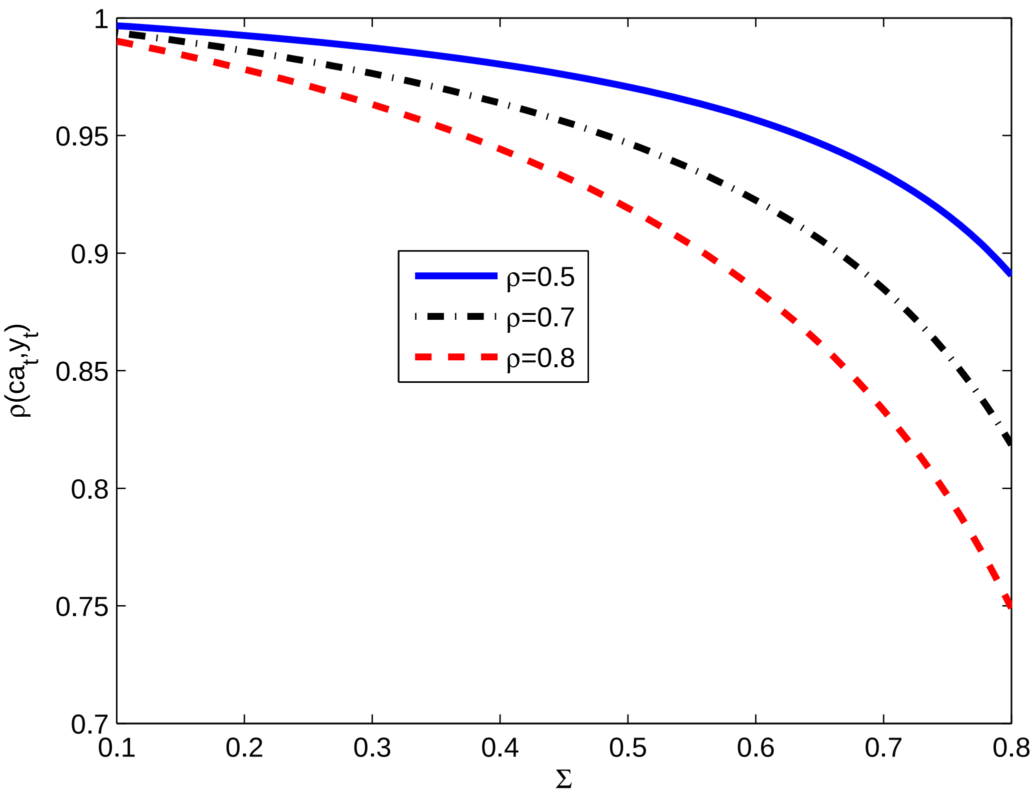


Figure 6: The Relative Volatility of Current Accounts to Net Income under RB and RI

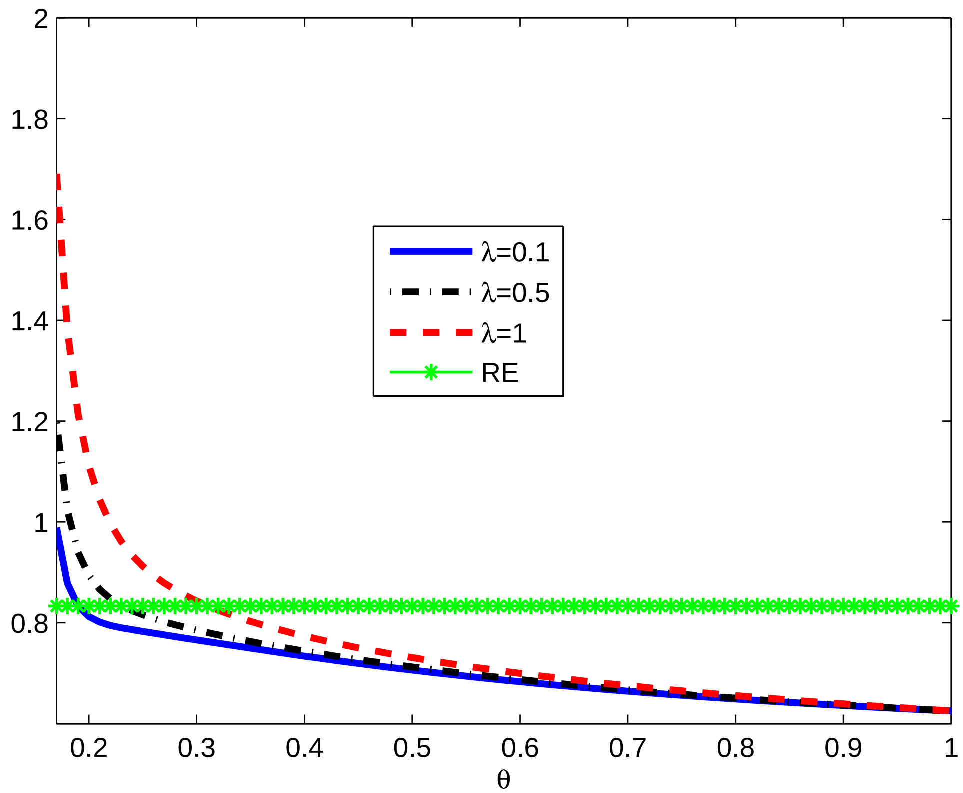


Figure 7: The Persistence of Current Accounts under RB and RI

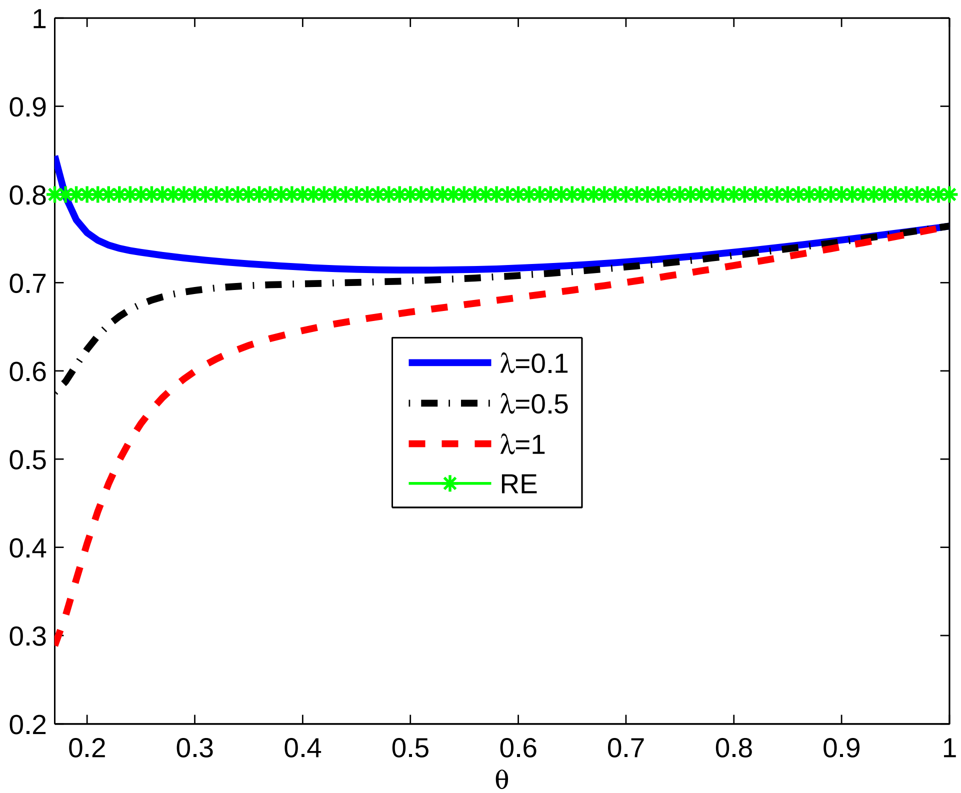


Figure 8: The Correlation between the Current Accounts and Net Income under RB and RI

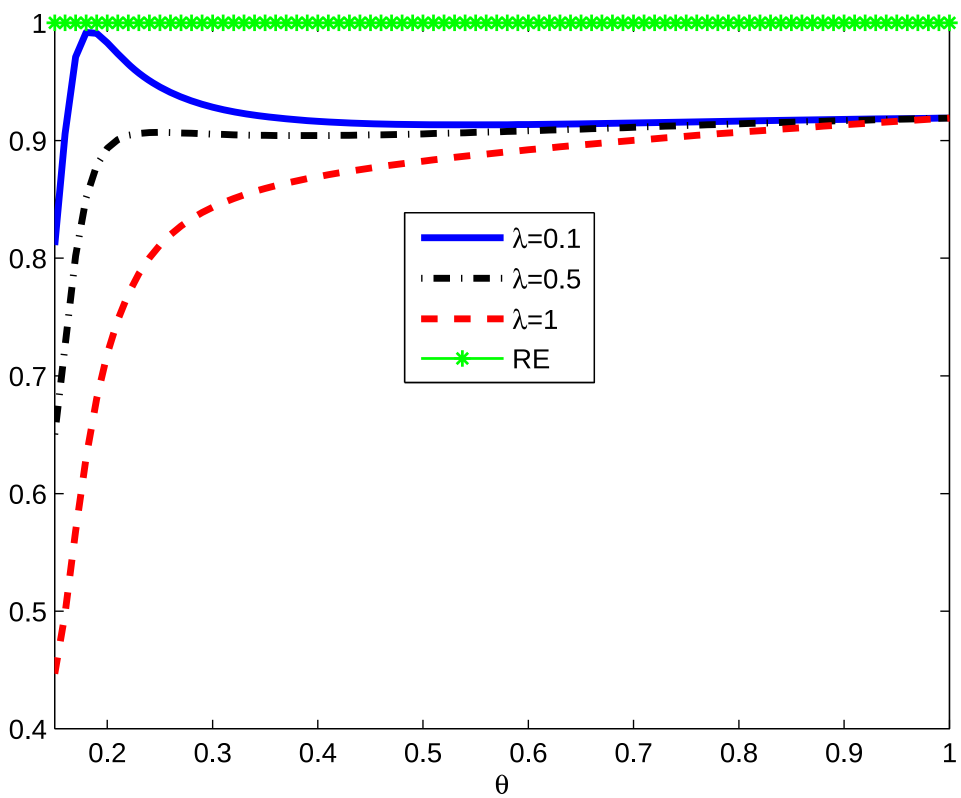


Figure 9: The Relative Volatility of Consumption Growth to Income Growth under RB and RI

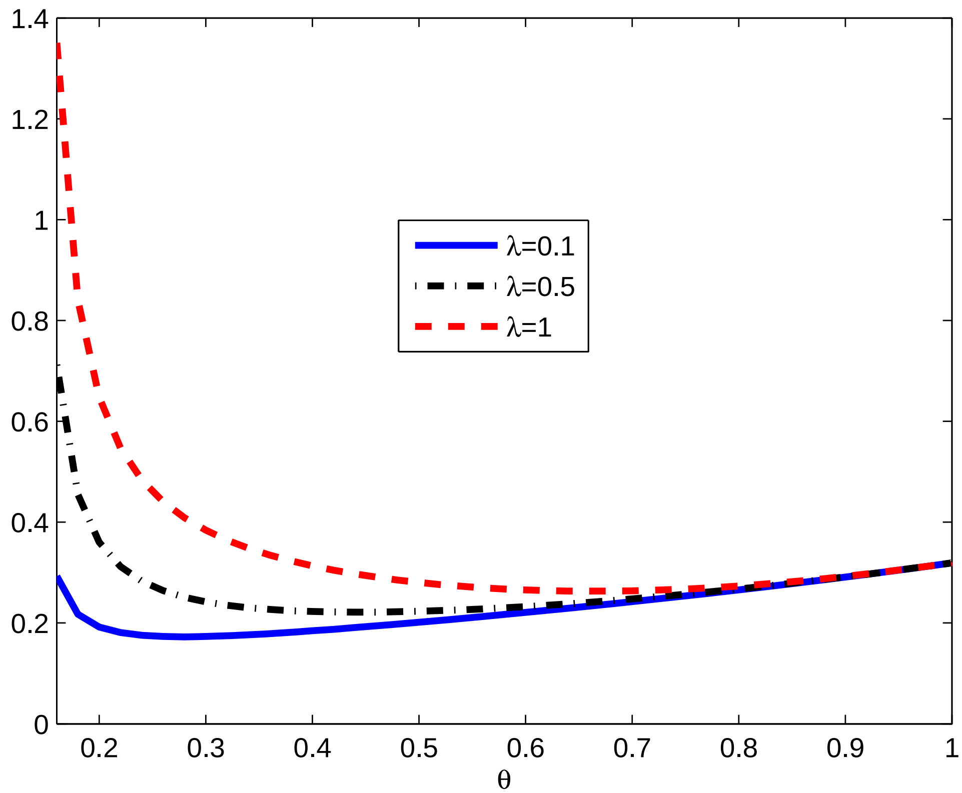
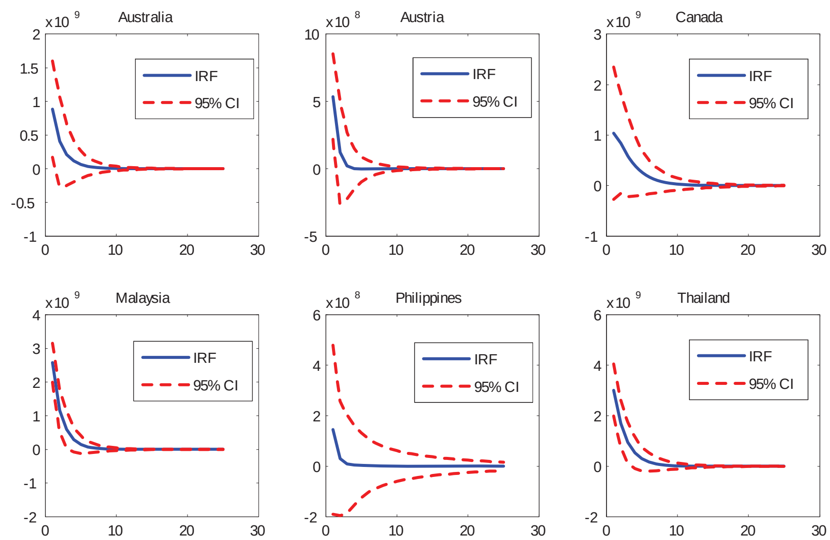


Figure 10: IRFs of the Current Account to Income Shocks

Panel A. **Type I** of IRFs



Panel B. **Type II** of IRFs

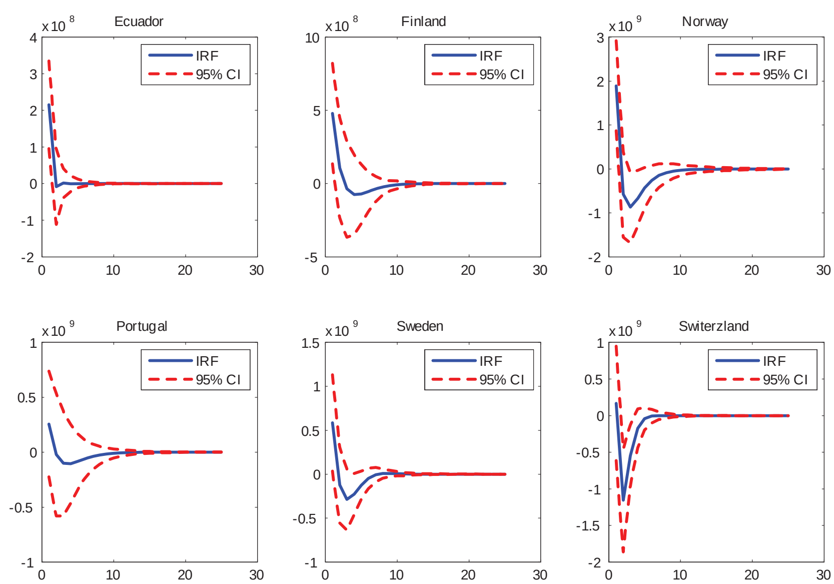
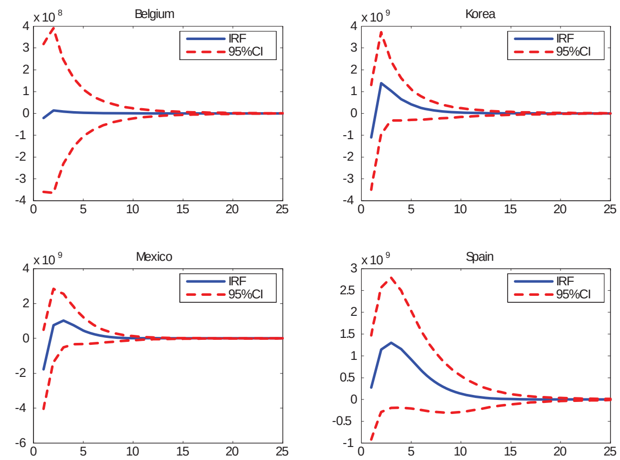
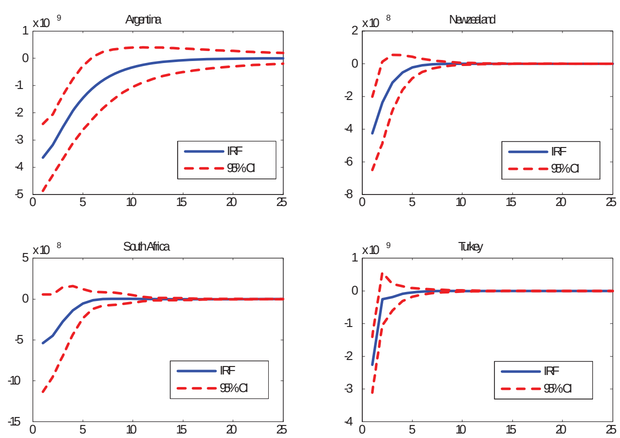


Figure 11: IRFs of the Current Account to Income Shocks

Panel C. **Type III** of IRFs



Panel D. **Type IV** of IRFs



Panel E. **Type V** of IRFs

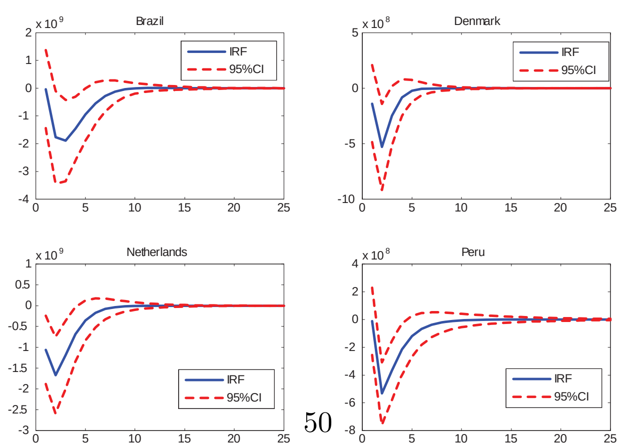


Table 1: Emerging vs. Developed Countries (Averages)

A: Emerging vs. Developed Countries (HP Filter)		
$\sigma(y)/\mu(y)$	3.19(0.20)	1.83(0.07)
$\sigma(\Delta y)/\mu(y)$	3.82(0.19)	2.07(0.06)
$\rho(y_t, y_{t-1})$	0.50(0.03)	0.44(0.03)
$\sigma(\Delta c)/\sigma(\Delta y)$	1.35(0.08)	0.98(0.04)
$\sigma(ca)/\sigma(y)$	1.53(0.09)	1.60(0.08)
$\rho(c, y)$	0.33(0.04)	0.46(0.04)
$\rho(ca_t, ca_{t-1})$	0.30(0.05)	0.41(0.03)
$\rho(ca, y)$	0.05(0.05)	0.06(0.05)
$\rho\left(\frac{ca}{y}, y\right)$	0.04(0.04)	0.15(0.04)
B: Emerging vs. Developed Countries (Linear Filter)		
$\sigma(y)/\mu(y)$	9.03(0.43)	4.37(0.18)
$\sigma(\Delta y)/\mu(y)$	3.82(0.19)	2.07(0.06)
$\rho(y_t, y_{t-1})$	0.80(0.02)	0.79(0.02)
$\sigma(\Delta c)/\sigma(\Delta y)$	1.35(0.08)	0.98(0.04)
$\sigma(ca)/\sigma(y)$	0.80(0.06)	1.35(0.06)
$\rho(c, y)$	0.68(0.04)	0.63(0.04)
$\rho(ca_t, ca_{t-1})$	0.53(0.04)	0.71(0.02)
$\rho(ca, y)$	0.13(0.05)	0.17(0.05)
$\rho\left(\frac{ca}{y}, y\right)$	0.03(0.05)	0.16(0.05)

Table 2: Summary of Statistics: Emerging Countries

	Argentina	Brazil	Ecuador	Korea	Malaysia	Mexico
$\sigma(y)/\mu(y)$	6.01(1.04)	4.73(0.49)	3.86(0.64)	13.65(1.85)	18.01(2.24)	4.01(0.36)
$\sigma(\Delta y)/\mu(y)$	4.51(0.61)	2.72(0.20)	4.95(0.76)	4.02(0.45)	5.55(1.07)	1.82(0.20)
$\rho(y_t, y_{t-1})$	0.71(0.05)	0.83(0.05)	0.01(0.11)	0.95(0.02)	0.95(0.02)	0.90(0.03)
$\sigma(\Delta c)/\sigma(\Delta y)$	1.55(0.05)	1.40(0.14)	0.78(0.25)	1.41(0.35)	0.97(0.14)	2.60(0.56)
$\sigma(ca)/\sigma(y)$	0.65(0.11)	0.73(0.14)	1.85(0.49)	0.47(0.13)	0.86(0.11)	0.88(0.10)
$\rho(c, y)$	0.99(0.00)	0.78(0.09)	−0.28(0.17)	0.38(0.24)	0.74(0.13)	0.90(0.03)
$\rho(ca_t, ca_{t-1})$	0.47(0.14)	0.79(0.06)	−0.18(0.29)	0.40(0.13)	0.80(0.10)	0.60(0.07)
$\rho(ca, y)$	−0.70(0.08)	0.21(0.27)	0.40(0.17)	0.08(0.18)	0.89(0.04)	0.29(0.15)
$\rho\left(\frac{ca}{y}, y\right)$	−0.21(0.16)	0.07(0.18)	0.17(0.15)	−0.01(0.11)	0.28(0.16)	−0.26(0.15)

Table 3: Summary of Statistics: Emerging Countries (*Continued*)

	Peru	Philippines	South Africa	Thailand	Turkey
$\sigma(y)/\mu(y)$	11.89(1.67)	11.95(2.19)	5.24(1.03)	16.00(1.86)	4.05(0.58)
$\sigma(\Delta y)/\mu(y)$	5.06(0.95)	3.54(0.67)	2.39(0.20)	4.78(0.81)	2.62(0.14)
$\rho(y_t, y_{t-1})$	0.91(0.04)	0.96(0.02)	0.89(0.05)	0.95(0.02)	0.76(0.08)
$\sigma(\Delta c)/\sigma(\Delta y)$	1.07(0.07)	0.66(0.08)	1.31(0.31)	1.05(0.14)	2.01(0.25)
$\sigma(ca)/\sigma(y)$	0.45(0.08)	0.37(0.05)	1.05(0.20)	0.69(0.16)	0.79(0.09)
$\rho(c, y)$	0.95(0.02)	0.94(0.03)	0.87(0.05)	0.26(0.33)	0.93(0.03)
$\rho(ca_t, ca_{t-1})$	0.57(0.12)	0.54(0.14)	0.72(0.11)	0.64(0.11)	0.45(0.18)
$\rho(ca, y)$	0.47(0.23)	0.64(0.16)	−0.58(0.15)	0.58(0.13)	−0.80(0.09)
$\rho\left(\frac{ca}{y}, y\right)$	0.32(0.13)	0.31(0.20)	−0.12(0.16)	0.27(0.13)	−0.44(0.11)

Table 4: Summary of Statistics: Developed Countries

	Australia	Austria	Belgium	Canada	Denmark	Finland	Netherlands
$\sigma(y)/\mu(y)$	6.15(0.65)	3.26(0.38)	1.56(0.14)	6.81(0.94)	1.60(0.11)	7.03(1.17)	5.35(0.51)
$\sigma(\Delta y)/\mu(y)$	2.22(0.35)	1.66(0.13)	1.42(0.14)	2.64(0.35)	1.64(0.20)	2.67(0.29)	2.26(0.20)
$\rho(y_t, y_{t-1})$	0.94(0.02)	0.86(0.04)	0.59(0.05)	0.92(0.02)	0.46(0.12)	0.92(0.02)	0.91(0.02)
$\sigma(\Delta c)/\sigma(\Delta y)$	0.93(0.13)	0.95(0.11)	0.82(0.13)	0.71(0.11)	1.39(0.23)	0.91(0.17)	0.79(0.12)
$\sigma(ca)/\sigma(y)$	0.47(0.08)	0.97(0.14)	1.99(0.34)	0.55(0.04)	1.79(0.19)	0.86(0.17)	0.74(0.15)
$\rho(c, y)$	0.75(0.07)	0.21(0.26)	0.51(0.12)	0.76(0.06)	0.02(0.25)	0.62(0.19)	0.91(0.03)
$\rho(ca_t, ca_{t-1})$	0.55(0.08)	0.73(0.10)	0.87(0.03)	0.81(0.08)	0.50(0.15)	0.84(0.06)	0.71(0.06)
$\rho(ca, y)$	−0.10(0.20)	0.50(0.19)	−0.40(0.18)	0.81(0.09)	0.00(0.14)	0.44(0.20)	0.21(0.23)
$\rho\left(\frac{ca}{y}, y\right)$	0.32(0.17)	0.20(0.20)	0.02(0.14)	0.06(0.14)	0.16(0.12)	0.33(0.18)	0.03(0.15)

Table 5: Summary of Statistics: Developed Countries (*Continued*)

	New Zealand	Norway	Portugal	Spain	Sweden	Switerland
$\sigma(y)/\mu(y)$	5.53(0.69)	3.20(0.19)	2.32(0.32)	3.36(0.28)	8.48(1.31)	2.09(0.29)
$\sigma(\Delta y)/\mu(y)$	2.44(0.22)	2.67(0.24)	1.88(0.23)	1.13(0.14)	2.44(0.23)	1.80(0.16)
$\rho(y_t, y_{t-1})$	0.90(0.03)	0.66(0.08)	0.68(0.07)	0.94(0.02)	0.96(0.01)	0.57(0.10)
$\sigma(\Delta c)/\sigma(\Delta y)$	1.10(0.14)	0.67(0.09)	1.28(0.20)	1.95(0.23)	0.81(0.10)	0.40(0.06)
$\sigma(ca)/\sigma(y)$	0.91(0.14)	2.54(0.45)	3.02(0.34)	1.49(0.29)	0.65(0.04)	1.53(0.21)
$\rho(c, y)$	0.91(0.03)	0.67(0.08)	0.34(0.18)	0.86(0.04)	0.84(0.08)	0.74(0.10)
$\rho(ca_t, ca_{t-1})$	0.50(0.11)	0.61(0.09)	0.79(0.06)	0.88(0.04)	0.90(0.03)	0.49(0.15)
$\rho(ca, y)$	−0.39(0.16)	0.43(0.17)	0.03(0.18)	−0.60(0.10)	0.92(0.03)	0.32(0.17)
$\rho\left(\frac{ca}{y}, y\right)$	−0.07(0.12)	0.27(0.18)	0.06(0.21)	0.01(0.17)	0.34(0.17)	0.31(0.17)

Table 6: Emerging vs. Developed Countries (Averages, $p = 0.1$)

	Emerging Countries	Developed Countries
Σ	0.524	0.205
p	0.100	0.100
ρ	0.802	0.793
$\frac{\sigma(y)}{\mu(y)}$	0.090	0.044
$\frac{\sigma(\zeta)}{\mu(y)}$	0.284	0.132

Table 7: Emerging Countries

	Arg	Bra	Ecu	Kor	Mal	Mex	Per	Phi	Sou	Tha	Tur
Σ	0.749	0.397	0.281	0.470	0.424	0.554	0.684	0.871	0.274	0.631	0.427
p	0.100	0.101	0.100	0.099	0.100	0.099	0.101	0.101	0.101	0.100	0.101
ρ	0.705	0.825	0.014	0.952	0.949	0.896	0.907	0.958	0.894	0.953	0.764
$\frac{\sigma(y)}{\mu(y)}$	0.060	0.047	0.039	0.137	0.180	0.040	0.119	0.119	0.052	0.160	0.040
$\frac{\sigma(\zeta)}{\mu(y)}$	0.127	0.124	0.038	0.475	0.624	0.124	0.377	0.418	0.161	0.557	0.095

Table 8: Developed Countries

	Aus	Aut	Bel	Can	Den	Fin	Net	New	Nor	Por	Spa	Swe	Swi
Σ	0.385	0.158	0.038	0.307	0.027	0.225	0.163	0.355	0.052	0.303	0.274	0.247	0.125
p	0.100	0.100	0.100	0.100	0.101	0.101	0.099	0.100	0.100	0.100	0.100	0.099	0.100
ρ	0.935	0.864	0.587	0.924	0.459	0.924	0.912	0.901	0.656	0.678	0.943	0.957	0.570
$\frac{\sigma(y)}{\mu(y)}$	0.062	0.033	0.016	0.068	0.016	0.070	0.054	0.055	0.032	0.023	0.034	0.085	0.021
$\frac{\sigma(\zeta)}{\mu(y)}$	0.208	0.093	0.028	0.224	0.024	0.232	0.172	0.173	0.063	0.047	0.115	0.296	0.037

Table 9: Implications of Different Models (Emerging Countries, $p = 0.1$)

	Data	RE	RB	RB+RI ($\theta = 0.9$)	RB+RI ($\theta = 0.8$)	RB+RI ($\theta = 0.7$)	RB+RI ($\theta = 0.5$)
				($\lambda = 1$)			
$\rho(ca, y)$	0.13	1.00	0.62	0.57	0.56	0.56	0.58
$\rho(ca_t, ca_{t-1})$	0.53	0.80	0.74	0.57	0.50	0.45	0.36
$\sigma(ca)/\sigma(y)$	0.80	0.71	0.49	0.52	0.55	0.59	0.79
$\sigma(\Delta c)/\sigma(\Delta y)$	1.35	0.28	0.90	0.89	0.89	0.91	1.36
				($\lambda = 0.5$)			
$\rho(ca, y)$	0.13	1.00	0.62	0.59	0.58	0.59	0.64
$\rho(ca_t, ca_{t-1})$	0.53	0.80	0.74	0.63	0.59	0.55	0.46
$\sigma(ca)/\sigma(y)$	0.80	0.71	0.49	0.50	0.52	0.53	0.64
$\sigma(\Delta c)/\sigma(\Delta y)$	1.35	0.28	0.90	0.85	0.81	0.79	0.99
				($\lambda = 0.1$)			
$\rho(ca, y)$	0.13	1.00	0.62	0.61	0.60	0.61	0.67
$\rho(ca_t, ca_{t-1})$	0.53	0.80	0.74	0.67	0.64	0.62	0.56
$\sigma(ca)/\sigma(y)$	0.80	0.71	0.49	0.49	0.50	0.51	0.57
$\sigma(\Delta c)/\sigma(\Delta y)$	1.35	0.28	0.90	0.84	0.79	0.75	0.82

Table 10: Implications of Different Models (Developed Countries,$p = 0.1$)

	Data	RE	RB	RB+RI ($\theta = 0.9$)	RB+RI ($\theta = 0.6$)	RB+RI ($\theta = 0.3$)	RB+RI ($\theta = 0.1$)
				($\lambda = 1$)			
$\rho(ca, y)$	0.17	1.00	0.94	0.94	0.91	0.87	0.83
$\rho(ca_t, ca_{t-1})$	0.71	0.79	0.78	0.76	0.70	0.64	0.58
$\sigma(ca)/\sigma(y)$	1.35	0.75	0.64	0.65	0.69	0.79	0.89
$\sigma(\Delta c)/\sigma(\Delta y)$	0.98	0.24	0.33	0.31	0.26	0.21	0.21
				($\lambda = 0.5$)			
$\rho(ca, y)$	0.17	1.00	0.94	0.94	0.93	0.91	0.90
$\rho(ca_t, ca_{t-1})$	0.71	0.79	0.78	0.77	0.73	0.71	0.70
$\sigma(ca)/\sigma(y)$	1.35	0.75	0.64	0.64	0.68	0.76	0.82
$\sigma(\Delta c)/\sigma(\Delta y)$	0.98	0.24	0.33	0.30	0.23	0.17	0.16
				($\lambda = 0.1$)			
$\rho(ca, y)$	0.17	1.00	0.94	0.94	0.93	0.93	0.94
$\rho(ca_t, ca_{t-1})$	0.71	0.79	0.78	0.77	0.74	0.74	0.76
$\sigma(ca)/\sigma(y)$	1.35	0.75	0.64	0.64	0.68	0.75	0.79
$\sigma(\Delta c)/\sigma(\Delta y)$	0.98	0.24	0.33	0.30	0.22	0.16	0.14

Table 11: Implications of Different Models (Emerging Countries, $p = 0.01$)

	Data	RE	RB	RB+RI ($\theta = 0.95$)	RB+RI ($\theta = 0.9$)	RB+RI ($\theta = 0.85$)	RB+RI ($\theta = 0.8$)
				($\lambda = 1$)			
$\rho(ca, y)$	0.13	1.00	-0.01	0.09	0.13	0.15	0.18
$\rho(ca_t, ca_{t-1})$	0.53	0.80	0.64	0.52	0.46	0.42	0.37
$\sigma(ca)/\sigma(y)$	0.80	0.71	0.61	0.65	0.69	0.74	0.82
$\sigma(\Delta c)/\sigma(\Delta y)$	1.35	0.28	2.00	2.09	2.22	2.44	2.84
				($\lambda = 0.5$)			
$\rho(ca, y)$	0.13	1.00	-0.01	0.07	0.10	0.12	0.15
$\rho(ca_t, ca_{t-1})$	0.53	0.80	0.64	0.58	0.55	0.52	0.48
$\sigma(ca)/\sigma(y)$	0.80	0.71	0.61	0.63	0.65	0.67	0.73
$\sigma(\Delta c)/\sigma(\Delta y)$	1.35	0.28	2.00	2.03	2.08	2.19	2.42
				($\lambda = 0.1$)			
$\rho(ca, y)$	0.13	1.00	-0.01	0.05	0.08	0.12	0.15
$\rho(ca_t, ca_{t-1})$	0.53	0.80	0.64	0.65	0.64	0.63	0.61
$\sigma(ca)/\sigma(y)$	0.80	0.71	0.61	0.62	0.63	0.65	0.69
$\sigma(\Delta c)/\sigma(\Delta y)$	1.35	0.28	2.00	2.00	2.03	2.10	2.27

Table 12: Implications of Different Models (Developed Countries,$p = 0.01$)

	Data	RE	RB	RB+RI ($\theta = 0.9$)	RB+RI ($\theta = 0.6$)	RB+RI ($\theta = 0.3$)	RB+RI ($\theta = 0.2$)
				($\lambda = 1$)			
$\rho(ca, y)$	0.17	1.00	0.90	0.85	0.79	0.75	0.72
$\rho(ca_t, ca_{t-1})$	0.71	0.79	0.77	0.66	0.56	0.49	0.43
$\sigma(ca)/\sigma(y)$	1.35	0.75	0.54	0.55	0.62	0.78	1.11
$\sigma(\Delta c)/\sigma(\Delta y)$	0.98	0.24	0.43	0.41	0.35	0.32	0.49
				($\lambda = 0.5$)			
$\rho(ca, y)$	0.17	1.00	0.90	0.87	0.84	0.83	0.84
$\rho(ca_t, ca_{t-1})$	0.71	0.79	0.77	0.69	0.62	0.60	0.59
$\sigma(ca)/\sigma(y)$	1.35	0.75	0.54	0.55	0.60	0.70	0.89
$\sigma(\Delta c)/\sigma(\Delta y)$	0.98	0.24	0.43	0.40	0.31	0.25	0.32
				($\lambda = 0.1$)			
$\rho(ca, y)$	0.17	1.00	0.90	0.89	0.86	0.88	0.94
$\rho(ca_t, ca_{t-1})$	0.71	0.79	0.77	0.72	0.66	0.70	0.75
$\sigma(ca)/\sigma(y)$	1.35	0.75	0.54	0.54	0.59	0.67	0.79
$\sigma(\Delta c)/\sigma(\Delta y)$	0.98	0.24	0.43	0.39	0.29	0.22	0.24