Ambiguity, Information Processing, and Financial Intermediation

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Abstract: This paper incorporates ambiguity and information processing constraints into the He and Krishnamurthy (2012) model of intermediary asset pricing. Financial intermediaries possess greater information processing capacity than households. In response, households optimally choose to delegate their investment decisions. The contractual relationship between households and intermediaries is subject to a moral hazard friction, which results in a financial constraint. We show that ambiguity aversion not only amplifies households' incentives to delegate but also tightens the financial constraint. The calibrated model can quantitatively explain both the unconditional and time-varying moments of observed asset prices, while endogenously generating an empirically consistent crisis frequency.

Keywords: Ambiguity, Rational Inattention, Portfolio Delegation, Intermediary Asset Pricing, Financial Crisis.
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1 Introduction

The intermediary asset pricing literature, pioneered by He and Krishnamurthy (2012, 2013) and Brunnermeier and Sannikov (2014), argues that financial intermediaries play a first-order role in determining asset prices, primarily due to market segmentation that limits household participation. However, since households face few explicit constraints to participating in most financial markets, some observers have questioned the relevance of the intermediary asset pricing literature. For example, Cochrane (2017) remarks,

> *Furthermore, if there is such an extreme agency problem, that delegated managers were selling during the buying opportunity of a generation, why do fundamental investors put up with it? Why not invest directly, or find a better contract?* (Cochrane (2017, p. 963))

We respond to Cochrane's skepticism by providing a micro-foundation for why households delegate. Specifically, we suppose both households and specialists face limits on their ability to process information, giving rise to rational inattention (Sims (2003)). Specialists who run the intermediaries are assumed to have greater information processing capacity. Households can effectively purchase this additional capacity by delegating their portfolio decisions to intermediaries.

Besides introducing rational inattention, we also assume that agents in the He and Krishnamurthy (2012) [henceforth HK] model are ambiguity averse (or equivalently, have preferences for robustness as in Hansen and Sargent (2008)). We do this for two reasons. First, delegation only occurs when households value information precision due to ambiguity aversion. We demonstrate that combining log utility in HK with ambiguity aversion provides a sufficient preference condition for portfolio delegation. Moreover, ambiguity interacts with rational inattention, thereby amplifying households' incentives to delegate. Second, incorporating ambiguity allows our model to yield a stationary wealth distribution, enabling quantitative assessments of the data. In contrast, the HK model features a degenerate wealth distribution in which specialists ultimately dominate. We show that ambiguity aversion tightens the financial constraint of intermediaries and magnifies its effects. Our calibrated model can quantitatively account for both the unconditional and time-varying moments of asset returns, with empirically plausible concerns for robustness.

The primary contribution of this paper is to endogenize households' optimal delegation decision. An important difference between our model and the HK model is that in our model, the mean dividend growth is stochastic and unobserved. Agents must therefore solve a filtering (learning) problem. Ambiguity aversion and rational inattention both influence this filtering problem. Following Hansen and Sargent (2011), we operationalize robust filtering by supposing agents distrust their priors, and so introduce a pessimistic drift distortion into the Kalman filter. At the same time, the rate at which they can learn is constrained by their information processing capacity. Greater capacity accelerates learning and produces a lower steady-state estimation risk (i.e., conditional variance of the unobserved state). Households, being less efficient in processing information, experience higher estimation risk. With ambiguity, this leads to a welfare loss, motivating households to optimally delegate their investments to specialists who possess greater information capacity. Households are willing to pay a lump-sum delegation fee to reduce estimation risk. We show that a small difference in information capacity can rationalize an empirically plausible upfront delegation fee.

Interestingly, we also show that ambiguity aversion and rational inattention *interact*. In the absence of ambiguity, households would not choose to delegate, as seen in the HK model with log utility, where estimation risk becomes irrelevant. Therefore, the HK model makes the extreme assumption that households *must* delegate to intermediaries to invest in risky assets. However, in our model, the introduction of ambiguity concerning the unobserved state effectively motivates households to delegate. Furthermore, ambiguity aversion amplifies households' incentives to delegate. They are willing to pay more when they are more ambiguity averse.

Our second contribution is empirical. We show that our model can quantitatively match observed data. In order to bring the model to the data in a meaningful way, it is crucial for the model to feature a stationary wealth distribution. Unfortunately, the HK model results in a degenerate distribution under empirically relevant parameter assumptions; that is, specialists are more patient than households.[1] Consequently, in the long run, specialists end up accumulating all the wealth. The assumption that specialists are more patient is required to generate a procyclical price-to-dividend ratio under log utility in HK. We adopt the same assumption to account for this observed state dependency.[2] However, introducing ambiguity about asset returns (or concerns about model misspecification) enables our model to possess a stationary wealth distribution. Despite both households and specialists having the same degree of ambiguity aversion, the impact of ambiguity aversion is inversely scaled by the time preference. Specialists become effectively more ambiguity averse since they care more about the future. As a result, their strategically pessimistic drift distortions are greater than those of households. In our model, this pessimism leads to more conservative portfolio strategies, causing specialists to invest less of their wealth in risky assets. We demonstrate that with reasonable parameter values, specialists' relatively greater pessimism offsets their greater patience, resulting in a stationary wealth distribution (Yan (2008)).

We show that ambiguity tightens the financial constraint and amplifies its impact due to state-dependent belief differences. The key ingredient in the HK model is that the delegation contract is subject to a moral hazard problem, resulting in a capital constraint faced by intermediaries, which requires specialists to maintain a minimum amount of 'skin in the game.' In contrast, in our model, belief differences are state-dependent. They widen during crises as the specialist's wealth

[1]Nonstationarity of the wealth distribution is endemic to intermediary asset pricing models since they are based on differences across agents. Many devices have been used to enforce a stationary distribution. Perhaps the most common one is simply assuming that specialists die off at an exogenous rate (Bernanke, Gertler, and Gilchrist (1999)). He and Krishnamurthy (2013) have identical time preferences but assume that households die off instantaneously.

[2]Intermediary asset pricing models generate cyclicality in risk premia by supposing that agents value assets differently, and that during downturns assets are reallocated to relatively low valuation agents. Typically this occurs because relatively high valuation agents are more exposed to higher yielding risky assets, so their relative wealth declines during downturns. Brunnermeier and Sannikov (2014) model valuation differences by simply assuming that assets yield higher returns when held by specialists. In contrast, HK model valuation differences by assuming that specialists have a relatively low rate of time preference.

declines, leading to a more binding capital constraint and an increase in specialists' relative risk exposure. Since subjective model uncertainty 'hides behind' objective risk, the increased leverage of specialists makes them endogenously more pessimistic than households. This relative pessimism drives up risk premia during crises.

Finally, we demonstrate that our model is capable of capturing both unconditional and time-varying asset returns, while endogenously generating an empirically consistent crisis frequency and persistence. Although we are able to derive explicit expressions for asset prices and the distribution of wealth, these processes are highly nonlinear and feature an occasionally and endogenously binding constraint, which makes the model challenging to fit to the data using conventional methods. In response, we use the simulation-based methodology of 'indirect inference' (Gourieroux, Monfort, and Renault (1993)). We find that the model performs well in matching unconditional moments. It matches upfront delegation fees, the mean equity premium, the Sharpe ratio, the mean and unconditional volatility of the risk-free rate, and the long-run frequency of crises. The model falls somewhat short in capturing state dependence. Although it can replicate the persistence of the equity premium and the price-to-dividend ratio, it generates only about a 150 basis point increase in the equity premium during crises and understates movements in the price-to-dividend ratio. We suspect the issue here is that our model-implied relative wealth variable is a poor proxy for the financial sector's capital. To check this, we use updated data from He, Kelly, and Manela (2017), who construct market equity capital ratios for approximately two dozen New York Fed primary dealers for the period 1970.01–2022.12. These firms include JP Morgan, Goldman Sachs, and Citigroup. With this data, the model generates greater fluctuations in risk premia and the price-to-dividend ratio. Finally, following Hansen and Sargent (2008), we discipline the degree of pessimism in our model by requiring that the agents' doubts be empirically plausible. In other words, all our calibrated robustness parameters have detection error probabilities (DEPs) greater than 10%.

Literature Review Our paper contributes to three branches of literature. First, it is connected to developments in intermediary asset pricing within the macro-finance literature. For example, He and Krishnamurthy (2012, 2013), Brunnermeier and Sannikov (2014), Gertler and Kiyotaki (2015), Adrian and Boyarchenko (2012), Vandeweyer and d'Avernas (2023) develop related theoretical models, with He and Krishnamurthy (2018) providing an extensive survey. In contrast to those models, which either assume segmented markets or exogenously low 'productivity' for households in holding capital, our paper establishes a micro-foundation for delegation based on information frictions. Our model implies that these frictions are more pronounced in more complex asset markets, which is consistent with the empirical evidence in Haddad and Muir (2021). Our paper is also related to the literature that explores the influence of beliefs on intermediary asset returns, including studies by Krishnamurthy and Li (2021), Li (2023), and Gertler, Kiyotaki, and Prestipino (2020). Unlike these models, where crises typically result from exogenous jump risks, our model generates crises endogenously through belief heterogeneity driven by ambiguity, leading to a nonlinear amplification effect during financial downturns. In our model, while crisis episodes may be initially triggered by negative shocks, these shocks are not themselves disasters but instead become amplified. This distinguishes our approach from the rare disasters literature such as Rietz (1988) and Barro (2006).

Second, our paper relates to the literature on delegated asset management. As the share of financial assets held by institutional investors has significantly increased (Lewellen (2011), Greenwood and Scharfstein (2013)), economists have debated whether professional asset managers possess skills. Some studies, such as French (2008), argue that passive market portfolios outperform actively managed ones, while others, such as Berk and Green (2004) and Berk and Van Binsbergen (2015), emphasize the skills of managers. However, their focus is solely on the heterogeneity of specialist skills, while our model centers on delegation driven by differences in skills between households and managers. Furthermore, these papers do not address the source of skill heterogeneity, which we attribute to information processing constraints. Our paper also differs from Taylor and Verrecchia (2015) and Huang, Qiu, and Yang (2020), where delegation occurs because intermediaries have access to private information. In our model, specialists have access to the same publicly available information as households, but they have a higher information processing capacity. Moreover, in our model, delegation is driven by the second moment of beliefs rather than the first moment. Kaniel and Kondor (2013) study the interaction of delegation with manager incentives and asset prices. Unlike our model, their model does not incorporate information; instead, delegation incurs a unit cost that is based on the past performance of managers. Our paper shares similarities with these studies in motivating delegation by information acquisition costs, but provides a micro-foundation for the costs based on rational inattention. Importantly, we emphasize the delegation incentives reinforced by ambiguity.

In terms of modeling strategy, our paper is related to the burgeoning literature on rational inattention (Sims (2003)) and heterogenous information. For example, Pagel (2018) and Andries and Haddad (2020) also base portfolio delegation on inattention, where inattention is based on information avoidance, while Gennaioli, Shleifer, and Vishny (2015) is based on trust. In contrast, in our model, attention is determined by information processing limits. Kacperczyk, Van Nieuwerburgh, and Veldkamp (2016) and Kacperczyk, Nosal, and Stevens (2019) examine the optimal allocation of attention over business cycles, where sophisticated investors have higher channel capacity. However, they do not allow agents to buy and sell this capacity. We could also interpret a specialist's information processing capacity as the 'expertise' of financial intermediaries, as in Eisfeldt, Lustig, and Zhang (2023) and Goldstein and Yang (2015). Our paper implies that agents endowed with higher capacities become specialists. Finally, Ordonez (2013) combines endogenous asymmetric information with financial frictions to explain cross-country variations in interest rates and output. He shows that countries facing higher financial frictions exhibit increased asymmetry. While the mechanisms differ, many of his results are similar to the implications of our model.

Third, our work is related to the vast literature on asset pricing under Knightian Uncertainty. Epstein and Schneider (2010) survey the early literature. The early literature followed Gilboa and Schmeidler (1989) by positing a fixed set of priors. This can result in inaction, as an agent's beliefs remain pinned at the boundary of the set (Dow and Werlang (1992)). Easley and O'Hara (2009) use this framework to study how regulations that mitigate ambiguity can encourage market participation. Hansen and Sargent (2008) refer to these as 'constraint preferences.' More recent literature focuses on what Hansen and Sargent call 'multiplier preferences,' parameterized by the Lagrange Multiplier on the set. Here the set of priors is not fixed. Klibanoff, Marinacci, and Mukerji (2005) provide an axiomatization and call it 'smooth ambiguity.' This is the approach we pursue here. It has the advantage of linking the ambiguity parameter to statistical decision theory. Our contribution is to combine ambiguity with rational inattention and intermediation frictions among heterogeneous agents. Hansen, Miao, and Xing (2022) and Luo and Young (2016) combine robustness and rational inattention in a discrete-choice setup. Bhandari, Borovička, and Ho (2019) and Maenhout, Vedolin, and Xing (2021) provide survey evidence supporting robustness-induced belief distortions. Illeditsch (2011) and Condie, Ganguli, and Illeditsch (2021) show that ambiguity can produce information inertia, leading investors to fail to process public information efficiently. Finally, the state-dependence in the degree of robustness in our model bears some resemblance to the business cycle models of Bidder and Smith (2012) and Jahan-Parvar and Liu (2014). However, the stochastic volatility in their models is exogenous. Here, it arises endogenously via equilibrium portfolio policies.

The remainder of the paper is organized as follows. Section 2 outlines the information and market structure, the objective functions of households and specialists, and the optimal delegation decision and portfolio choices. Section 3 imposes market-clearing and solves for equilibrium asset prices. Section 4 outlines our indirect inference estimation strategy and compares model predictions to U.S. asset market data. We then describe the simulation methodology we use to compute detection error probabilities, demonstrating that our agents' doubts are empirically plausible. Finally, Section 5 provides a few concluding remarks and offers extensions for future research. A technical Appendix contains proofs and derivations.

2 Information, Market Structure, and Preferences

This section outlines the main ingredients of our model. We first briefly summarize the HK model (Section 2.1). We then extend their model by assuming that the dividend growth rate is stochastic and unobserved, which confronts agents with a filtering problem. We show how rational inattention and ambiguity aversion influence the solution to this filtering problem (Section 2.2). Next, we turn to the control problem of agents, and show how ambiguity about asset returns gives rise to robust portfolio policies (Section 2.3). Finally, we explore how rational inattention and ambiguity interact to produce portfolio delegation (Section 2.4). Interestingly, we show that even with log utility, which normally makes estimation risk irrelevant, ambiguity about the unobserved dividend growth state makes agents care about estimation risk, and hence can lead households to delegate their investment decisions to intermediaries that posses greater channel capacities.

2.1 The HK Model

Consider an infinite horizon continuous-time Lucas (1978) endowment economy populated by two types of agents: specialists and households. There are two assets: one risky and one risk-free. Only specialists can invest in the risky asset. However, in contrast to traditional segmented-markets models (e.g., Basak and Cuoco (1998)), households can only indirectly invest in the risky asset by delegating some of their wealth to the specialist. At every time t, households invest in intermediaries run by specialists. Households cannot observe the portfolio choices of the specialist, nor can they observe his 'effort' level. These unobserved choices produce a moral hazard problem. Figure 1 depicts the economy's market structure. The intermediary sector is indicated in the middle block.

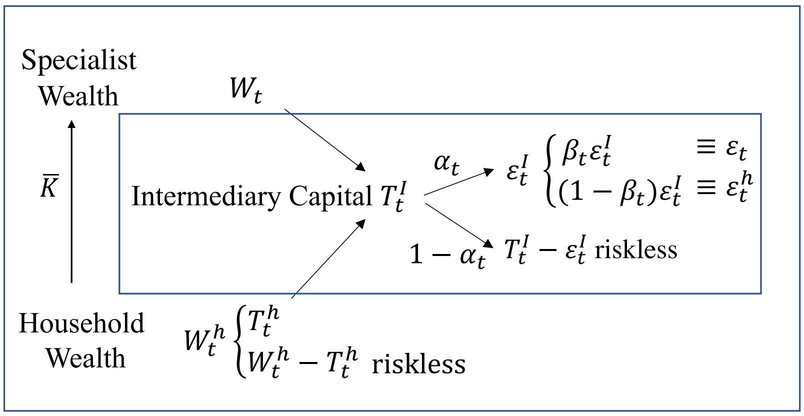

Figure 1: Market Structure and Intermediation Relationship

Specialist wealth is W_t, and household wealth is W_t^h. Households allocate T_t^h to purchase intermediary equities, with the remaining fraction used to buy riskless bonds. Intermediaries absorb a total of T_t^I dollars, consisting of T_t^h from households and W_t from their own wealth. They then allocate a fraction α_t to the risky asset and $1-\alpha_t$ to the riskless bond. Assuming there is a no short-selling constraint for the intermediary, we expect α_t to be greater than 1, i.e., specialists use leverage. In this case, specialists invest more than the total intermediary capital into risky equity and borrow $(\alpha_t - 1)\,T_t^I$ from the bond market. The total risky asset dollar exposure of the intermediary is ε_t^I, where $\varepsilon_t^I = \alpha_t T_t^h$. Through an affine contract developed by HK, $\beta_t \in [0,1]$ is the share of returns going to the specialist, while $1-\beta_t$ goes to households. Thus, at time t, the specialist bears a total risk exposure of $\varepsilon_t = \beta_t \varepsilon_t^I$, and the household is offered an exposure of $\varepsilon_t^h = (1-\beta_t)\varepsilon_t^I = \left(\frac{1-\beta_t}{\beta_t}\right)\varepsilon_t$ because $\varepsilon_t^I = \varepsilon_t^h + \varepsilon_t$.

In practice, wealth management typically involves *two* fees — a one-time fixed cost $\bar{K}$, and an ongoing flow cost, typically expressed as a percentage of profits. HK only consider the flow cost, denoted as K_t, which is an endogenous variable. Later in Section 2.4 we model the fixed cost $\bar{K}$ using robust filtering and information processing constraints. It will be determined by differences in channel capacity. Households make two decisions. They first decide whether to delegate or not. Delegation involves paying a fixed cost $\bar{K}$ in order to access the channel capacity of the intermediary.

Once delegation occurs, households then need to pay a flow cost K_t per unit of time, as in HK. We will show in Section 3.2 that this flow cost only arises during financial crises when the capital constraint binds. The reason is that during a crisis, intermediary capital is scarce. Therefore, specialists have to charge a fee to allocate the capital efficiently.

The population measures of households and specialists are normalized to one. Both are infinitely lived and have log preferences over consumption. Denote households' (specialists') consumption rate as C_t^h (C_t). The household's objective is to:

$$\max_{\{C_t^h, \varepsilon_t^h\}} \mathbb{E}\left[\int_0^\infty e^{-\rho^h t} \ln C_t^h dt\right], \tag{1}$$

while the specialist's objective is to:

$$\max_{\{C_t, \varepsilon_t, \beta_t\}} \mathbb{E}\left[\int_0^\infty e^{-\rho t} \ln C_t dt\right], \tag{2}$$

where ρ^h and ρ denote the time discount rates for households and specialists, respectively. Households are assumed to be more impatient than specialists, i.e., $\rho^h > \rho$. The exogenous dividend follows a geometric Brownian motion: $dD_t/D_t = gdt + \sigma dZ_{D,t}$, where g and σ are constants, and $Z_{D,t}$ is a Brownian motion. The endogenous risky asset return is defined as:

$$dR_t = \frac{D_t dt + dP_t}{P_t} = \mu_{R,t} dt + \sigma_{R,t} dZ_{D,t}, \tag{3}$$

where P_t is the risky asset price, $\mu_{R,t}$ is the expected return, and $\sigma_{R,t}$ is the volatility of the risky asset. The riskless asset is in zero-net supply and has a return r_t. The risk premium is defined as: $\pi_{R,t} \equiv \mu_{R,t} - r_t$. The budget constraint for the household is given as:

$$dW_t^h = \varepsilon_t^h (dR_t - r_t dt) - k_t \varepsilon_t^h dt + r_t W_t^h dt - C_t^h dt. \tag{4}$$

Households obtain an exposure ε_t^h from the intermediary with an excess return indicated by the first term in the budget constraint, i.e., $\varepsilon_t^h(dR_t - r_t dt)$. For simplicity, define the equilibrium per-unit-of-exposure intermediation fee households have to pay as $k_t \equiv K_t/\varepsilon_t^h$. The second term in the household's budget constraint represents this flow cost. The third term is the risk-free interest earned by the household on his own wealth. The last term is the consumption expense.

The specialist's budget constraint is as follows:

$$dW_t = \varepsilon_t (dR_t - r_t dt) + \max_{\beta_t \in \left[\frac{1}{1+m}, 1\right]} \left(\frac{1-\beta_t}{\beta_t}\right) k_t \varepsilon_t^* dt + r_t W_t dt - C_t dt. \tag{5}$$

Specialists bear a risky exposure ε_t by investing their own wealth into the intermediary, earning returns denoted by the first term. The second term is the total variable intermediation fee $K_t = k_t \varepsilon_t^h = \left(\frac{1-\beta_t}{\beta_t}\right) k_t \varepsilon_t^*$ received from the households, where ε_t^* is the equilibrium specialist exposure.

Households choose ε_t^h taking the expected ε_t^* as given, as they cannot observe specialists' portfolio choice. Specialists solve an optimal contracting problem by choosing the contract share β_t to maximize this intermediation fee, taking households' expected exposure ε_t^* as given. $\beta_t \geq \frac{1}{1+m}$ arises from an incentive constraint due to moral hazard friction, a key aspect in HK. Intuitively, households are reluctant to invest more than $m\varepsilon_t$ in intermediaries unless specialists maintain $\frac{1}{1+m}$ shares of capital, where m measures the inverse severity of agency problems. We provide a more detailed description of the incentive constraint in Section 3.2. Solving the inner maximization problem in Equation (5), the optimal contract can be obtained as $\beta_t^* = \frac{1}{1+m}$ if $k_t > 0$ and $\beta_t^* \in \left[\frac{1}{1+m}, 1\right]$ if $k_t = 0$. For simplicity, we define the per-unit-of-specialist-wealth fee as $q_t \equiv K_t/W_t$, and the specialist's budget constraint can be expressed as:

$$dW_t = \varepsilon_t(dR_t - r_t dt) + (q_t + r_t)W_t dt - C_t dt. \tag{6}$$

The key endogenous state variable in HK is the scaled relative wealth of the specialist: $x_t \equiv W_t/D_t$. It is assumed to be governed by the following stochastic process:

$$\frac{dx_t}{x_t} = \mu_{x,t}dt + \sigma_{x,t}dZ_{D,t}, \tag{7}$$

where $\mu_{x,t}$ and $\sigma_{x,t}$ are endogenously determined drift and diffusion coefficients (functions of x_t), respectively. They capture the nonlinear dynamics of the model. In HK, x_t is not a stationary process; it will converge to $1/\rho$ in the long run (specialists eventually dominate). However, we will show later in Section 3.4 that with ambiguity, x_t follows a stationary distribution.

2.2 Capacity Constrained Robust Filtering

As mentioned earlier, our model combines robust filtering and robust control. Our approach to modeling ambiguity is based on Hansen and Sargent's (2008) work on robustness. Agents are assumed to have a correctly specified benchmark model of asset returns, which they distrust in a way that cannot be captured by a conventional finite-dimensional Bayesian prior. Instead of committing to a single model/prior, agents consider a *set* of unstructured alternative models and optimize against the worst-case model. Since the worst-case model depends on an agent's own actions, agents view themselves as being immersed in a dynamic zero-sum game. To prevent agents from being overly pessimistic, the hypothetical 'evil agent' who selects the worst-case model is required to pay a penalty that is proportional to the relative entropy between the benchmark model and the worst-case model. In our model, agents entertain doubts about *both* the model and the underlying, time-varying, dividend growth state. Hence, they must simultaneously solve both a robust control and a robust filtering problem.

A key benefit of our log preference specification is that we can separate these two problems. Here we focus on the robust filtering problem, which we later use to motivate the portfolio delegation decision. We shall see that even with log preferences, agents care about estimation risk when they distrust their priors. The analysis in this section can be interpreted as combining the recursive robust filtering approach of Hansen and Sargent (2007) with the channel capacity-constrained Merton model of Turmuhambetova (2005) and an asset pricing model of Peng (2005).[3]

We assume that the dividend growth rate follows a diffusion process of the form:

$$\frac{dD_t}{D_t} = g_t dt + \sigma dZ_{D,t}, \tag{8}$$

$$dg_t = \rho_g \left(\bar{g} - g_t\right) dt + \sigma_g dZ_{g,t}, \tag{9}$$

$$ds_t = g_t dt + \sigma_s dZ_{s,t}, \tag{10}$$

where $Z_{D,t}$, $Z_{g,t}$, and $Z_{s,t}$ are independent standard Brownian motions, and σ is the volatility of dividend growth. In contrast to HK, we assume that the mean dividend growth, g_t, is both stochastic and unobserved. It follows an Ornstein-Uhlenbeck process with a long-run mean of $\bar{g}$, a mean reversion rate of ρ_g, and volatility of σ_g. Agents can observe a noisy signal about g_t, denoted as s_t, with an inverse signal precision of σ_s. The conditional mean and variance of g_t are characterized by $\hat{g}_t = \mathbb{E}_t\left[g_t\right]$ and $Q_t = \mathbb{E}_t\left[\left(g_t - \hat{g}_t\right)^2\right]$, respectively. The posterior variance Q_t characterizes the estimation risk, which quantifies the state uncertainty agents face. The application of standard Kalman filtering suggests that:

$$d\hat{g}_t = \rho_g \left(\bar{g} - \hat{g}_t\right) dt + \frac{Q_t}{\sigma} d\hat{Z}_{D,t} + \frac{Q_t}{\sigma_s} d\hat{Z}_{s,t}, \tag{11}$$

$$dQ_t = \left[\sigma_g^2 - 2\rho_g Q_t - Q_t^2 \left(\frac{1}{\sigma^2} + \frac{1}{\sigma_s^2}\right)\right] dt, \tag{12}$$

where $d\hat{Z}_{D,t} = \frac{1}{\sigma}\left(\frac{dD_t}{D_t} - \hat{g}_t dt\right)$ and $d\hat{Z}_{s,t} = \frac{1}{\sigma_s}\left(ds_t - \hat{g}_t dt\right)$ are innovations corresponding to (8) and (10), respectively.

First, consider the capacity constraint. To model rational inattention due to finite capacity, we follow Sims (2003) and impose the following constraint on the investor's information processing ability:

$$\mathbb{H}\left(g_{t+\Delta t} | \mathcal{I}_t\right) - \mathbb{H}\left(g_{t+\Delta t} | \mathcal{I}_{t+\Delta t}\right) \leq \kappa \Delta t, \tag{13}$$

where κ is the investor's information channel capacity. κ can also be interpreted as the degree of attention that agents are able to allocate. $\mathcal{I}_t$ is the processed information at time t; $\mathbb{H}\left(g_{t+\Delta t} | \mathcal{I}_t\right)$ denotes the entropy of the state prior to observing the new signal at $t + \Delta t$; and $\mathbb{H}\left(g_{t+\Delta t} | \mathcal{I}_{t+\Delta t}\right)$ is the entropy after observing the new signal $s_{t+\Delta t}$. Equation (13) implies that the reduction in uncertainty upon observing the new signal is bounded by the investor's information processing capacity κ. Therefore, κ imposes an upper bound on the signal-to-noise ratio — that is, the change in entropy — that can be transmitted in any given period. The Kalman gain is constrained by the agent's channel capacity, which limits the rate of learning, as shown in Duncan (1970), Liptser and

[3] Using data on CDS credit spreads, Boyarchenko (2012) shows that state uncertainty and robust filtering were important in the early phase of the financial crisis. She also provides evidence that the importance of model uncertainty increased relative to state uncertainty as the crisis unfolded.

Shiryaev (2001), and Luo (2017):[4]

$$\frac{1}{2}\frac{Q_t}{\sigma_s^2} \leq \kappa. \tag{14}$$

Since the capacity constraint (14) always binds, these information processing constraints help determine the variances of the endogenous noises as $\sigma_s^2 = Q_t/(2\kappa)$. We have:

$$dQ_t = \left(\sigma_g^2 - 2\rho_g Q_t - \frac{Q_t^2}{\sigma^2} - 2\kappa Q_t\right) dt.$$

We only focus on the steady state, where $dQ_t = 0$; thus,

$$Q = \frac{-(\kappa+\rho_g) + \sqrt{(\kappa+\rho_g)^2 + \sigma_g^2/\sigma^2}}{1/\sigma^2}. \tag{15}$$

We now turn to the robustness part of the problem. Following Hansen and Sargent (2007), the robust filter can be implemented by introducing a drift distortion into the Kalman filtering equation for the conditional mean. The robust filtering equation can be written as:

$$d\hat{g}_t = \rho_g(\bar{g} - \hat{g}_t)\,dt + \frac{Q_t}{\sigma}\left(\omega_t dt + d\hat{Z}_t\right) + \frac{Q_t}{\sigma_s} d\hat{Z}_{s,t}, \tag{16}$$

where $d\hat{Z}_t = d\hat{Z}_{D,t} - \omega_t dt$. The process ω_t is a robust distortion to the conditional mean. It reflects the agent's distrust of his own priors.

Denote the information processing capacities for specialists and households as κ^s and κ^h respectively. We assume $\kappa^s > \kappa^h$, that is, specialists have a higher information processing capacity than households. We can then write the capacity-constrained robust filter for agent $i = \{s, h\}$ as:[5]

$$d\hat{g}_t^i = \left[\rho_g\left(\bar{g} - \hat{g}_t^i\right) + \frac{Q^i}{\sigma}\omega_t^i\right] dt + \frac{Q^i}{\sigma} d\hat{Z}_t^i + \sqrt{2\kappa^i Q^i}\, d\hat{Z}_{s,t}^i, \tag{17}$$

where $d\hat{Z}_t^i = d\hat{Z}_{D,t}^i - \omega_t^i dt$, $d\hat{Z}_{D,t}^i = \frac{1}{\sigma}\left(\frac{dD_t}{D_t} - \hat{g}_t^i dt\right)$, and $d\hat{Z}_{s,t}^i = \sqrt{\frac{2\kappa^i}{Q^i}}\left(ds_t^i - \hat{g}_t^i dt\right)$. Note that a larger channel capacity, κ, accelerates learning (i.e., causes Q_t to decrease faster). It also produces a lower steady state estimation risk Q. We shall see in Section 2.4 that this can motivate portfolio delegation under ambiguity aversion when agents have concerns about state uncertainty.

2.3 Robust Control

In addition to having doubts about the unobserved state, g_t, agents also have doubts about the model generating asset returns. In response, they formulate robust portfolio policies (Anderson,

[4]Specifically, from time 0 to T, the amount of mutual information between the true states and the noisy signals in our univariate case can also be written as: $\mathcal{I}\left(g_0^T; s_0^T\right) = \mathcal{I}\left(g_0^T; \hat{g}_0^T\right) = \frac{1}{2}\int_0^T \mathbb{E}\left[\left\|\sigma_s^{-1}(g_t - \hat{g}_t)\right\|^2\right] dt = \frac{1}{2}\int_0^T \left(Q_t\sigma_s^{-2}\right) dt$. As a result, in the steady state when $T \to \infty$, we have: $\lim_{T\to\infty}\sup \frac{1}{T}\mathcal{I}\left(g_0^T; \hat{g}_0^T\right) = \frac{1}{2}Q_t\sigma_s^{-2} \leq \kappa$, which is just (14).

[5]For ease of notation, throughout the remainder of the paper, we use the superscript h to indicate households, while we use no superscript or the superscript s to indicate specialists.

Hansen, and Sargent (2003), Maenhout (2004)). We assume model misspecification concerns are focused on equilibrium return processes, rather than the observable state variable dynamics. With log preferences, this means agents do not need to hedge against changes in the investment opportunity set. This enables us to obtain explicit analytical expressions for decision rules and equilibrium returns.[6] Ambiguity about asset returns implies that the budget constraints in Equations (4) and (6) are viewed as merely a useful approximating model, associated with a benchmark probability measure $\mathbb{P}$. To ensure robustness, the agent surrounds the approximating model with a convex set of unstructured alternatives and then optimizes against the worst-case model within the set.[7] The agent recognizes that the worst-case model depends on his own actions. If we let $\mathbb{Q}$ denote the probability measure of the worst-case model, then Girsanov's Theorem implies that the conditional relative entropy (or Kullback-Leibler distance) between the benchmark and worst-case models is given by a drift distortion, ν_t:

$$\int \ln\left(\frac{d\mathbb{Q}_t}{d\mathbb{P}_t}\right) d\mathbb{Q}_t = \frac{1}{2}\mathbb{E}^{\mathbb{Q}} \int_0^t \left(\nu_\tau\right)^2 d\tau.$$

The ν_t process conveniently parameterizes the alternative models. A hypothetical evil agent chooses ν_t subject to a relative entropy cost. Using this distortion, we can define a change of measure for agent i as $dZ_t^i = d\hat{Z}_{D,t}^i - \nu_t^i dt$, where dZ_t^i is a Brownian motion under $\mathbb{Q}$. This drift distortion operationalizes an agent's defensive pessimism.

Under the distorted probability measure $\mathbb{Q}$, the household's problem becomes:[8]

$$V_t^h = \sup_{\{C_t^h, \varepsilon_t^h\}} \inf_{\{\nu_t^h, \omega_t^h\}} \mathbb{E}\left[\int_0^\infty e^{-\rho^h t}\left(\ln C_t^h + \frac{1}{2\theta_1^h}\left(\nu_t^h\right)^2 + \frac{1}{2\theta_2^h}\left(\omega_t^h\right)^2\right) dt\right] \tag{18}$$

subject to (17) and

$$dW_t^h = \left[\varepsilon_t^h(\pi_{R,t} - k_t) + r_t W_t^h - C_t^h\right] dt + \sigma_{R,t}\varepsilon_t^h\left(\nu_t^h dt + dZ_t^h\right), \tag{19}$$

where $V_t^h = V^h\left(\hat{g}_t^h, W_t^h, x_t\right)$ is the household's value function. Note that the household's dynamic programming problem features three state variables: $\hat{g}_t^h$ summarizes his current beliefs, W_t^h is his current wealth, and x_t captures the endogenous dynamics of equilibrium asset prices. Following Hansen and Sargent (2007), we assume that state and model uncertainty are constrained by separate relative entropy penalties, θ_2^h and θ_1^h. The relative entropies induced by concerns about state and model uncertainty are captured by ω_t^h and ν_t^h, respectively.

[6]A version of the model relaxing this assumption is available upon request, yet the main results remain unchanged.

[7]In principle, the agent might distrust some asset returns more than others, but that is not the case here. Uppal and Wang (2003) pursue this idea and show that more focused ambiguity can help explain observed underdiversification.

[8]Note, following Hansen, Sargent, Turmuhambetova, and Williams (2006), we discount increments to relative entropy. This allows doubts to persist as the sample grows and delivers stationary decision rules.

The specialist's problem is very similar. It can be written as:

$$V_t = \sup_{\{C_t,\varepsilon_t\}} \inf_{\{\nu_t^s,\omega_t^s\}} \mathbb{E}\left[\int_0^\infty e^{-\rho t}\left(\ln C_t + \frac{1}{2\theta_1}\left(\nu_t^s\right)^2 + \frac{1}{2\theta_2}\left(\omega_t^s\right)^2\right) dt\right] \tag{20}$$

subject to (17) and

$$dW_t = \left[\varepsilon_t \pi_{R,t} + (q_t + r_t)W_t - C_t\right] dt + \sigma_{R,t}\varepsilon_t \left(\nu_t^s dt + dZ_t^s\right), \tag{21}$$

where $V_t = V\left(\hat{g}_t^s, W_t, x_t\right)$ is the specialist's value function. We denote the specialist's ambiguity aversion about state and model uncertainty as θ_2 and θ_1, and the corresponding relative entropies as ω_t^s and ν_t^s. The following lemma summarizes the solutions to these two robustness problems. Appendix 5.1 provides details of the derivations.

Lemma 1. *Given log preferences, the household's and specialist's value functions take the additively separable form:*

$$\begin{aligned} V^h\left(\hat{g}_t^h, W_t^h, x_t\right) &= \frac{1}{\rho^h}\ln W_t^h + F^h\left(\hat{g}_t^h; Q^h\right) + Y^h\left(x_t\right), && (22)\\ V\left(\hat{g}_t^s, W_t, x_t\right) &= \frac{1}{\rho}\ln W_t + F^s\left(\hat{g}_t^s; Q^s\right) + Y\left(x_t\right), && (23)\end{aligned}$$

where $F^h\left(\hat{g}_t^h; Q^h\right)$ and $F^s\left(\hat{g}_t^s; Q^s\right)$ are defined in Equations (57) and (64); Y_t^h and Y_t (functions of the aggregate state x_t) solve the ODEs of Equations (59) and (66) given in Appendix 5.1.

The consumption policies and the optimal risk exposures are:

$$\begin{aligned} C_t^h &= \rho^h W_t^h, \text{ and } C_t = \rho W_t, && (24)\\ \varepsilon_t^h &= \frac{\pi_{R,t} - k_t}{\gamma^h \sigma_{R,t}^2} W_t^h, \text{ and } \varepsilon_t = \frac{\pi_{R,t}}{\gamma \sigma_{R,t}^2} W_t, && (25)\end{aligned}$$

and the optimal entropy-constrained drift distortions of the control and filtering problems are:

$$\nu_t^h = -\frac{\theta_1^h}{\rho^h}\frac{\varepsilon_t^h \sigma_{R,t}}{W_t^h}, \text{ and } \omega^h = -\frac{\theta_2^h}{\rho^h\left(\rho^h + \rho_g\right)}\frac{Q^h}{\sigma}, \tag{26}$$

$$\nu_t^s = -\frac{\theta_1}{\rho}\frac{\varepsilon_t \sigma_{R,t}}{W_t}, \text{ and } \omega^s = -\frac{\theta_2}{\rho\left(\rho + \rho_g\right)}\frac{Q^s}{\sigma}, \tag{27}$$

where $\gamma^h = 1 + \theta_1^h/\rho^h$ and $\gamma = 1 + \theta_1/\rho$ are defined as the household's and specialist's effective degree of ambiguity aversion, respectively. θ_1^h, θ_2^h summarize the household's degree of model and state uncertainty, and θ_1, θ_2 summarize the specialist's degree of model and state uncertainty.

In our calibration, we impose $\theta_1 = \theta_1^h$ and $\theta_2 = \theta_2^h$. Both agents have the same degrees of ambiguity aversion, so our model dynamics do not rely on the assumption of differences in investors' risk appetites. Even though both agents have the same degree of ambiguity aversion, specialists are *effectively* more ambiguity averse than households because they are more patient ($\rho^h > \rho$ as in

HK), so they care more about the future ($\theta_1/\rho > \theta_1^h/\rho^h$).[9]

2.4 Portfolio Delegation

Portfolio delegation is based on the fact that if $\kappa^s > \kappa^h$, then $V^h(\kappa^s) > V^h(\kappa^h)$. This difference in value functions motivates trade in channel capacity. Households are willing to pay to use a specialist's channel. In contrast to the investment decision, the filtering efforts of the specialist in our model are not subject to incentive constraints.[10] To the best of our knowledge, our paper is the first to motivate portfolio delegation through trading in information capacity.

In contrast to HK, we do not restrict households from directly investing in risky assets. In fact, we assume that the decision is made once at time-0, and it is based on an ex-ante expected value calculation, given the equilibrium prices. Since the flow delegation cost rises during crises, households might have an incentive to opt out during crises to avoid this cost and then attempt to enter a new contract once things return to normal. Our commitment assumption prevents this. Once the household delegates, he no longer needs to filter.[11] The probability measure used to compute this expectation is the probability measure associated with a delegated competitive equilibrium, reflecting our assumption that households make unilateral delegation decisions. As a result, the $F^h\left(\hat{g}_t^h; Q^h\right)$ component of V^h influences the delegation decision. Since signals are unbiased, all that matters from an ex-ante perspective is the steady state conditional second moment of beliefs. Households prefer delegation when the estimation risk Q is lower. Additionally, the $Y^h(x_t)$ component also affects delegation, as households would not need to pay the flow cost if they invest directly, where we take $k_t = 0$ in the optimization problems. $Y^h(x_t)$ becomes smaller on average if households delegate, as they would need to pay a flow fee of $k_t > 0$ during crises. This discourages delegation. In principle, the first channel dominates if the differences in κ are sufficiently large. In the following Proposition, we establish a lower bound on households' optimal delegation decisions. Detailed derivations are provided in Appendix 5.2.

Proposition 1. *In the steady state, the household's expected value function difference arising from delegation is given by*

$$F^h(\kappa^s) - F^h(\kappa^h) = \frac{\theta_2^h\left(Q^{h2} - Q^{s2}\right)}{2\sigma^2\rho^{h3}\left(\rho^h + \rho_g\right)^2}, \tag{28}$$

where $Q^i = \frac{-\left(\kappa^i+\rho_g\right)+\sqrt{\left(\kappa^i+\rho_g\right)^2+\sigma_g^2/\sigma^2}}{1/\sigma^2}$. *A higher channel capacity reduces the steady state conditional variance, that is,*

$$\frac{dQ^i}{d\kappa^i} < 0. \tag{29}$$

[9] Luo and Young (2016) show that when channel capacity is endogenous, more patient agents choose to pay more attention to fundamentals because they care more about the future. This is one way to justify our assumption that intermediaries have higher channel capacities.

[10] It would be interesting to allow households to view the filtering of specialists as being ambiguous. This would connect our paper to Easley, O'Hara, and Yang (2014), who argue that the proprietary information and trading strategies of hedge funds are better interpreted as giving rise to ambiguity rather than asymmetric information.

[11] Yin (2021) provides evidence in support of this. Using data from the Survey of Consumer Finances, he finds that households who delegate pay less attention to their portfolios.

As long as $\theta_2^h > 0$ *and* $\kappa^s > \kappa^h$,

$$F^h(\kappa^s) > F^h(\kappa^h), \tag{30}$$

households choose to delegate by paying a fixed cost

$$\bar{K} = F^h(\kappa^s) - F^h(\kappa^h). \tag{31}$$

Notice that what matters here is the *interaction* between state uncertainty θ_2^h of households and differences in channel capacity κ^i. Delegation arises in equilibrium due to two reasons. First, specialists have a higher information processing capacity ($\kappa^s > \kappa^h$), leading a lower steady state posterior variance: $Q^s > Q^h$. Second, households are ambiguity averse: $\theta_2^h > 0$. Without ambiguity about the state uncertainty, $\theta_2^h = 0$, differences in estimation variance $Q^h - Q^s$ alone do not generate any incentive to delegate, as expected under any expected utility framework (e.g., log utility in HK). Moreover, ambiguity aversion amplifies households' incentives to delegate. Capacity differences become more important when robust concerns about state uncertainty is greater ($d\bar{K}/d\theta_2^h > 0$). Intuitively, households are more willing to pay when they are more ambiguity averse.

In fact, ambiguity aversion gives rise to a form of preference for early resolution of uncertainty. As a result, delegation reduces estimation risk, increases household welfare gain, and leads to endogenous portfolio delegation. In our model, log utility combined with ambiguity aversion provide a *sufficient* preference condition for portfolio delegation. Although log preferences are not *necessary* for our result, without them, it becomes much more difficult to compute value functions, which then complicates the calculation of the delegation fee. Likewise, since without log utility even an ambiguity neutral agent would care about estimation risk (e.g., under recursive preference with early resolution of uncertainty), ambiguity per se is not essential to our delegation result. Ambiguity simply reinforces it.

3 Market Equilibrium

We now combine the policy functions derived in the previous section with market clearing conditions to determine equilibrium asset prices. We demonstrate that ambiguity aversion tightens the capital constraint and amplifies its effects in driving time-varying risk premiums. Additionally, we show that the introduction of model uncertainty results in a stationary wealth distribution.

3.1 Equilibrium Definition

Here we provide a detailed definition of market equilibrium in our model economy:

Definition. An equilibrium for the economy is a set of prices $\{P_t, r_t, R_t, k_t\}$, and households' decisions $\{C_t^h, \varepsilon_t^h\}$, and specialists' decisions $\{C_t, \varepsilon_t, \beta_t\}$ such that:

1. Given the prices and quantities, agents' consumption and portfolio decisions solve the objective functions (1) and (2).

2. The intermediation market reaches equilibrium with the risk exposure clearing condition:

$$\varepsilon_t^h = \frac{1-\beta_t}{\beta_t}\varepsilon_t. \tag{32}$$

3. The stock market clears:

$$\varepsilon_t + \varepsilon_t^h = P_t. \tag{33}$$

4. The goods market clears:

$$C_t + C_t^h = D_t. \tag{34}$$

3.2 Capital Constraint

The key assumption in HK is that the contractual relationship between households and intermediaries is subject to a moral hazard friction, which results in a minimum capital constraint. The incentive/financial constraint is given as follows:

$$\varepsilon_t^h \leq m\varepsilon_t, \tag{35}$$

where ε_t and ε_t^h are the specialist's and household's risky exposures as given in Lemma 1. This constraint implies that the household is less willing to supply capital to the intermediary when the specialist is less exposed. Therefore, (35) can also be interpreted as a risk-sharing/capital constraint. Here, m represents an inverse measure of the severity of the agency problem. The lower m is, the more severe the agency problem, and the less capital specialists can raise from households. If we substitute the optimal exposure policies given in Lemma 1 into the financial constraint, we can express the constraint in terms of wealth:

$$W_t^h \leq \tilde{m}W_t, \tag{36}$$

where $\tilde{m} \equiv \frac{\gamma^h}{\gamma}m$ is the effective capital constraint. When the constraint is not binding, households allocate their entire wealth to the intermediary, so that $T_t^h = W_t^h$. In contrast, when the constraint begins to bind, households allocate only a portion of their total wealth to the intermediary, i.e., $T_t^h = \tilde{m}W_t \left(< W_t^h\right)$, thus imposing a capital constraint on the specialist. Without ambiguity, the HK model implies $\tilde{m} = m$. From this, we obtain the following result:

Proposition 2. *If $\theta_1/\rho > \theta_1^h/\rho^h$, then ambiguity aversion tightens the capital constraint, i.e., $\tilde{m} < m$.*

The intuition is as follows. Since $\theta_1/\rho > \theta_1^h/\rho^h$ implies $\gamma > \gamma^h$, the specialist is effectively more ambiguity averse than the household. Agents worry more about model uncertainty and become more pessimistic when they are more patient. Since the constraint binds when the household wants to invest while the specialist does not, ambiguity tightens the constraint because this makes the specialist less willing to invest compared to the household.[12]

3.3 Equilibrium Asset Prices

The following propositions summarize the influence of ambiguity and the capital constraint on the dynamics of asset prices. Figure 2 plots the associated policy functions.

(a) The price/dividend ratio.

Since bonds are in zero net supply, the asset market clears when aggregate wealth equals the market value of the risky asset,

$$W_t^h + W_t = P_t. \tag{37}$$

In equilibrium, from the goods market clearing condition (34) and the optimal consumption rules of households and specialists, we have

$$\rho W_t + \rho^h W_t^h = D_t. \tag{38}$$

The equilibrium price/dividend ratio is thus given by:

$$\frac{P_t}{D_t} = \frac{1}{\rho^h} + \left(1 - \frac{\rho}{\rho^h}\right) x_t = \frac{1 + \Delta\rho x_t}{\rho^h}, \tag{39}$$

where $\Delta\rho \equiv \rho^h - \rho > 0$.

Notice that ambiguity aversion only affects the price/dividend ratio through its influence on x_t. Since specialists are relatively patient, the price/dividend ratio falls as their relative wealth decreases. Therefore, since crises are characterized by declines in specialists' wealth, the model generates pro-cyclical movements in the price/dividend ratio.

When the risk-sharing constraint just starts to bind, the threshold level of the state x^c can be written as: $mW_t = W_t^h = P_t - W_t$. Together with the equilibrium price/dividend ratio, this allows us to restate the capital constraint in terms of the specialist's scaled wealth:

$$x^c = \frac{1}{\tilde{m}\rho^h + \rho}. \tag{40}$$

When $x_t \leq x^c$, the economy is within the constrained region; otherwise, when $x_t > x^c$, the economy is unconstrained. Note that the ambiguity aversion of both households and specialists influence x^c through the effective capital constraint $\tilde{m}$. As the relative ambiguity of the specialist γ/γ^h increases, x^c increases.

(b) Specialist's share of return and portfolio choice.

Later we shall see that the specialist's share of the intermediary's return plays an important role in the dynamics of asset prices. Appendix 5.3 shows that this share can be characterized as follows:

[12] This is consistent with the risk premium channel in Borovička (2020). It is also consistent with Miao and Rivera (2016), who show that the dividend payout threshold increases with robustness in their robust version of the DeMarzo-Sannikov model.

in the unconstrained region and constrained region, the specialist's share of the intermediary's return is:

$$\beta_t^U = \left[1 + \frac{\gamma}{\gamma^h}\left(\frac{1}{\rho^h x_t} - \frac{\rho}{\rho^h}\right)\right]^{-1}, \text{ and } \beta_t = \frac{1}{1+m}, \tag{41}$$

respectively. In other words, within the unconstrained region, denoted using the superscript U throughout the paper, the specialist's share of the intermediary's return declines as his wealth share declines. However, a higher relative ambiguity between specialists and households mitigates this effect, as specialists prefer to take on less exposure than households in the presence of greater ambiguity. Nevertheless, once the constraint binds, his share remains fixed at $\frac{1}{1+m}$.

The specialist invests all his wealth into the intermediary in equlibrium. The specialist then makes a portfolio choice to invest a share α_t of the total intermediary wealth $T_t^I = W_t + T_t^h$ into the risky asset, and the rest into the riskless bond. Thus, the intermediary's total exposure is: $\varepsilon_t^I = \alpha_t T_t^I$, or in other words,

$$\varepsilon_t + \varepsilon_t^h = \alpha_t(W_t + T_t^h). \tag{42}$$

Given the specialist's optimal ε_t choice, this can be interpreted as a constraint on the intermediary's portfolio. It requires the specialist to choose α_t in order to reach the optimal risk exposure ε_t. Similarly, households obtain their desired exposure ε_t^h by choosing how much of their wealth T_t^h to contribute to the intermediary. Notice that when $\alpha_t > 1$, the intermediary is leveraged.

Appendix 5.3 shows that, in the unconstrained and constrained regions, the specialist's optimal portfolio choice of the intermediary is as follows:

$$\alpha_t^U = 1, \text{ and } \alpha_t = \frac{1/x_t + \rho^h - \rho}{(1+\tilde{m})\rho^h}. \tag{43}$$

That is, when the constraint does not bind, the specialist invests all of the intermediary's equity capital in the risky asset. Notice that when the constraint binds, the intermediary becomes leveraged ($\alpha_t > 1$), and its exposure increases as the specialist's scaled wealth declines. Once the constraint is binding, $\alpha_t > 1$ means the specialist holds above 100% of the total equity and borrows $(\alpha_t - 1)(W_t + T_t^h)$ riskless bonds. Also, when specialists are effectively more ambiguity averse than households, so that $\tilde{m} < m$, ambiguity aversion magnifies leverage.

(c) Return volatility of the risky asset.

Return volatility is a central feature of financial crises. Appendix 5.4 shows that the return volatility is

$$\sigma_{R,t} = \frac{1}{P_t/D_t}\frac{\sigma}{\rho^h - \Delta\rho\beta_t}. \tag{44}$$

Using Equations (39) and (41), the return volatility can be explicitly determined as:

$$\sigma_{R,t}^U = \frac{\sigma}{1+\Delta\rho x_t}\frac{\left(\rho^h\gamma^h - \rho\gamma\right)x_t + \gamma}{\rho\left(\gamma^h - \gamma\right)x_t + \gamma}, \text{ and } \sigma_{R,t} = \frac{\sigma\rho^h}{1+\Delta\rho x_t}\frac{1+m}{m\rho^h + \rho}. \tag{45}$$

In general, the effects of the specialist's scaled wealth on return volatility are subtle. However, return volatility unambiguously increases once the constraint binds.

When the constraint is not binding, declines in x_t have offsetting effects on $\sigma_{R,t}$. The induced decline in P_t/D_t increases $\sigma_{R,t}$. However, the decline in the specialist's return share offsets this. Once the constraint binds and the return share is fixed, only the P_t/D_t effect remains, and $\sigma_{R,t}$ unambiguously increases as x_t falls further.

(d) The risk premium and financial constraint.

In addition to increases in volatility, financial crises are also accompanied by large increases in risk premia. Appendix 5.4 shows that, in the unconstrained and constrained regions, the risk premium is given by,

$$\pi^U_{R,t} = \frac{\sigma^2\gamma\gamma^h}{(1+\Delta\rho x_t)}\frac{\left[\left(\rho^h\gamma^h-\rho\gamma\right)x_t+\gamma\right]}{\left[\rho\left(\gamma^h-\gamma\right)x_t+\gamma\right]^2}, \text{ and } \pi_{R,t} = \frac{\sigma^2\rho^h\gamma}{x_t\left(1+\Delta\rho x_t\right)}\frac{1+m}{\left(m\rho^h+\rho\right)^2}. \tag{46}$$

It is easy to see that $\partial\pi^U_{R,t}/\partial\theta_1 > 0$ and $\partial\pi_{R,t}/\partial\theta_1 > 0$, which show that ambiguity amplifies the risk premium, both unconditionally and during crises. Notice that the risk premium is highly state dependent. Risk premia increase during crises, and ambiguity amplifies this increase. The higher risk premium is necessary to incentivize specialists, who possess low wealth and, consequently, limited risk capacity, to bear the risk exposure. Ambiguity aversion amplifies this agency friction in risk-sharing during crises.

(e) The market price of risk and uncertainty.

The market price of risk is defined as the Sharpe ratio. In our model, the conventional Sharpe ratio measures a combination of risk aversion and ambiguity aversion. Ambiguity increases it unambiguously. Barillas, Hansen, and Sargent (2009) call this induced increase 'the price of model uncertainty.' Combining Equations (45) and (46), it is straightforward to obtain the market price of risk in the unconstrained and constrained region as:

$$\frac{\pi^U_{R,t}}{\sigma^U_{R,t}} = \frac{\sigma\gamma\gamma^h}{\rho\left(\gamma^h-\gamma\right)x_t+\gamma}, \text{ and } \frac{\pi_{R,t}}{\sigma_{R,t}} = \frac{\sigma\gamma}{\left(m\rho^h+\rho\right)x_t}. \tag{47}$$

Similar to the risk premium, in the constrained region, *only* the specialist's ambiguity γ has a direct effect on the Sharpe ratio. This reflects a central aspect of intermediary asset pricing literature, where specialists are the 'marginal investors' in pricing assets.

(f) The risk-free interest rate.

The risk-free interest rate can be obtained from households' Euler equations. As shown in Appendix 5.4 the real interest rate in the unconstrained and constrained region is given by:

$$r^U_t = \hat{g}^s_t + \rho^h - \rho\Delta\rho x_t - \sigma^2\frac{\gamma^2\left(1-\rho x_t\right)+\rho\gamma^{h2}x_t}{\left[\rho\left(\gamma^h-\gamma\right)x_t+\gamma\right]^2}, \tag{48}$$

$$r_t = \hat{g}^s_t + \rho^h - \rho\Delta\rho x_t - \sigma^2\frac{\left(1-\rho x_t\right)\rho\left(1+\gamma m\right)+\rho^h m^2\left(\rho^h-\rho\gamma^h\right)x_t}{\left(1-\rho x_t\right)\left(\rho+m\rho^h\right)^2 x_t}. \tag{49}$$

In the unconstrained region, the interest rate mainly reflects the disparity in agents' time discount rates. In the limiting case when the economy consists only of specialists, $\lim_{x\to 1/\rho} r_t^U = \hat{g}_t^s + \rho - \sigma^2$, where the specialist's time discount rate ρ determines the interest rate. When the specialist's wealth share declines, and households become more involved, the interest rate converges to ρ^h. Since $\rho^h > \rho$, the interest rate in the unconstrained region increases as the specialist's wealth x declines. In the constrained region, this effect is reversed. As the specialist's wealth further drops, households start to withdraw equity from the intermediary and 'flight to safety,' further depressing the interest rate. With ambiguity, this effect is magnified, as both agents have greater incentives to increase their demand for riskless bonds.

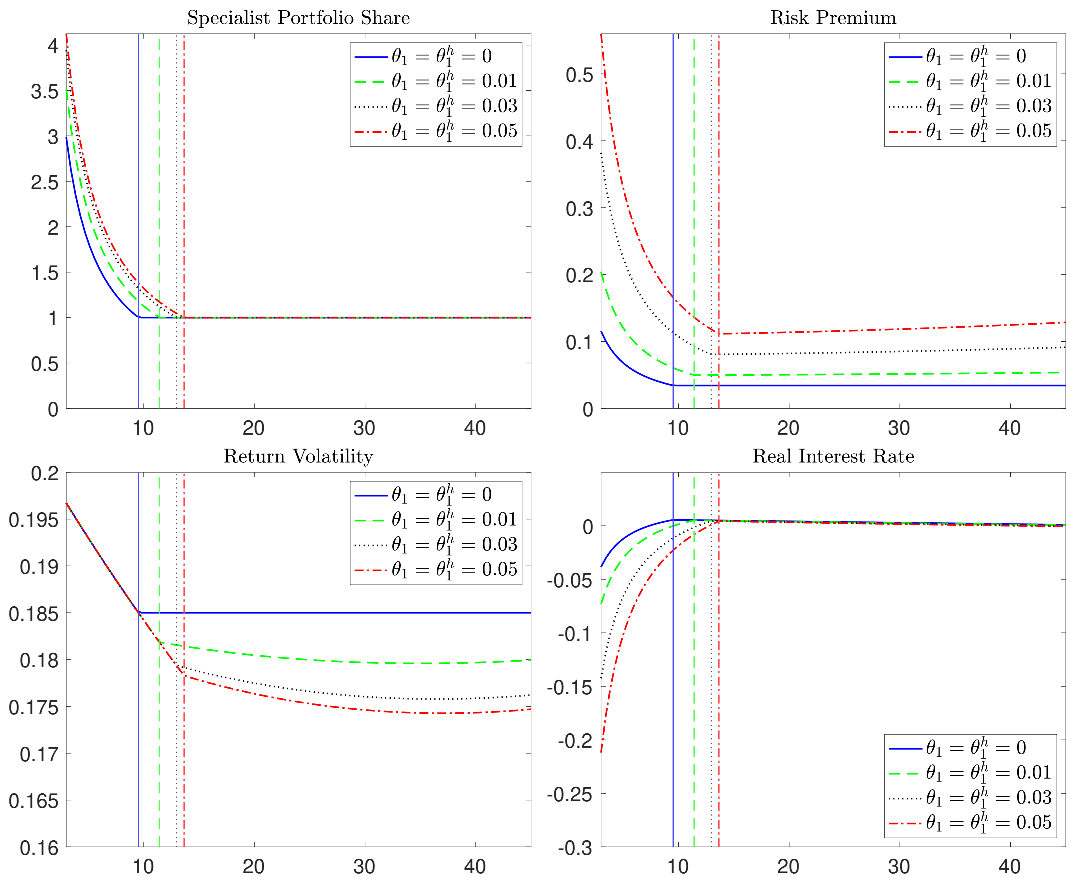

Figure 2: Policy Functions and Asset Prices

This graph plots the specialist's risky portfolio share α_t, the risk premium $\pi_{R,t}$, return volatility $\sigma_{R,t}$, and the risk-free rate r_t (while keeping $\hat{g}$ at $\bar{g}$) for different ambiguity parameters ($\theta_1 = \theta_1^h$). The threshold value x^c (vertical lines) separates the constrained (left) and unconstrained (right) regions. The solid blue line shuts down ambiguity as in HK. Parameter values are from Table 1. The dotted black line represents our benchmark case ($\theta_1 = \theta_1^h = 0.03$). Note that $x_t \in [3, 45]$ for illustrative purposes, although the actual support of x_t is $(1/(m\rho^h + \rho, 1/\rho)$, as indicated in Proposition 3.

Figure 2 depicts the intermediary's optimal portfolio policy along with equilibrium asset prices as functions of the specialist's scaled wealth x_t. We use the benchmark parameter values estimated in the following section, which are contained in Table 1. In all cases, we assume that households and specialists have the same degree of ambiguity aversion, $\theta_1 = \theta_1^h$. Since $\rho < \rho^h$, this implies that specialists are effectively more ambiguity averse in all plots, that is, $\gamma > \gamma^h$. To illustrate the effects of ambiguity, each panel contains four lines, pertaining to four alternative values of $\theta_1 = \theta_1^h$. For comparison, the blue solid line in each plot pertains to the no ambiguity case of HK.

The top left plot shows that leverage increases when the constraint binds, and ambiguity both amplifies this effect and causes it to occur at higher levels of specialist wealth. The top right panel plots the risk premium. Evidently, the risk premium increases as x_t decreases within the constrained region, as in HK. However, ambiguity magnifies this effect. For example, at the HK constraint, the equity premium is only 3% in the HK model, whereas it is about 12% in our benchmark model. It should be noted, though, that the lower bound on x in our benchmark model is above the HK constraint, so our model would never generate such a high risk premium. Still, it is clear that the risk premium in our model is 3-4 times higher than in the HK model, even during tranquil periods. The bottom panels depict two other commonly observed features of financial crises, namely, increased volatility and a decline in the risk-free rate. Ambiguity influences return volatility through the contract share. When the constraint binds, the specialist's share of the intermediary's return is fixed and independent of ambiguity. However, when the constraint is slack, higher ambiguity mitigates return volatility because specialists want to take less exposure to risks as they become relatively more ambiguity averse. Remember, this is a real model, so there is nothing wrong per se with having a negative real interest rate during crisis episodes. Indeed, short-term real rates *were* negative for a prolonged period following the 2008 financial crisis. Also, there is no issue of sovereign default here, so financial crises depress the interest rate in response to a flight to safety, rather than causing the sort of sharp increase that is so commonly seen in many developing countries. As with the risk premium, the very low interest rates associated with very low values of x are not empirically relevant since x has a positive lower bound.

3.4 The Stationary Wealth Distribution

To map the plots in Figure 2 into empirical predictions, we need to compute the stationary distribution of x_t, which is endogenous. As noted earlier, without ambiguity aversion, a stationary distribution does *not* exist in the HK model because when $\rho^h > \rho$, specialists eventually dominate households as they are more patient and accumulate all the wealth, and so the capital constraint never binds. However, here we show that when specialists are effectively more ambiguity averse, the greater impatience of households can be offset by the greater ambiguity of the specialists.

In what follows, denote the equilibrium specialist's scaled wealth process as given by the following diffusion equation:

$$\frac{dx_t}{x_t} = \mu_x(x_t)\,dt + \sigma_x(x_t)\,d\hat{Z}^s_{D,t}. \tag{50}$$

Note that the drift and volatility functions are state-dependent. In the unconstrained (constrained) region, the drift and volatility are denoted by $\mu^U_{x,t}$ and $\sigma^U_{x,t}$ ($\mu_{x,t}$ and $\sigma_{x,t}$). Explicit expressions for these functions are provided in Appendix 5.5. Whether this diffusion process yields a stationary distribution depends on the boundary properties of $\mu_x(x_t)$ and $\sigma_x(x_t)$. Intuitively, $\mu_x(x_t)$ should be negative for sufficiently large x_t and positive for sufficiently small x_t. To derive precise conditions under which a stationary equilibrium exists, we exploit the boundary classification results in Chapter 15 of Karlin and Taylor (1981). Using their notation, define the following function

$$s\left(y\right)=\exp\left\{-\int^{y}\left[\frac{2\mu_{X}(v)}{\sigma_{X}^{2}(v)}\right]dv\right\}, \tag{51}$$

where $\mu_X\left(x\right)=\mu_x\left(x_t\right)x_t$ and $\sigma_X\left(x\right)=\sigma_x\left(x_t\right)x_t$. Assuming it exists, let $f(x)$ be the stationary density associated with the diffusion in Equation (50). The following proposition uses the steady state Kolmogorov-Fokker-Planck (KFP) equation to provide an explicit characterization of $f(x)$.

Proposition 3. *If the following two parameter restrictions are satisfied:*

$$(i)\left(\rho^{h}-\rho\right)+m\sigma^{2}\left(\gamma-\gamma^{h}\right)>0,$$
$$(ii)\gamma\sigma^{2}\left(\gamma-\gamma^{h}\right)>\left(\rho^{h}-\rho\right)\gamma^{h2},$$

then a non-degenerate stationary distribution for relative wealth, x, exists, and its support is $(1/(\rho+m\rho^{h}),1/\rho)$. The two endpoints are 'entrance' boundaries, which are unattainable in finite mean time. The solution of the KFP equation is given by

$$f(x)=C_{1}\left[\frac{1}{s(x)\sigma_{X}^{2}(x)}\right]\cdot\mathbb{1}(x\leq x^{c})+C_{2}\left[\frac{1}{s^{U}(x)(\sigma_{X}^{U}(x))^{2}}\right]\cdot\mathbb{1}(x\geq x^{c}), \tag{52}$$

where $\mathbb{1}$ is an indicator function, and the integration constants C_1 and C_2 satisfy:

1. *$\int f(x)dx=1$;*
2. *continuous at x^c.*

The proof is provided in Appendix 5.6. Notice that the first parameter restriction is always satisfied given our assumptions ($\rho^h>\rho$ and $\theta_1^h=\theta_1$). However, the second restriction, which applies at the upper boundary, requires that the greater ambiguity of the specialist (as scaled by the volatility, σ^2) dominates the greater impatience of the household, so that the relative wealth distribution gets pulled to the left.[13] This is a version of the 'survival index' discussed in Yan (2008). Alternatively, from the perspective of Borovička (2020), at the upper boundary, the portfolio channel must dominate the saving channel. Our benchmark parameter values (discussed in the next section) satisfy this restriction.

We verify Proposition 3 in two ways. First, we numerically solve the steady state KFP equation using our benchmark parameter values. The result is depicted in the right panel of Figure 3. The dashed vertical line is the binding capital constraint, x^c. Note that, in contrast to the U-shaped

[13] Time preference differences are not necessary for a non-degenerate distribution in our model. The crucial condition is that specialists are more effectively ambiguity averse than households, as indicated by conditions (i) and (ii) with $\gamma>\gamma^h$. We can relax HK's assumption by allowing specialists to be equally or even less patient than households ($\rho\geq\rho^h$), while ensuring that specialists maintain a higher degree of ambiguity aversion ($\theta>\theta^h$), as supported by Haddad and Muir (2021). The intuition is that as specialists become more ambiguity averse, households accumulate wealth in the unconstrained region, shifting x to the left. Once the constraint starts binding, specialists collect increasing flow fees k as compensation, pushing x back to the right and resulting in a stationary distribution.

(risk-neutral) stationary distribution in Brunnermeier and Sannikov (2014), the probability of being in the tails is quite low in our model. The fact that the mode of the distribution appears to be at the constraint is coincidental. A lower value of θ_1 shifts it to the right since the ambiguity of specialists declines, whereas a lower value of ρ shifts it to the left since the (effective) ambiguity of specialists increases. For the benchmark parameters, the long-run average probability that the constraint binds is 6.1%, or about once every 16 years. This is roughly double the probability of a 'disaster' (Barro (2006)). However, here a binding capital constraint does not necessarily correspond to a disaster. Risk premia begin to rise in the constrained region, but they do not spike as in a disaster unless x falls significantly below x^c. If anything, our model likely underpredicts disaster probabilities given that the density drops off sharply to the left of x^c.

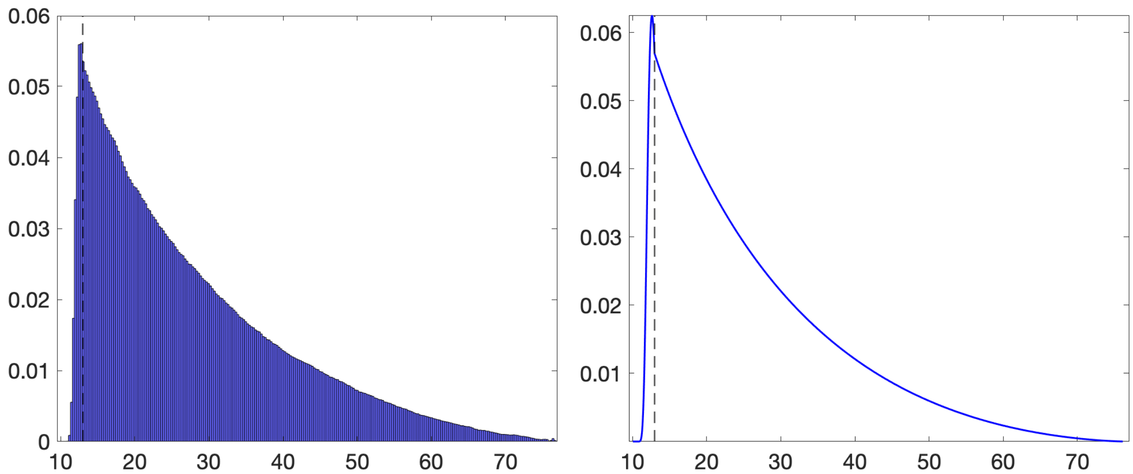

Figure 3: Stationary Distribution

This figure plots a histogram of the specialist's scaled wealth x using the benchmark parameters over 200 years and 10,000 independent sample paths. The vertical black dashed line separates the constrained (left) and unconstrained (right) regions. The right panel displays a numerical solution of the KFP equation. Both panels plot x over the support of $[1/(\rho + m\rho^h), 1/\rho]$.

The second check we perform is to run a Monte Carlo simulation, consisting of 10,000 repetitions, each lasting 1,000 years. We plot the last 200 years over all repetitions as the stationary distribution. The left side of Figure 3 reports the result. Evidently, it closely matches the numerical solution to the KFP equation. As an additional check, we also ran Monte Carlo simulations where restriction (*ii*) of Proposition 3 was violated. As predicted, the distribution becomes degenerate, with the entire mass piled up at $x = 1/\rho$.

4 Quantitative Implications

4.1 Indirect Inference

Due to the endogenously binding capital constraint, our model would be difficult to estimate using traditional methods. However, it is relatively easy to generate sample paths from the model.

Hence, it is a good candidate for the simulation-based estimation methodology of indirect inference (Gourieroux, Monfort, and Renault (1993), Smith (1993)). The basic idea behind indirect inference is to specify a set of (easily estimated) reduced-form 'auxiliary functions,' which are designed to capture features of the data of interest, i.e., comovement, volatility, and persistence. The reduced-form parameters are estimated twice — once using the actual data, and again using repeated samples generated from the structural model. The model is then evaluated based on how close (averaged) estimates from the simulated data are to those from the actual data. If there are more auxiliary function parameters than model parameters, then some sort of minimum distance/weighting matrix must be specified. The minimum distance can be used to test the null that the structural model is correctly specified. Note that the auxiliary functions can be misspecified.

We use two sets of auxiliary functions. The first set captures the model's ability to explain unconditional moments. Because many models are capable of explaining unconditional moments, we also want to assess our model's ability to explain state dependence in risk prices and return premia. This is more challenging. Our second set of auxiliary functions is, therefore, based on our model's ability to match simple auto-regressions of equity premia and price/dividend ratios. Table 1 through 3 summarize our benchmark parameters and the data/model-implied measurements.

Several of our model's parameters can be calibrated to outside data. For example, m is related to the specialist's return share. Following HK, we set $m = 4$, reflecting the assumption that specialists maintain a 20% return share (i.e., $1/(1+m)$). HK cite evidence in support of this. The mean dividend growth parameter is calibrated to match U.S. dividend data. We use the CRSP value-weighted monthly portfolios of all stocks from 1929.01–2022.12 to estimate annual dividends. This implies $\bar{g} = 0.018$, $\sigma_g = 0.007$, and $\rho_g = 0.06$. The model-based mean, volatility, and autocorrelation of dividend growth are 1.26%, 15.47%, and 0.24, which are fairly close to 1.54%, 10.80%, and 0.19 in the data. The information processing capacities are determined by two considerations. First, we choose the household's capacity as $\kappa^h = 0.01$ to align with the literature on rational inattention, e.g., Luo (2010).[14] Second, we calibrate the specialist's channel capacity, $\kappa^s = 0.04$, so that the implied delegation fee is consistent with empirical data. Empirical evidence suggests that upfront management fees are approximately 1.5–2.5% of invested assets (Greenwood and Scharfstein (2013))). More specifically, to calculate the corresponding fixed cost, we follow Cochrane (1989) and use $\bar{K}/V^{h\prime}(W_t^h)$ to measure the money loss due to limited capacity. Our model implies that this fee is approximately 2.46% of wealth.[15]

This leaves $(\sigma, \rho, \rho^h, \theta_1)$ as the free parameters. The first set of auxiliary functions consists of the following five unconditional moments: the mean equity premium, the mean risk-free rate, the mean price/dividend ratio, and the standard deviations of the risky return and price/dividend ratio for the period 1929.01–2022.12. The second set of auxiliary functions includes the intercept

[14]This value is consistent with Luo (2010) in which κ^h is calibrated to match individual households' risky asset holdings as observed in the data. κ^h is also chosen to keep $\hat{g}$ close to $\bar{g}$. This prevents DEPs from becoming implausibly low. Notice from Equation (26) that the magnitude of the household's filtering distortion ω^h depends positively on estimation variance, Q^h, which in turn depends negatively on capacity, κ^h (Equation (15)). The magnitude of filtering DEP, therefore, depends on the difference between $\hat{g}$ and $\bar{g}$.

[15]More specifically, Lemma 1 implies $\frac{\bar{K}}{V^{h\prime}(W_t^h)}/W_t^h = \rho^h \bar{K}$.

and autoregressive coefficient in simple univariate AR(1) processes for the equity premium and price/dividend ratio. For the equity premium data, we use estimates from a simple dividend yield forecasting model (Fama and French (1988), Cochrane (2011)). Specifically, we estimate the risk premium by projecting the excess return based on the one-year lagged dividend yield and time trend. Monthly price/dividend data is sourced from Robert Shiller's webpage, while stock market excess returns and risk-free rates are obtained from Kenneth French's data library. All nominal quantities are adjusted for inflation using the annual CPI. The weighting matrix used to match the 4 parameters to the 10 targets is described in Appendix 5.7.

In Section 4.5, we further discipline the ambiguity aversion parameters ($\theta_1 = \theta_1^h$ and $\theta_2 = \theta_2^h$) using DEPs. Following Hansen and Sargent (2008), we require DEPs to exceed 10% to reflect reasonable concerns for robustness. In our calibration, the specialist's DEP for the robust control parameter θ_1 is 11.69%, while the household's DEP for θ_1^h is 16.74%. The specialist's DEP for the robust filtering parameter θ_2 is 28.25%, and the household's DEP for θ_2^h is 34.59%.

4.2 Estimates

Table 1 contains calibrated and estimated benchmark parameter values. The grid search constrained $\theta_1 = \theta_1^h$, $\theta_2 = \theta_2^h$, $\rho^h > \rho$, and imposed restriction (ii) in Proposition 3. This ensures a stationary wealth distribution and assumes that, absent discounting effects, the ambiguity aversion of households and specialists is the same. For each candidate parameter value, we simulate 10,000 independent observations over 1,000 years. We then average across repetitions and use the final 200 years to compute moments and regressions. Our model is simulated at monthly frequencies and aggregated to appropriate frequencies for comparison with the data.

Table 1: Benchmark Parameters

Panel A. Preferences and Intermediation		
ρ	Specialist Time Discount Rate	0.013
ρ^h	Household Time Discount Rate	0.023
θ_1 (θ_1^h)	Specialist (Household) Model Uncertainty Preference	0.03
θ_2 (θ_2^h)	Specialist (Household) State Uncertainty Preference	0.12
κ^s	Specialist Information Channel Capacity	0.04
κ^h	Household Information Channel Capacity	0.01
m	Intermediation Multiplier	4
Panel B. Equity Market		
$\bar{g}$	Mean Dividend Growth Rate	0.018
σ	Dividend Growth Volatility	0.185
σ_g	Unobserved Dividend Growth Volatility	0.007
ρ_g	Mean Reversion Rate of Unobserved Dividend Growth	0.06

The data moments and model estimates are summarized in Table 2. There are several points to notice. When agents are concerned about model misspecification, our model produces an 8.53%

equity premium per year, 17.74% return volatility per year, and a 48.12 Sharpe ratio, which are very close to the same moments (7.53%, 18.67%, and 40.34) in the data. Second, our model generates an average risk-free rate of 0.23%, with a standard deviation of 1.19% per year. Both moments are close to their data counterparts (0.17% and 1.09%). Hence, we avoid the risk-free rate puzzle. Third, and perhaps most important, remember that the HK model does not produce recurrent crises. Asymptotically, the constraint never binds. In contrast, in our model, the constraint binds 6.08% of the time. This implies crises occur on average about once every 16 years. It is important to keep in mind that the crises here are not exogenous events; they are *endogenous.* Although crises occur in response to negative shocks, the shocks themselves are not disasters. Nor are the crises here i.i.d. Their conditional probability depends on the distribution of wealth. This is consistent with recent work in the rare disasters literature, which emphasizes the non-i.i.d. nature of disasters (Tsai and Wachter (2015) provide a survey). In our model, a small negative shock in dividends will be amplified through ambiguity because ambiguity tightens the capital constraint. This amplification effect arises from the heterogeneous beliefs between the two agents and endogenously produces higher probabilities and long persistence of financial crises. The only significant discrepancy is the model's price/dividend ratio. The mean is about 50% too high, and the volatility is too low. We could easily match the mean by raising the rates of time preference. However, for given values of θ_1, this would reduce predicted risk premia. Increasing the θ_1 to compensate would then reduce detection error probabilities.

Table 2: Measurements and Estimates

	Data	Model		Data	Model
Risk Premium (%)	7.53	8.53	P/D Mean (%)	34.12	55.43
Return Volatility (%)	18.67	17.74	P/D Volatility (%)	16.96	3.68
Sharpe Ratio	40.34	48.12	Delegation Fee $\bar{K}$ (%)	2.50	2.46
Interest Rate (%)	0.17	0.23	Prob. of Crisis (%)		6.08
Interest Rate Volatility (%)	1.09	1.19	Specialist DEP for θ_1 (%)		11.69
Dividend Growth Mean (%)	1.54	1.26	Household DEP for θ_1^h (%)		16.74
Dividend Growth Volatility (%)	10.80	15.47	Specialist DEP for θ_2 (%)		28.25
Dividend Growth AR(1)	0.19	0.24	Household DEP for θ_2^h (%)		34.59

This table reports annualized unconditional moments. We simulated 1,000 years and 10,000 independent sample paths using our benchmark parameters. We report means from the final 200 years over all sample paths. DEPs are calculated based on the final 50 years. The data include the period 1929.01–2022.12.

Table 3 shows that our model successfully captures the time-varying and state-dependent equity premium and price/dividend ratio. Specifically, the autoregressive coefficients in the model are 0.986 and 0.997, which closely align with the data's coefficients (0.975 and 0.997) for the risk premium and price/dividend ratio, respectively. Moreover, the implied long-run means are also in close agreement: 0.001 and 0.151 in the model, compared to 0.001 and 0.122 in the data.

Table 3: Persistence

	Data	Model
Risk Premium: AR(1) Coefficient	0.975	0.986
Risk Premium: Intercept	0.001	0.001
P/D: AR(1) Coefficient	0.997	0.997
P/D: Intercept	0.122	0.151

This table reports AR(1) coefficients and constants estimated from risk premium and price/dividend ratio monthly data (1929.1–2022.12), as well as the model-implied coefficients and constants.

4.3 Sample Paths

Although it is encouraging that the model matches the unconditional moments of asset returns, many other models can do this as well. The value-added of the HK model is to explain the nonlinear, state-dependent dynamics of asset returns. For example, a significant portion of the average equity premium is generated during infrequent crisis episodes when it rises dramatically. Generating strong counter-cyclicality in risk prices and risk premia is more challenging. Perhaps the best way to assess this ability is to simply look at sample paths produced by the model. Figure 4 displays a representative sample path using our benchmark parameters. The time unit is a year. The red rectangular areas indicate times when the constraint binds. The top left panel plots the specialist's scaled wealth, which is the model's key endogenous state variable. The top right panel plots the drift distortions of the specialist and household, while the bottom left panel plots their difference. The bottom right panel plots the risk premium, with the black horizontal line depicting the sample mean.

These plots nicely reveal the model's key forces. As in HK, the top left plot shows that crises occur when the specialist's wealth declines. The top right and bottom left plots depict the new mechanism produced by model uncertainty. Note that as the specialist's wealth declines, he becomes relatively pessimistic, i.e., his relative drift distortion increases. This occurs because he endogenously becomes more exposed to the risky asset. This endogenous belief heterogeneity amplifies the rise in risk premia, as depicted in the bottom right plot. The risk premium rises by about 150 basis points, from about 8% to 9.7%. It is interesting to observe that the constraint can easily bind for as long as a decade, capturing the observed *persistence* of crises. For example, Nakamura, Steinsson, Barro, and Ursúa (2013) report a mean disaster duration of 6 years based on consumption data from a sample of 24 countries. In contrast, He and Krishnamurthy (2013) emphasize that most crisis models have difficulty capturing the duration of crises. Finally, note that these crisis episodes contribute significantly to average equity premia and Sharpe ratios. As you would expect from Figure 2, when the constraint is slack, the equity premium and Sharpe ratio are nearly constant.

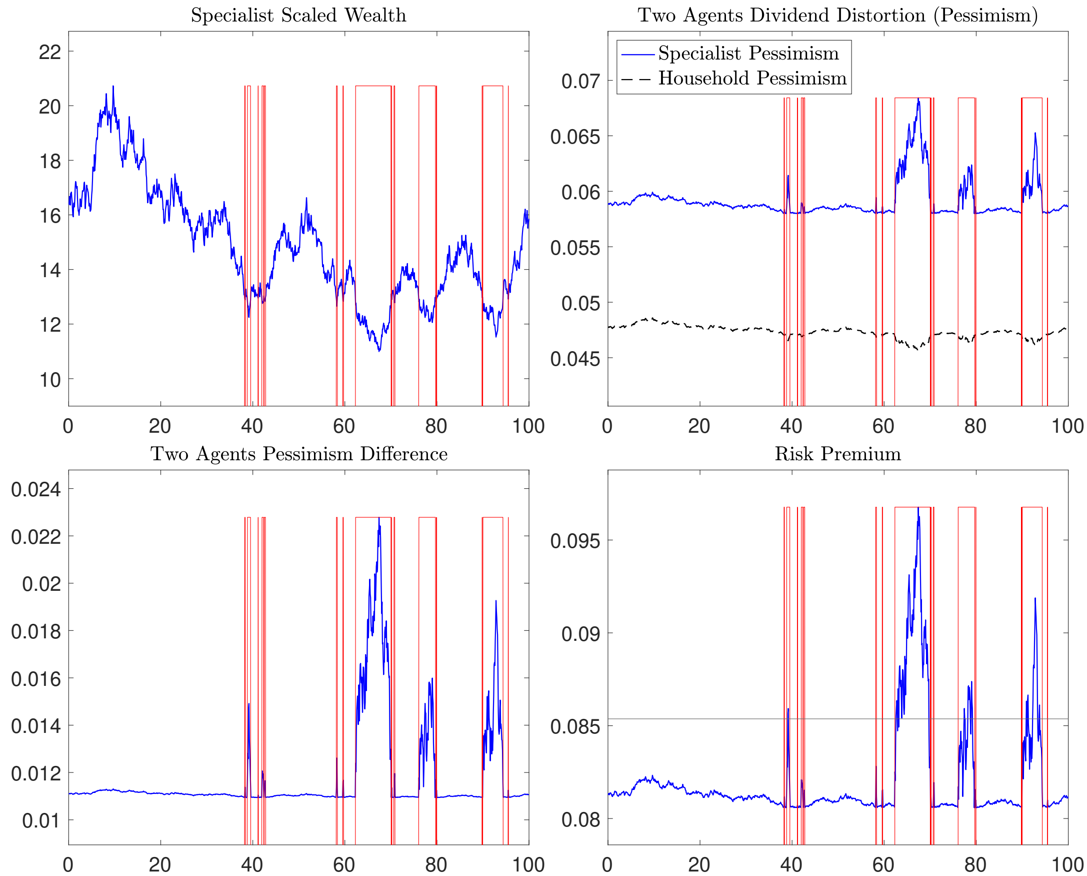

Figure 4: A Representative Sample Path

This figure plots a representative sample path over 100 years using the benchmark parameters. The horizontal axis represents years. Red rectangular regions indicate the constrained region. The black horizontal line in the bottom right panel represents the unconditional mean for the risk premium. Specialist scaled wealth and risk premium are defined in Equations (50) and (46), respectively. The dividend drift distortion from both agents are defined as $-\sigma\nu_t^s$ and $-\sigma\nu_t^h$.

4.4 Implications From Observed Capital Ratios

A 100–150 basis point increase in the risk premium is significant but still less than the increases observed during actual crises, where spreads often increase by several hundred basis points. We suspect that the failure of our model to fully match the magnitude of the increase arises from the fact that our model-generated wealth process produces less variation than observed in the data. We can check this by using data from He, Kelly, and Manela (2017). They collect data on market-value capital ratios for the New York Fed's primary dealers. These institutions actively trade in most, if not all, asset markets. Although there is heterogeneity across dealers, they compute a simple value-weighted average to correspond with our model's assumed 'representative' specialist.[16] We can then use this observed capital ratio in place of x_t in the equilibrium pricing equations of our

[16]There is also considerable heterogeneity across assets. However, their empirical results suggest that the model's assumption of a single risky asset might not be a bad approximation, since the estimated price of risk is quite similar across asset classes.

model. Figure 5 depicts the results for the period 1970.01–2022.12.

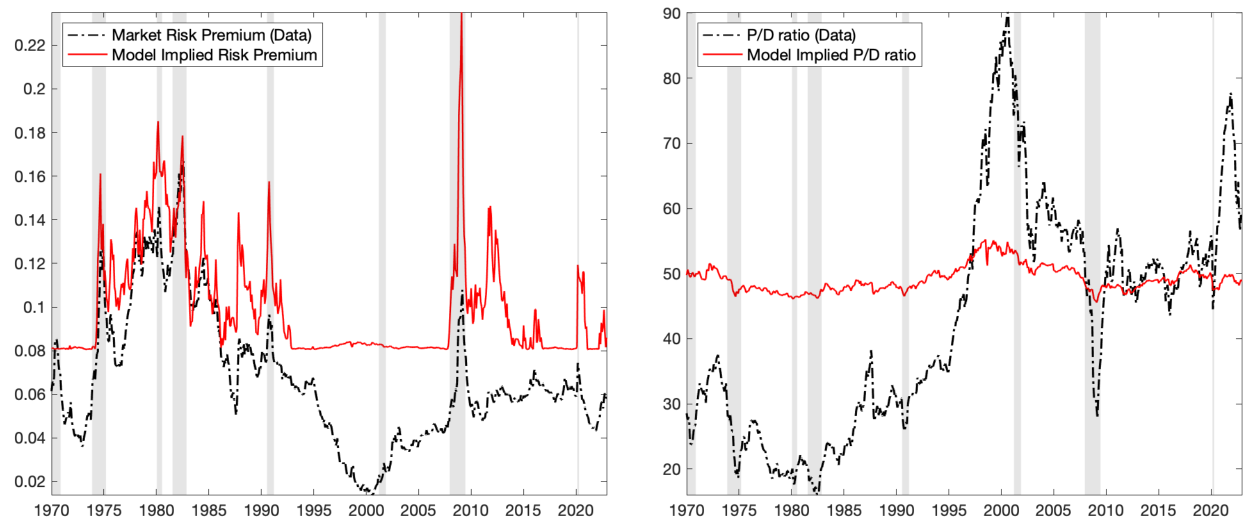

Figure 5: Model Implied Risk Premium and Price/Dividend Ratio
This figure plots the model-implied risk premium and price/dividend ratio by imputing He, Kelly, and Manela (2017) data (solid red lines) against the estimated risk premium from our dividend yield forecasting model and price/dividend ratio from Shiller's webpage (dashed black lines) for the period 1970.01–2022.12. The grey shaded bars depict NBER recessions.

The left panel displays the risk premium, and the right panel displays the price/dividend ratio. For comparison, the red solid lines are from the model, and the black dashed lines plot the data. With the data of He, Kelly, and Manela (2017), our model generates greater time-varying risk premium and price/dividend ratio. Perhaps not surprisingly, given Figure 2, during normal times, the model generates little variation in the equity premium and price/dividend ratio. Notice, however, that during recessions, the model generates significant spikes in the equity premium. For example, during the financial crisis of 2008-09, the model generates an increase in the equity premium of nearly 15 percentage points. Interestingly, the most apparent discrepancy occurs during the dotcom boom of the late 1990s, which corresponds to a very low equity premium. The inability of Intermediary Asset Pricing models to generate booms in asset prices and low risk premia is well-known. Krishnamurthy and Li (2021) show that adding 'sentiments' can remedy this deficiency. Therefore, a potential extension of our model to capture the larger variation in x_t is to incorporate stochastic volatility in dividends. We leave this for future research.

4.5 Detection-Error Probabilities

We have seen that the ambiguity aversion parameters $(\theta_1, \theta_1^h, \theta_2, \theta_2^h)$ play an important role in our model's ability to fit the unconditional moments and state dependent dynamics of asset returns. It is possible to view these parameters as solely a reflection of preferences and allow them to be unrestricted. However, following Hansen and Sargent (2008), we prefer to interpret them as reflecting both the presence of ambiguity and the agent's preference for robustness.[17] Under this interpretation, it is important to discipline the magnitude of $(\theta_1, \theta_1^h, \theta_2, \theta_2^h)$. In particular, we do not want to allow agents to hedge against empirically implausible alternative models.

Plausibility is quantified using detection error probabilities (DEPs). Agents are viewed as statisticians who attempt to discriminate among models using likelihood ratio statistics after seeing a finite sample, T, of observations. Letting $L_{\mathbb{P}}$ and $L_{\mathbb{Q}}$ represent the likelihood of the null/approximating model and the alternative/distorted model, the log-likelihood ratio is given by $l = \log\left(\frac{L_{\mathbb{P}}}{L_{\mathbb{Q}}}\right)$. Therefore, maximizing the likelihood ratio will successfully select model $\mathbb{P}$ when $l > 0$ (or equivalently, $L_{\mathbb{P}} > L_{\mathbb{Q}}$) and select model $\mathbb{Q}$ when $l < 0$ (or equivalently, $L_{\mathbb{Q}} > L_{\mathbb{P}}$). DEPs are based on an equally-weighted average of Type I and Type II errors:

$$\text{DEP} = \frac{1}{2}\text{Prob}\left(l < 0 \middle| \mathbb{P}\right) + \frac{1}{2}\text{Prob}\left(l > 0 \middle| \mathbb{Q}\right). \tag{53}$$

That is, the probability that agents select model $\mathbb{P}$ ($\mathbb{Q}$) when model $\mathbb{Q}$ ($\mathbb{P}$) is actually the true data generating process. Hence, a DEP$\in [0, 0.5]$ is analogous to a p-value. Agents assign equal prior probabilities of 0.5 to both models. Therefore, a DEP equal to 0.5 means the two models are identical and cannot be distinguished. When likelihood ratio statistics are large, DEPs are small, and models are easy to distinguish. DEPs converge to zero when models are very different or when there is a lot of data; in this case, agents have no concerns about ambiguity. Therefore, as stated in Hansen and Sargent (2008), DEPs greater than about 10% indicate a reasonable preference for ambiguity, i.e., agents want to make robust decisions against plausible alternative models. We compute DEPs using the Monte Carlo simulation strategy outlined in Chapter 9 of Hansen and Sargent (2008). Appendix 5.8 provides the proof and closed-form solutions. Figure 6 depicts the results using our benchmark parameters after observing $T = 50$ years of simulated data.[18]

As in Hansen and Sargent (2007), each agent has two robustness parameters, one pertaining to the control problem, θ_1^i, and one pertaining to the filtering problem, θ_2^i. The left panel of Figure 6 plots DEPs as a function of the robust control parameter, θ_1^i, holding constant the filtering parameter at its benchmark value. On the other hand, the right panel shows DEPs as a function of the robust filtering parameter, θ_2^i, with the robust control parameter held at its benchmark value. We can see that DEPs remain above 5% for values of θ^i as large as 0.05 in our sample of 50 years. In contrast, the frictionless model of Barillas, Hansen, and Sargent (2009) can only attain the Hansen-Jaganathan bound with DEPs below 0.05.

[17] In discrete-time models, it is possible to distinguish the presence of ambiguity from the agent's aversion to ambiguity (see e.g., Klibanoff, Marinacci, and Mukerji (2005), Ju and Miao (2012) and Gallant, Jahan-Parvar, and Liu (2019)). However, in the continuous-time limit, one must rescale θ_1 to maintain ambiguity aversion (see Hansen and Miao (2018) for details). Given the hidden state in our model, we could in principle distinguish ambiguity from ambiguity aversion. We leave this for future work.

[18] $T = 50$ years is intended to match the actual data from 1970–2022. Note, T cannot be infinity since DEPs will converge to zero.

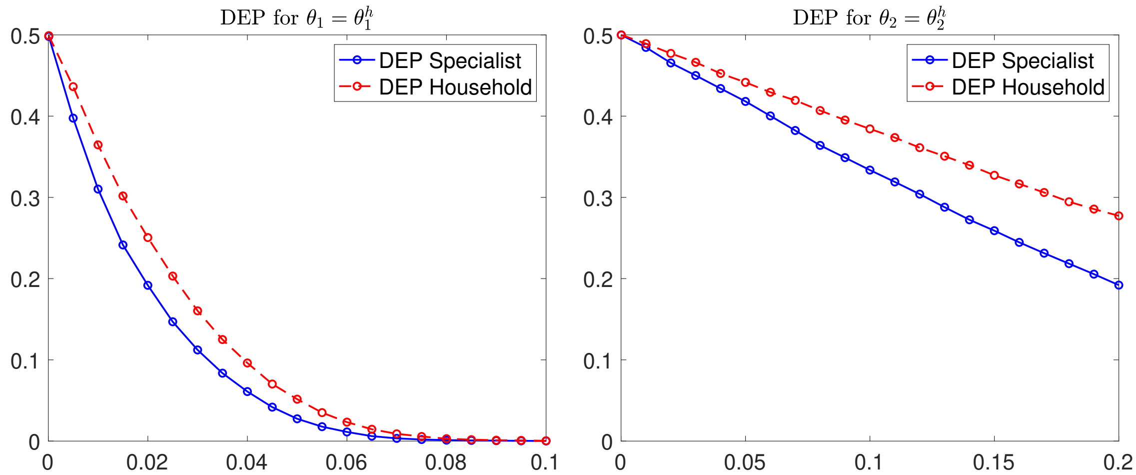

Figure 6: Detection Error Probability
This figure plots detection error probabilities for the specialist and household. In the left panel, we fix $\theta_2 = \theta_2^h$ at the benchmark value in Table 1 and then plot the DEP against $\theta_1 (= \theta_1^h)$. In the right panel, we fix $\theta_1 = \theta_1^h$ at the benchmark value in Table 1 and then plot the DEP against $\theta_2 (= \theta_2^h)$. Other parameters are from Table 2.

5 Conclusion

This paper introduces two new elements into the Intermediary Asset Pricing literature. First, we formalize the notion of 'complexity' in Cochrane's critique and motivate portfolio delegation by assuming agents face information processing constraints. Portfolio delegation allows households to purchase the higher channel capacity of financial intermediaries. Second, we assume that households and specialists are ambiguity averse. We demonstrate that capacity differences become more important in the presence of greater ambiguity, and ambiguity tightens the incentive constraint and amplifies its effects in driving asset returns. Our model can quantitatively match both the unconditional asset pricing moments and the time-varying prices of risk. Additionally, our model endogenously generates an empirically plausible probability and persistence of financial crises.

Although Lucas-style models are convenient for studying asset pricing, they have the drawback of eliminating feedback from asset prices to the real economy. It is widely believed that the financial crisis of 2008-09 featured such feedback. Therefore, a useful extension of our model would be to introduce ambiguity and information processing constraints into the production-based asset pricing model He and Krishnamurthy (2019).

Another extension would be to combine our analysis with the Complex Asset Markets model of Eisfeldt, Lustig, and Zhang (2023). They also study the pricing implications of a model that combines 'experts' and 'non-experts.' In contrast to our model, where channel capacity differences are exogenous, agents in their model can *choose* whether to become experts. Becoming an expert is beneficial because it reduces idiosyncratic investment risk. The key mechanism in their model is endogenous entry and exit, and their induced selection effects. However, in their model, funds cannot be reallocated across investors, and there is no trade in expertise. As a result, there is no moral hazard problem or capital constraint. Combining endogenous expertise, equilibrium entry and exit, portfolio delegation, and moral hazard would be challenging but also potentially quite fruitful.

Appendix

5.1 Solving the Agents' Optimization Problems

Proof of Lemma 1. Using Ito's Lemma, the HJB equation is:

$$\rho^h V^h = \sup_{\{C_t^h,\varepsilon_t^h\}} \inf_{\{\nu_t^h,\omega_t^h\}} \left[\ln C_t^h + \mathcal{D}V^h + \nu_t^h \sigma_{R,t}^h \varepsilon_t^h V_w^h + \omega_t^h \frac{Q_t}{\sigma} V_g^h + \frac{1}{2\theta_1^h} \left(\nu_t^h\right)^2 + \frac{1}{2\theta_2^h} \left(\omega_t^h\right)^2 \right], \quad (54)$$

where $\mathcal{D}[\cdot]$ is the Dynkin operator,

$$\mathcal{D}\left[V^h\right] = V_w^h \left[\varepsilon_t^h (\pi_{R,t} - k_t) + r_t W_t^h - C_t^h\right] + \frac{1}{2} V_{ww}^h (\varepsilon_t^h)^2 \sigma_{R,t}^2 + V_{wg}^h \varepsilon_t^h \sigma_{R,t} \frac{Q^h}{\sigma} + V_g^h \rho_g \left(\bar{g} - \hat{g}_t^h\right)$$
$$+ \frac{1}{2} V_{gg}^h \left(\frac{Q^{h2}}{\sigma^2} + 2\kappa^h Q^h\right) + V_x^h \mu_{x,t} x_t + \frac{1}{2} V_{xx}^h \sigma_{x,t}^2 x_t^2 + V_{wx}^h \sigma_{x,t} x_t \varepsilon_t^h \sigma_{R,t} + V_{xg}^h \sigma_{x,t} x_t \frac{Q^h}{\sigma}.$$

Solving the infimization part first yields $\nu_t^h = -\theta_1^h \varepsilon_t^h \sigma_{R,t} V_w^h$ and $\omega_t^h = -\theta_2^h \frac{Q^h}{\sigma} V_g^h$. Substituting them back into the HJB equation gives:

$$\rho^h V^h = \sup_{\{C_t^h,\varepsilon_t^h\}} \left[\ln C_t^h + \mathcal{D}V^h - \frac{\theta_1^h}{2} (\varepsilon_t^h \sigma_{R,t} V_w^h)^2 - \frac{\theta_2^h}{2} \left(\frac{Q^h}{\sigma} V_g^h\right)^2 \right]. \quad (55)$$

Note that ambiguity makes the agent dislike the variance of continuation utility. The optimal household consumption and portfolio rules under robustness are

$$C_t^h = \frac{1}{V_w^h}, \text{ and } \varepsilon_t^h = \frac{-V_w^h}{V_{ww}^h - \theta_1^h V_w^{h2}} \frac{(\pi_{R,t} - k_t)}{\sigma_{R,t}^2}, \quad (56)$$

respectively. Guessing and subsequently verifying the value function in the form of Equation (22), where

$$F^h\left(\hat{g}_t^h; Q^h\right) = A^h \left(\hat{g}_t^h - \bar{g}\right) + B^h, \quad (57)$$

we have $V_w^h = \frac{1}{\rho^h W_t^h}$, $V_{ww}^h = -\frac{1}{\rho^h (W_t^h)^2}$, $V_{gg}^h = 0$, and all the cross terms are zero, i.e., $V_{wx}^h = 0$, $V_{wg}^h = 0$, and $V_{xg}^h = 0$. Substituting these expressions into the FOCs in (56) yields the consumption and portfolio rules for households in Lemma 1. Further substituting these back into the HJB equation, we can obtain the following system of ODEs for $F^h\left(\hat{g}_t^h; Q^h\right)$ and $Y^h(x_t)$:

$$\rho^h F^h = \frac{\hat{g}_t^h - \bar{g}}{\rho^h} + \rho_g \left(\bar{g} - \hat{g}_t^h\right) F_g^h - \frac{\theta_2^h}{2} \frac{Q^{h2}}{\sigma^2} F_g^{h2}, \quad (58)$$

$$\rho^h Y^h = \ln \rho^h - 1 + \frac{\hat{r}_t^h}{\rho^h} + \frac{(\pi_{R,t} - k_t)^2}{2\rho^h \gamma^h \sigma_{R,t}^2} + Y_x^h \mu_{x,t} x_t + \frac{1}{2} Y_{xx}^h \sigma_{x,t}^2 x_t^2, \quad (59)$$

where $\hat{r}^h(x_t) = r_t - \left(\hat{g}_t^h - \bar{g}\right)$ is only a function of x_t. Solving (58) gives

$$A^h = \frac{1}{\rho^h (\rho^h + \rho_g)}, \text{ and } B^h = -\frac{\theta_2^h Q^{h2}}{2\rho^{h3} (\rho^h + \rho_g)^2 \sigma^2}. \quad (60)$$

Similarly, the specialist's problem can be solved following a similar procedure. The HJB equation for the specialist is given by:

$$\rho V = \sup_{\{C_t,\varepsilon_t\}} \inf_{\{\nu_t,\omega_t\}} \left[\ln C_t + \mathcal{D}V + \nu_t^s \sigma_{R,t}\varepsilon_t V_w + \omega_t \frac{Q^s}{\sigma} V_g + \frac{1}{2\theta_1}\left(\nu_t^s\right)^2 + \frac{1}{2\theta_2}\left(\omega_t^s\right)^2 \right], \tag{61}$$

where

$$\begin{aligned}\mathcal{D}\left[V\right] = {} & V_w\left[\varepsilon_t \pi_{R,t} + (q_t + r_t)W_t - C_t\right] + \frac{1}{2}V_{ww}{\varepsilon_t}^2\sigma_{R,t}^2 + V_{wg}\frac{\sigma_{R,t}}{\sigma}\varepsilon_t Q^s + V_g \rho_g\left(\bar{g} - \hat{g}_t^s\right) \\ & + \frac{1}{2}V_{gg}\left(\frac{Q^{s2}}{\sigma^2} + 2\kappa Q^s\right) + V_x \mu_{x,t} x_t + \frac{1}{2}V_{xx}\sigma_{x,t}^2 x_t^2 + V_{wx}\sigma_{x,t}x_t\varepsilon_t\sigma_{R,t} + V_{xg}\sigma_{x,t}x_t\frac{Q^s}{\sigma}.\end{aligned}$$

Solving the infimization part first yields $\nu_t^s = -\theta_1 \varepsilon_t \sigma_{R,t} V_w$ and $\omega_t^s = -\theta_2 \frac{Q^s}{\sigma} V_g$. Substituting them back into the HJB equation gives:

$$\rho V = \sup_{\{C_t,\varepsilon_t\}} \left[\ln C_t + \mathcal{D}V - \frac{\theta_1}{2}\left(\sigma_{R,t}\varepsilon_t V_w\right)^2 - \frac{\theta_2}{2}\left(\frac{Q^s}{\sigma}V_g\right)^2 \right]. \tag{62}$$

Optimal specialist consumption and portfolio rules are therefore

$$C_t = \frac{1}{V_w}, \text{ and } \varepsilon_t = \frac{-V_w}{V_{ww} - \theta_1 V_w^2}\frac{\pi_{R,t}}{\sigma_{R,t}^2}, \tag{63}$$

respectively. Guessing the value function takes the form of Equation (23), where $F^s\left(\hat{g}_t^s, Q_t^s\right)$ satisfies

$$F^s\left(\hat{g}_t^s; Q^s\right) = A^s\left(\hat{g}_t^s - \bar{g}\right) + B^s, \tag{64}$$

and under this conjectured form, Equation (63) yields the optimal consumption and portfolio rules for the specialist in Lemma 1. Further substituting these back into the HJB equation, we can obtain $F^s\left(\hat{g}_t^s; Q^s\right)$ and $Y\left(x_t\right)$ that solve the following system of ODEs:

$$\rho F^s = \frac{\hat{g}_t^s - \bar{g}}{\rho^s} + \rho_g\left(\bar{g} - \hat{g}_t^s\right)F_g^s - \frac{\theta_2}{2}\frac{Q^{s2}}{\sigma^2}F_g^{s2}, \tag{65}$$

$$\rho Y = \ln\rho - 1 + \frac{q_t + \hat{r}_t^s}{\rho^h} + \frac{\pi_{R,t}^2}{2\rho\gamma\sigma_{R,t}^2} + Y_x \mu_{x,t} x_t + \frac{1}{2}Y_{xx}\sigma_{x,t}^2 x_t^2, \tag{66}$$

where $\hat{r}^s\left(x_t\right) = r_t - \left(\hat{g}_t^s - \bar{g}\right)$. Solving (65) finally gives

$$A^s = \frac{1}{\rho\left(\rho + \rho_g\right)}, \text{ and } B^s = -\frac{\theta_2 Q^{s2}}{2\rho^3\left(\rho + \rho_g\right)^2\sigma^2}. \tag{67}$$

5.2 Solving the Optimal Delegation Problem

In the steady state, when $dQ_t^i = 0$, we have:

$$Q^{i2} + 2\sigma^2 \left(\kappa^i + \rho_g\right) Q^i - (\sigma_g \sigma)^2 = 0. \tag{68}$$

It is straightforward to show that

$$\frac{dQ^i}{d\kappa^i} = -\frac{\sigma^2 Q^i}{Q^i + \sigma^2 \left(\kappa^i + \rho_g\right)} < 0. \tag{69}$$

In the last section, we established that $F^h\left(\hat{g}_t^h; Q^h\right)$ takes the form of Equation (57). Evaluating this expression at $\hat{g} = \bar{g}$, we can deduce that the household's expected value function difference arising from delegation is given by:

$$F^h(\kappa) - F^h(\kappa^h) = \frac{\theta_2^h \left(Q^{h2} - Q^{s2}\right)}{2\sigma^2 \rho^{h3} \left(\rho^h + \rho_g\right)^2}, \tag{70}$$

which defines the delegation fee as provided in Proposition 1.

5.3 Proofs of Capital Constraints

In section 3.2, we defined the capital constraint as follows:

$$\begin{cases} \text{Unconstrained:} & m\varepsilon_t > \varepsilon_t^h,\ \beta_t \geq \frac{1}{1+m}, \\ \text{Constrained:} & m\varepsilon_t = \varepsilon_t^h,\ \beta_t = \frac{1}{1+m}. \end{cases}$$

The exposure fee k_t is determined by the supply and demand of risky asset exposure. The specialist's exposure supply is a step function:

$$\begin{cases} \text{Unconstrained:} & \varepsilon_t^h \in [0, m\varepsilon_t]\,, \text{ for any } \beta_t \in \left[\frac{1}{1+m}, 1\right] & \text{and } k_t = 0, \\ \text{Constrained:} & m\varepsilon_t \text{ with } \beta_t = \frac{1}{1+m} & \text{and } k_t > 0. \end{cases} \tag{71}$$

In contrast, the household's exposure demand depends negatively on k_t, and is $\varepsilon_t^h = \frac{\pi_{R,t} - k_t}{\gamma^h \sigma_{R,t}^2} W_t^h$. Notably, both the exposure supply and demand curves are influenced by ambiguity aversion. Figure 7 illustrates the equilibrium intermediary fee under two distinct cases. The left panel depicts the situation where the capital constraint is binding ($k_t > 0$), reflecting the scarcity of intermediary capital. Conversely, the right panel depicts the case when the constraint is not binding, resulting in intermediary capital being plentiful, and thus $k_t = 0$. In summary, the equilibrium intermediation flow fees can be characterized as follows:

$$k_t^U = 0 \text{ and } k_t > 0,$$
$$q_t^U = 0 \text{ and } q_t > 0,$$

in the unconstrained and constrained region, respectively.

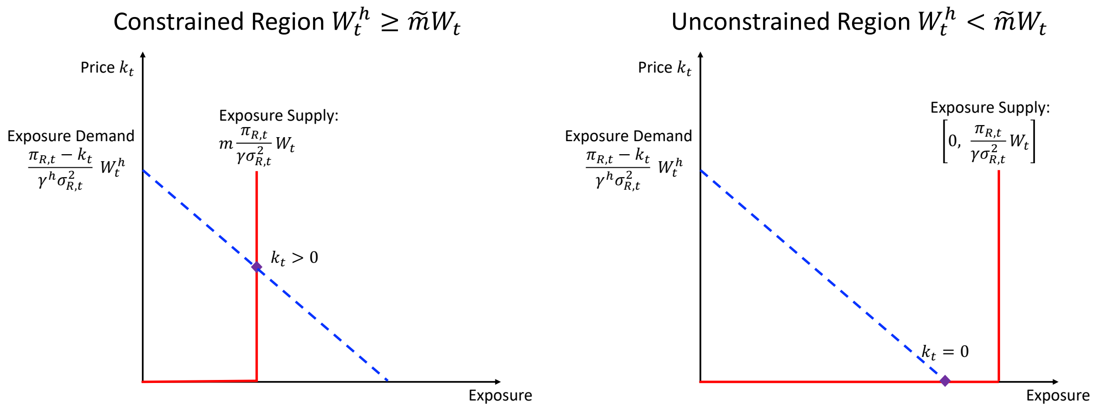

Figure 7: Equilibrium Intermediation Fee

Solving the optimal contract. In the unconstrained case, let's begin by analyzing the derivation of the optimal share of the return contract. Recall that this share is defined as $\beta_t \equiv \frac{\varepsilon_t}{\varepsilon_t + \varepsilon_t^h}$. By substituting the optimal risky exposure obtained from Equation (25), where $k_t = 0$, we can determine the choice of the share contract as follows:

$$\beta_t^U = \frac{\varepsilon_t}{\varepsilon_t + \varepsilon_t^h} = \frac{W_t}{W_t + \frac{\gamma}{\gamma^h} W_t^h} \text{ and } k_t = 0. \tag{72}$$

Now the specialist and household no longer hold the equity claims according to their wealth contributions as the benchmark case, but with a distortion term $\frac{\gamma}{\gamma^h}$, which equals the inverse of distortion on the capital constraint. Note that although the agency friction parameter m does not directly affect β_t^U in the unconstrained region, both robustness parameters distort the contract share alternatively. Replacing W_t^h with asset market clearing condition (37) yields:

$$\beta_t^U = \frac{W_t}{W_t + \frac{\gamma}{\gamma^h}(P_t - W_t)} = \frac{x_t}{x_t + \frac{\gamma}{\gamma^h}(P/D - x_t)} = \frac{1}{1 + \frac{\gamma}{\gamma^h}\left(\frac{1}{\rho^h x_t} - \frac{\rho}{\rho^h}\right)},$$

where we used the equilibrium price/dividend ratio (39). Additionally, due to the imposed assumption that $0 \leq \beta_t^U \leq 1$, x_t must be constrained within the range $[0, 1/\rho]$.

In the constrained region, the share of return is determined by the incentive constraint of the specialist. To prevent the specialist from shirking, households need to pay a positive intermediation fee and exposure price to the intermediary. As a result,

$$\beta_t = \frac{1}{1+m} \text{ and } k_t > 0.$$

Solving optimal portfolio holdings. Recall that the specialist's portfolio share is defined as $\alpha_t \equiv \varepsilon_t^I / T_t^I = (\varepsilon_t + \varepsilon_t^h)/(W_t + T_t^h)$. In the unconstrained region, $T_t^h = W_t^h$, households invest all their wealth in intermediation. This leads to $\alpha_t = (\varepsilon_t + \varepsilon_t^h)/(W_t + W_t^h)$. The equilibrium conditions

(33) and (37) yield

$$\alpha_t^U = 1.$$

In the constrained region, where $T_t^h = \tilde{m} W_t (\leq W_t^h)$, households allocate only a portion of their wealth to intermediation. Consequently, the specialist's portfolio share is given by:

$$\alpha_t = \frac{\varepsilon_t^I}{W_t + T_t^h} = \frac{P_t}{(1+\tilde{m})W_t} = \frac{P/D}{(1+\tilde{m})x_t} = \frac{\frac{1}{x_t} + \rho^h - \rho}{(1+\tilde{m})\rho^h}.$$

The risk exposure for the specialist is defined as

$$\varepsilon_t = \beta_t \varepsilon_t^I = \beta_t P_t, \tag{73}$$

where we used the market clearing condition (33). Combining this with the optimal β_t derived in Equation (41) gives

$$\varepsilon_t^U = \frac{1}{1 + \frac{\gamma}{\gamma^h}\left(\frac{1}{\rho^h x_t} - \frac{\rho}{\rho^h}\right)} P_t, \text{ and } \varepsilon_t = \frac{1}{1+m} P_t.$$

5.4 Solving for Asset Prices

The return volatility. The cumulative return process of the stock follows

$$dR_t = \frac{D_t dt + dP_t}{P_t} = \mu_{R,t} dt + \sigma_{R,t} d\hat{Z}_{D,t}^s. \tag{74}$$

Note that after delegation, both households and specialists use the specialist's channel capacity, which leads to only specialists engaging in filtering. Therefore, changes in returns are dependent on the shocks within the specialist's filtering process, $\hat{Z}_{D,t}^s$. Rewriting the price/dividend ratio (39) as $P_t = \frac{D_t + (\rho^h - \rho) W_t}{\rho^h}$, and then applying Ito's lemma, we obtain:

$$dR_t = \frac{D_t dt + dP_t}{P_t} = \frac{dD_t + \left(\rho^h - \rho\right) dW_t + \rho^h D_t dt}{\rho^h P_t}, \tag{75}$$

where the dividend process and the specialist's budget constraint under the physical measure are as follows:

$$dD_t = \hat{g}_t^s D_t dt + \sigma D_t d\hat{Z}_{D,t}^s, \tag{76}$$

$$dW_t = \left[\varepsilon_t \pi_{R,t} + (q_t + r_t) W_t - C_t\right] dt + \sigma_{R,t} \varepsilon_t d\hat{Z}_{D,t}^s. \tag{77}$$

The return volatility can be derived from matching the diffusion terms such that

$$\sigma_{R,t} = \frac{\sigma D_t + \left(\rho^h - \rho\right) \varepsilon_t \sigma_{R,t}}{\rho^h P_t} \Leftrightarrow \frac{\sigma D_t}{\rho^h P_t - (\rho^h - \rho)\varepsilon_t} = \frac{1}{P_t / D_t} \frac{\sigma}{\rho^h - (\rho^h - \rho)\beta_t}. \tag{78}$$

Using Equations (39) and (41), we obtain

$$\sigma_{R,t}^{U}=\frac{\sigma}{1+\Delta\rho x_t}\frac{\left(\rho^h\gamma^h-\rho\gamma\right)x_t+\gamma}{\rho\left(\gamma^h-\gamma\right)x_t+\gamma}\text{ and }\sigma_{R,t}=\frac{\sigma\rho^h}{1+\Delta\rho x_t}\frac{1+m}{m\rho^h+\rho},\tag{79}$$

as presented in the main text.

The risk premium. Rewriting the specialist's optimality condition (25), and combining it with (73), we obtain the following expression:

$$\pi_{R,t}=\frac{\gamma\sigma_{R,t}^2\varepsilon_t}{W_t}=\frac{\gamma\sigma_{R,t}^2\beta_t P_t}{W_t}=\frac{\gamma\sigma_{R,t}^2\beta_t(P_t/D_t)}{x_t}.\tag{80}$$

In the unconstrained region, we have

$$\pi_{R,t}^{U}=\frac{\gamma\sigma_{R,t}^{U2}\beta_t^U(P_t/D_t)}{x_t}=\frac{\sigma^2\gamma\gamma^h}{(1+\Delta\rho x_t)}\frac{\left[\left(\rho^h\gamma^h-\rho\gamma\right)x_t+\gamma\right]}{\left[\rho\left(\gamma^h-\gamma\right)x_t+\gamma\right]^2}.\tag{81}$$

By contrast, in the constrained region,

$$\pi_{R,t}=\frac{\gamma\sigma_{R,t}^2\beta_t(P_t/D_t)}{x_t}=\frac{\sigma^2\gamma\rho^h}{x_t\left(1+\Delta\rho x_t\right)}\frac{1+m}{\left(m\rho^h+\rho\right)^2}.\tag{82}$$

Solving the exposure price and intermediation fee. In the unconstrained region, $k_t^U=0$, while in the constrained region, $k_t\geq 0$. When the household's desired exposure demand equals the specialist's exposure supply (see Equation (25)), i.e., $\varepsilon_t^h(k_t)=m\varepsilon_t$, we arrive at:

$$\frac{\pi_{R,t}-k_t}{\gamma^h\sigma_{R,t}^2}W_t^h=m\frac{\pi_{R,t}}{\gamma\sigma_{R,t}^2}W_t,\tag{83}$$

which gives

$$k_t=\left(1-\tilde{m}\frac{W_t}{W_t^h}\right)\pi_{R,t}=\left(1-\frac{\tilde{m}\rho^h x_t}{1-\rho x_t}\right)\pi_{R,t},$$

where we use the fact that $\tilde{m}=\left(\gamma^h/\gamma\right)m$. Hence, the optimal per-unit-of-exposure intermediation fee can be summarized as

$$k_t^U=0,\text{ and }k_t=\frac{\sigma^2(1+m)}{\left(m\rho^h+\rho\right)^2}\left(\gamma-\frac{\rho^h\gamma^h m x_t}{1-\rho x_t}\right)\frac{\rho^h}{\left(1+\Delta\rho x_t\right)x_t},\tag{84}$$

by plugging in the risk premium derived above.

Likewise, the per-unit-of-specialist-wealth fee can be derived as follows:

$$q_t\equiv\frac{K_t}{W_t}=\frac{mk_t\varepsilon_t}{W_t}=\frac{mk_t}{\gamma}\frac{\beta_t P_t}{W_t}=\frac{m}{1+m}\frac{P_t/D_t}{x_t}k_t.\tag{85}$$

Finally, we can summarize

$$q_t^U = 0 \text{ and } q_t = \frac{\sigma^2 m}{\left(m\rho^h + \rho\right)^2 x_t^2}\left(\gamma - \frac{\rho^h \gamma^h m x_t}{1-\rho x_t}\right). \tag{86}$$

Solving for the risk free rate. From the household's Euler equation, we have

$$r_t = \rho^h - \frac{\mathcal{L}\left[u'\left(C_t^h\right)\right]}{u'\left(C_t^h\right)} = \rho^h - \frac{\mathcal{L}\left[\left(W_t^h\right)^{-1}\right]}{\left(W_t^h\right)^{-1}}, \tag{87}$$

where

$$\mathcal{L}\left[(W_t^h)^{-1}\right] = \mathcal{L}\left[(P_t - W_t)^{-1}\right] = -(P_t - W_t)^{-2}\mathbb{E}\left[d(P_t - W_t)\right] + (P_t - W_t)^{-3}\mathbb{E}\left[d^2(P_t - W_t)\right].$$

From Equations (37) and (38), we have $P_t - W_t = \frac{D_t}{\rho^h} - \frac{\rho}{\rho^h}W_t$, which leads to $d\left(P_t - W_t\right) = \frac{dD_t - \rho dW_t}{\rho^h}$. Combining with (76) and (77), we have:

$$\begin{aligned} d\left(P_t - W_t\right) &= \frac{(\hat{g}_t^s dt + \sigma \hat{Z}_{D,t}^s)D_t}{\rho^h} - \frac{\rho}{\rho^h}\left[\left(\frac{\varepsilon_t \pi_{R,t}}{W_t} + q_t + r_t - \rho\right)W_t dt + \sigma_{R,t}\varepsilon_t d\hat{Z}_{D,t}^s\right] \\ &= \left[\frac{1}{\rho^h}\hat{g}_t^s D_t - \frac{\rho}{\rho^h}\left(\frac{\varepsilon_t \pi_{R,t}}{W_t} + q_t - \rho + r_t\right)W_t\right]dt + \left(\frac{\sigma}{\rho^h}D_t - \frac{\rho}{\rho^h}\sigma_{R,t}\varepsilon_t\right)d\hat{Z}_{D,t}^s. \end{aligned}$$

This gives

$$\frac{\mathcal{L}\left[\left(W_t^h\right)^{-1}\right]}{\left(W_t^h\right)^{-1}} = \frac{\mathcal{L}\left[\left(P_t - W_t\right)^{-1}\right]}{\left(P_t - W_t\right)^{-1}} = \frac{-\frac{1}{\rho^h}\hat{g}_t^s D_t + \frac{\rho}{\rho^h}\left(\frac{\varepsilon_t \pi_{R,t}}{W_t} + q_t - \rho + r_t\right)W_t}{P_t - W_t} + \frac{\left(\frac{\sigma}{\rho^h}D_t - \frac{\rho}{\rho^h}\sigma_{R,t}\varepsilon_t\right)^2}{\left(P_t - W_t\right)^2}.$$

Therefore, the risk-free rate is given by:

$$r_t = \hat{g}_t^s + \rho^h - \rho\varDelta\rho x_t - \rho q_t x_t - \frac{\rho x_t\left(\left[\frac{\pi_{R,t}}{\gamma\sigma_{R,t}}\right]^2 - 2\sigma\frac{\pi_{R,t}}{\gamma\sigma_{R,t}}\right) + \sigma^2}{1 - \rho x_t}. \tag{88}$$

Using the expressions for $\pi_{R,t}/\sigma_{R,t}$ and q_t in the constrained and unconstrained regions by Equations (47) and (84), we obtain the risk free interest rate in the main text.

5.5 Solving the Stochastic Process of Aggregate State

In order to derive the unconditional mean and variance of risk premium and interest rate, we need to know the distribution of the state variable, $x_t = W_t/D_t$. Using Ito's formula, we have

$$\frac{dx_t}{x_t} = \frac{dW_t}{W_t} - \frac{dD_t}{D_t} - \frac{dD_t}{D_t}\frac{dW_t}{W_t} + \left(\frac{dD_t}{D_t}\right)^2.$$

Substituting the dividend process (76) and specialist's wealth process (77) into the above equation and matching the drift and diffusion coefficients of the aggregate state process, $\mu_{x,t}$ and $\sigma_{x,t}$, we get:

$$\begin{aligned}
\mu_{x,t} &= \sigma^2 - \rho + q_t + r_t - \hat{g}_t^s + \frac{1}{\gamma^2}\frac{\pi_{R,t}^2}{\sigma_{R,t}^2} - \frac{\sigma}{\gamma}\frac{\pi_{R,t}}{\sigma_{R,t}}, \\
&= \sigma^2 + (1-\rho x_t)(\Delta\rho + q_t) + \frac{\left(\frac{\pi_{R,t}}{\gamma\sigma_{R,t}}\right)^2(1-2\rho x_t) + (3\rho x_t - 1)\,\sigma\left(\frac{\pi_{R,t}}{\gamma\sigma_{R,t}}\right) - \sigma^2}{1-\rho x_t}, \\
\sigma_{x,t} &= \frac{\pi_{R,t}}{\gamma\sigma_{R,t}} - \sigma.
\end{aligned}$$

Putting the expressions of q_t, $\pi_{R,t}/\sigma_{R,t}$ and r_t from Equations (86), (47) and (49) back, we could obtain the scaled wealth process in the main text, where in the unconstrained region,

$$\mu_{x,t}^U = \Delta\rho\,(1-\rho x_t) + \sigma^2 + \sigma^2\frac{A_0^U x_t^2 + A_1^U x_t + A_2^U}{(1-\rho x_t)\left[\rho\left(\gamma^h - \gamma\right)x_t + \gamma\right]^2}, \tag{89}$$

$$\sigma_{x,t}^U = \sigma\left[\frac{\gamma^h}{\rho\left(\gamma^h-\gamma\right)x_t + \gamma} - 1\right], \tag{90}$$

where $A_0^U = \rho^2\left(\gamma^h - \gamma\right)\left(2\gamma^h + \gamma\right)$, $A_1^U = \rho\left(2\gamma^2 - 3\left(\gamma^h\right)^2 + 2\gamma\gamma^h\right)$, and $A_2^U = \left(\gamma^h\right)^2 - \gamma^2 - \gamma\gamma^h$.

In the constrained region they are given by

$$\mu_{x,t} = \Delta\rho\,(1-\rho x_t) + \sigma^2 + \sigma^2\frac{A_0 x_t^2 + A_1 x_t + A_2}{\left(m\rho^h + \rho\right)^2(1-\rho x_t)\,x_t^2}, \tag{91}$$

$$\sigma_{x,t} = \sigma\left[\frac{1}{\left(m\rho^h + \rho\right)x_t} - 1\right], \tag{92}$$

where $A_0 = \rho^h\left(\rho\gamma^h - \rho^h\right)m^2 + \rho\left(\rho\gamma + \rho^h\right)m + 2\rho^2$, $A_1 = -\rho^h\gamma^h m^2 - \left(\rho^h + 2\rho\gamma\right)m - 3\rho$, and $A_2 = \gamma m + 1$.

5.6 Existence of Stationary Distribution

By integrating the steady state KFP equation twice on both sides of x^c, we obtain the general solution (Karlin and Taylor (1981), pg. 221):

$$\begin{aligned}
f(x) = &\left\{C_1\left[\frac{1}{s(x)\sigma_X^2(x)}\right] + C_3\left[\frac{S(x)}{s(x)\sigma_X^2(x)}\right]\right\}\cdot \mathbb{1}(x \le x^c) \\
&+ \left\{C_2\left[\frac{1}{s(x)\sigma_X^2(x)}\right] + C_4\left[\frac{S(x)}{s(x)\sigma_X^2(x)}\right]\right\}\cdot \mathbb{1}(x \ge x^c),
\end{aligned} \tag{93}$$

where $s(x) = \exp\left\{-\int^x \left[\frac{2\mu_X(v)}{\sigma_X^2(v)}\right] dv\right\}$ and $S(x) = \int^x s(v)dv$ is the scale function.[19] To determine the constants of integration and derive necessary and sufficient existence conditions, we must examine the properties of $2\mu_X(v)/\sigma_X^2(v)$ at the two boundaries. Let's first consider the right (unconstrained) boundary, where $x = 1/\rho$. From Appendix 5.5 we have:

$$\mu_X^U(x_t) = \left[\Delta\rho(1-\rho x_t) + \sigma^2 + \sigma^2 \frac{A_0^U x_t^2 + A_1^U x_t + A_2^U}{\gamma^2(1-\rho x_t)(1-\rho H x_t)^2}\right] x_t, \tag{94}$$

$$\sigma_X^U(x_t) = -\sigma H \left(\frac{1-\rho x_t}{1-\rho H x_t}\right) x_t, \tag{95}$$

where $H = 1 - \frac{\gamma^h}{\gamma}$ and theA_j^U coefficients are given in Appendix 5.5. It is immediately clear that $(\sigma_X^U(1/\rho))^2 = 0$, and $\mu_X^U(1/\rho) = \sigma^2(1+0/0)/\rho$ contains 0/0. By applying L'Hopital's rule, we get:

$$\lim_{x\to 1/\rho} \frac{A_0^U x_t^2 + A_1^U x_t + A_2^U}{\gamma^2(1-\rho x_t)(1-\rho H x_t)^2} = \frac{2A_0^U x_t + A_1^U}{-\rho\gamma^{h2}} = -1,$$

which gives $\mu_X^U(1/\rho) = 0$ as well. Hence, to evaluate $\lim_{x\to 1/\rho} \mu_X^U(x)/(\sigma_X^U(x))^2$, we first simplify the expression:

$$\frac{\mu_X^U(x_t)}{\left(\sigma_X^U(x_t)\right)^2} = \frac{\left[1 + \frac{\Delta\rho}{\sigma^2}(1-\rho x_t)\right]\gamma^2(1-\rho H x_t)^2(1-\rho x_t) + A_0^U x_t^2 + A_1^U x_t + A_2^U}{\gamma^2 H^2 (1-\rho x_t)^3}.$$

It turns out that $\frac{\mu_x^U(1/\rho)}{(\sigma_x^U(1/\rho))^2}$ is 0/0 as well, so we must apply L'Hopital's rule again to both the numerator and denominator, which yields:

$$[\text{num}]' = 2A_0^U x_t + A_1^U - \gamma^2\rho(1-\rho H x_t)\left[H\left(2\frac{\Delta\rho}{\sigma^2}(1-\rho x_t)(1-2\rho x_t) - 3\rho x_t + 2\right) - 2\rho\frac{\Delta\rho}{\sigma^2}x_t + 2\frac{\Delta\rho}{\sigma^2} + 1\right]$$
$$[\text{den}]' = -3\gamma^2 H^2 (1-\rho x_t)^2 x + \gamma^2 H^2 (1-\rho x)^3.$$

Clearly, at the right boundary,

$$[\text{num}(1/\rho)]' = \frac{2}{\rho}A_0^U + A_1^U - \rho\gamma^2(1-H)^2 = 0$$
$$[\text{den}(1/\rho)]' = 0.$$

Taking second derivatives of the numerator and denominator,

$$[\text{num}]'' = 2\left[A_0^U + \gamma^2\rho^2\left[H\left[H\left(-6\rho\frac{\Delta\rho}{\sigma^2}x_t(1-\rho x_t) + \frac{\Delta\rho}{\sigma^2} - 3\rho x_t + 1\right) - 6\rho\frac{\Delta\rho}{\sigma^2}x_t + 4\frac{\Delta\rho}{\sigma^2} + 2\right] + \frac{\Delta\rho}{\sigma^2}\right]\right]$$
$$[\text{den}]'' = 6\gamma^2 H^2 \rho(1-\rho x_t)x - 3\gamma^2 H^2(1-\rho x)^2 - 3\gamma^2 H^2 (1-\rho x)^2 \rho,$$

[19] As noted by Karlin and Taylor (1981, p. 195), the lower limits of integration in these expressions are unimportant as long as they lie within the support of the distribution.

we find:

$$[\text{num}(1/\rho)]'' = \frac{2\rho^2}{\sigma^2}(\gamma^{h2}\Delta\rho + \gamma\gamma^h\sigma^2 - \gamma^2\sigma^2) \tag{96}$$

$$[\text{den}(1/\rho)]'' = 0. \tag{97}$$

Since we need $\lim_{x\to 1/\rho} \frac{2\mu_x^U(x_t)}{\sigma_x^U(x_t)^2} < 0$, we need $[\text{num}(1/\rho)]''$ to be negative. Hence, if the second restriction (ii) given in Proposition 3 is satisfied, $\mu_x^U(1/\rho)/(\sigma_x^U(1/\rho))^2 = -\infty$. Otherwise, $\mu_x^U(1/\rho)/(\sigma_x^U(1/\rho))^2 = \infty$. From Karlin and Taylor (1981), the nature of the boundary depends on the scale function $S(1/\rho)$ and the 'speed function' $M(1/\rho)$, where

$$M(x) = \int^x \frac{1}{(\sigma_X^U(v))^2} \exp\left\{\int^v \frac{2\mu_X^U(u)}{(\sigma_X^U(u))^2} du\right\} dv.$$

It is apparent that if the restriction in Proposition 3 is violated, then $S < \infty$ and $M = \infty$. In this case, $x = 1/\rho$ is a 'regular' (absorbing) boundary, and a non-degenerate distribution fails to exist. However, if the restriction is satisfied, then $S = \infty$ and $N < \infty$, where

$$N(1/\rho) = \int_x^{1/\rho} S[x, v]dM(v).$$

Hence, from Karlin and Taylor (1981, pgs. 234-36), $x = 1/\rho$ is an 'entrance' boundary, which is unattainable in finite mean-time. Finally, since $S(1/\rho) = \infty$, it is clear that for the density to remain bounded we must have $C_4 = 0$ for $x \geq x^c$.

Let's now turn to the left (constrained) boundary. This occurs when $x = 1/(\rho + m\rho^h)$. Referring to Appendix 5.5 we have

$$\mu_X(x_t) = \left[\Delta\rho(1 - \rho x_t) + \sigma^2 + \sigma^2 \frac{A_0 x_t^2 + A_1 x_t + A_2}{(m\rho^h + \rho)^2 (1 - \rho x_t) x_t^2}\right] x_t, \tag{98}$$

$$\sigma_X(x_t) = \sigma\left[\frac{1}{(m\rho^h + \rho) x_t} - 1\right] x_t. \tag{99}$$

The lower boundary of x is $1/(\rho + m\rho^h)$ instead of 0 because at this point $\sigma_X = 0$, and x can never move below the lower bound. One can readily verify, using the expressions for the constrained A_i coefficients given in Appendix 5.5, that if the first restriction (i) given in Proposition 3 is satisfied, then $\mu_X > 0$ at the left-boundary. As a result, once again we have $S = \infty$ and $N < \infty$ at the left-boundary. Again from Karlin and Taylor (1981), this implies the left-boundary is an entrance boundary. Finally, since $S = \infty$ at the left-boundary, we must set $C_3 = 0$ for $x \leq x^c$.

5.7 Indirect Inference

The distance is measured by the quadratic form

$$\mathbb{G}_T(\Psi) = G_T(\Psi)'\Omega_T G_T(\Psi), \;\; \theta_1 = \theta_1^h \text{ and } \rho < \rho^h,$$

where $\Psi = \left[\rho \ \rho^h \ \theta_1 \ \sigma\right]'$ is a 4×1 vector. $G_T(\Psi)$ is the moment condition involving the data and parameters. We use $T = 10,000$ to calculate the sample averages. Ω_T is an 10×10 symmetric and positive definite weighting matrix,

$$\Omega_T = \begin{bmatrix} \frac{1}{\lambda_1^2} & 0 & 0 & 0 \\ 0 & \frac{1}{\lambda_2^2} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \frac{1}{\lambda_{10}^2} \end{bmatrix}.$$

The sample moment conditions are defined as follows: $G_{iT}(\Psi) = \left(\hat{\lambda}_i - \lambda_i\right), i = 1, \ldots, 10$, where λ_1 = AR(1) coefficient of equity premium, λ_2 = AR(1) intercept of equity premium, λ_3 = AR(1) coefficient of price/dividend ratio, λ_4 = AR(1) intercept of price/dividend ratio, λ_5 = mean equity premium, λ_6 = mean risk free rate, λ_7 = mean price/dividend ratio, λ_8 = price/dividend ratio volatility, λ_9 = mean return volatility, and λ_{10} = probability that the constraint binds (the crisis happens). π_τ is the time series of equity premium with $\tau = 2,400$ (200 years). $\hat{\cdot}$ denotes the model estimated corresponding values.

We set the priors of the parameters as follows: $\rho \in [.005, .03]$, $\rho^h \in [.005, .03]$, $\theta_1 \in [.01, .1]$, and $\sigma \in [.1, .25]$. Finally, we use indirect inference to obtain the estimated parameters $\hat{\Psi}_{GMM} = \arg\min_{\Psi} \{G_T(\Psi)' \Omega_T G_T(\Psi)\}$.

5.8 Detection Error Probability

Both specialists and households seek robust decision rules against different models and different states. As a result, perturbations about the distribution of shocks to dividends capture the robust control preference; and perturbations about the distribution of hidden state capture the robust filtering preference. The null/approximating models under two types of ambiguity are therefore:

$$\frac{dD_t}{D_t} = \hat{g}_t^i dt + \sigma d\tilde{Z}_{D,t}^i, \tag{100}$$

$$d\hat{g}_t^i = \rho_g \left(\bar{g} - \hat{g}_t^i\right) dt + \frac{Q^i}{\sigma} d\tilde{Z}_{g,t}^i + \sqrt{2\kappa^i Q^i} d\hat{Z}_{s,t}^i, \tag{101}$$

where $\tilde{Z}_{D,t}^i$ and $\tilde{Z}_{g,t}^i$ are i.i.d under model $\mathbb{P}$. The alternative/distorted models are:

$$\frac{dD_t}{D_t} = \left[\hat{g}_t^i + \sigma \nu^i(x_t)\right] dt + \sigma d\check{Z}_{D,t}^i, \tag{102}$$

$$d\hat{g}_t^i = \left[\rho_g \left(\bar{g} - \hat{g}_t^i\right) + \frac{Q^i}{\sigma} \omega^i\right] dt + \frac{Q^i}{\sigma} d\check{Z}_{g,t}^i + \sqrt{2\kappa^i Q^i} d\hat{Z}_{s,t}^i, \tag{103}$$

where $\check{Z}_{D,t}^i$ and $\check{Z}_{g,t}^i$ are i.i.d. model $\mathbb{Q}$.

Let $L_{\mathbb{P}}$ and $L_{\mathbb{Q}}$ be the likelihood of the null model and the alternative model, respectively. The log-likelihood ratio is defined as $l = \log\left(\frac{L_{\mathbb{P}}}{L_{\mathbb{Q}}}\right)$. When the null/approximating model $\mathbb{P}$ generates the data, the log-likelihood ratios for specialists and households are as follows:

$$(l|\mathbb{P})^s = \frac{1}{2}\int_0^T \left(w_t^{s'} w_t^s\right) dt - \int_0^T w_t^{s'} d\epsilon_t^s, \tag{104}$$

$$(l|\mathbb{P})^h = \frac{1}{2}\int_0^T \left(w_t^{h'} w_t^h\right) dt - \int_0^T w_t^{h'} d\epsilon_t^h \tag{105}$$

where $w_t^s = \begin{bmatrix} \nu_t^s \\ \omega^s \end{bmatrix}$, $w_t^h = \begin{bmatrix} \nu_t^h \\ \omega^h \end{bmatrix}$, $\epsilon_t^s = \begin{bmatrix} \tilde{Z}_{D,t}^s \\ \tilde{Z}_{g,t}^s \end{bmatrix}$, and $\epsilon_t^h = \begin{bmatrix} \tilde{Z}_{D,t}^h \\ \tilde{Z}_{g,t}^h \end{bmatrix}$. Note that $\nu^i(x_t)$ is a function of x_t, where $dx_t = \mu_{x,t}x_t dt + \sigma_{x,t}x_t d\tilde{Z}_{D,t}^i$, while ω^i is a constant. We provide the closed-form solution for $\nu^i(x_t)$ below, and ω^i is provided in Lemma 1.

If the alternative/distorted model $\mathbb{Q}$ is the actual data-generating process, we have:

$$(l|\mathbb{Q})^s = -\frac{1}{2}\int_0^T \left(w_t^{s'} w_t^s\right) dt - \int_0^T w_t^{s'} d\epsilon_t^s, \tag{106}$$

$$(l|\mathbb{Q})^h = -\frac{1}{2}\int_0^T \left(w_t^{h'} w_t^h\right) dt - \int_0^T w_t^{h'} d\epsilon_t^h. \tag{107}$$

where $w_t^s = \begin{bmatrix} \nu_t^s \\ \omega^s \end{bmatrix}$, $w_t^h = \begin{bmatrix} \nu_t^h \\ \omega^h \end{bmatrix}$, $\epsilon_t^s = \begin{bmatrix} \check{Z}_{D,t}^s \\ \check{Z}_{g,t}^s \end{bmatrix}$, and $\epsilon_t^h = \begin{bmatrix} \check{Z}_{D,t}^h \\ \check{Z}_{g,t}^h \end{bmatrix}$. Under model $\mathbb{Q}$, the evolution of x_t is governed by $dx_t = \left[\mu_{x,t}x_t + \sigma_{x,t}x_t\nu_t^i(x_t)\right] dt + \sigma_{x,t}x_t d\check{Z}_{D,t}^i$.

Finally, the detection error probability is defined as

$$\text{DEP}^s = \frac{1}{2}\text{Prob}\left((l|\mathbb{P})^s < 0\right) + \frac{1}{2}\text{Prob}\left((l|\mathbb{Q})^s > 0\right), \tag{108}$$

$$\text{DEP}^h = \frac{1}{2}\text{Prob}\left((l|\mathbb{P})^h < 0\right) + \frac{1}{2}\text{Prob}\left((l|\mathbb{Q})^h > 0\right). \tag{109}$$

We now derive the closed-form solution for $\nu^i(x_t)$. Based on Lemma 1, the relative entropy from the robust control problems of the household and the specialist can be formulated as follows:

$$\nu_t^h = -\frac{\theta_1^h \sigma_{R,t}\varepsilon_t^h}{\rho^h W_t^h} = -\frac{\theta_1^h}{\rho^h\gamma^h}\frac{\pi_{R,t} - k_t}{\sigma_{R,t}} = \frac{1-\gamma^h}{\gamma^h}\left(\frac{\pi_{R,t}}{\sigma_{R,t}} - \frac{k_t}{\sigma_{R,t}}\right), \tag{110}$$

$$\nu_t^s = -\frac{\theta_1 \sigma_{R,t}\varepsilon_t}{\rho W_t} = -\frac{\theta_1}{\rho\gamma}\frac{\pi_{R,t}}{\sigma_{R,t}} = \frac{1-\gamma}{\gamma}\frac{\pi_{R,t}}{\sigma_{R,t}}. \tag{111}$$

Substituting the optimal solutions for $\frac{\pi_{R,t}}{\sigma_{R,t}}$, k_t, and $\sigma_{R,t}$ derived in Equations (47), (45) and (84), we obtain the following expressions:

$$\left(\nu_t^h\right)^U = \frac{\sigma\gamma\left(1-\gamma^h\right)}{\rho\left(\gamma^h - \gamma\right)x_t + \gamma}, \text{ and } \nu_t^h = \frac{\sigma\left(1-\gamma^h\right)m\rho^h}{\left(m\rho^h + \rho\right)\left(1-\rho x_t\right)}, \tag{112}$$

$$\left(\nu_t^s\right)^U = \frac{\sigma\gamma^h\left(1-\gamma\right)}{\rho\left(\gamma^h - \gamma\right)x_t + \gamma}, \text{ and } \nu_t^s = \frac{\sigma\left(1-\gamma\right)}{\left(m\rho^h + \rho\right)x_t}. \tag{113}$$

for the unconstrained case and constrained cases, respectively.

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