Strategic Asset Allocation and Optimal Consumption for Inattentive Investors with Non-tradable Labor Income*

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Abstract

This paper explores how the introduction of rational inattention (RI) – that agents process information subject to finite channel capacity – affects strategic asset allocation in the presence of nontradable labor income in an otherwise standard intertemporal model of portfolio choice. We solve the model explicitly and then show that introducing RI increases the relative importance of the income-hedging demand of the risky asset via increasing the long-run consumption risk facing inattentive investors. We also examine how the correlation between labor income and the equity return affects the stochastic properties of the joint dynamics of consumption growth and the equity return. Finally, we show that RI increases the implied equity premium because investors with non-tradable labor income and RI face greater long-run consumption risk and thus require higher compensation in equilibrium.

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Keywords: Rational Inattention, Long-run Consumption Risk, Hedging Demand, Nontradable Labor Income, Strategic Portfolio Choice.

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1. Introduction

This paper examines how non-tradable labor income and information-processing constraints affect the optimal asset allocation and consumption of inattentive investors in an otherwise standard intertemporal portfolio choice model. For most individual investors, human wealth, the expected present value of their current and future labor earnings, is a major fraction of their total wealth. The key difference between financial wealth and human wealth is that the latter is a non-tradable asset because it is difficult to sell claims against future labor income due to the moral hazard problem. Recently many studies have examined the effects of non-tradable labor income on the optimal share of financial wealth invested in the risky asset. For example, Heaton and Lucas (2000) studied how the presence of background risks influences portfolio allocations and found that labor income risk is weakly positively correlated with equity returns. Viceira (2001) examined the effects of labor income risk on optimal consumption and portfolio choice for both employed and retired investors. Cocco, Gomes, and Maenhout (2005) solved a life cycle model of consumption and portfolio choice with non-tradable labor income and borrowing constraints, and found that ignoring labor income generates large utility costs. Bodie, Merton, and Samuelson (1992) studied how endogenous labor supply affects the allocation of financial wealth between the riskless and risky assets. However, to obtain explicit solutions, they assume that wages are non-stochastic, or wages are stochastic but are perfectly correlated with the equity return. Chan and Viceira (2005) generalized the work of Bodie et al. (1992) to an incomplete market setting, and find that when labor income risk is idiosyncratic, endogenous labor supply can have significant positive effects on the share invested in the risky asset relative to the case in which labor income is exogenous.

An implicit but key assumption in the above full-information rational expectations (RE) models of portfolio choice is that investors have *unlimited* information-processing capacity and thus can observe the state related to their financial decisions without error. Consequently, they can react *instantaneously and completely* to any innovations to equity returns. However, the assumption that ordinary investors can observe the relevant state without errors is too strong to be consistent with the reality because perfect observation requires infinite information processing ability that normal agents may not possess in the real world. In fact, ordinary people face many competing demands for their information capacity every period, so the amount of capacity devoted to processing financial information could be much lower than their total capacity. Hence, it is reasonable to assume that ordinary people only have *finite* information-processing capacity. As a result, they cannot observe the state perfectly and thus have to react to the innovations *slowly and incompletely.* Sims (2003) first introduced this type of information capacity constraints into economics and called it "rational inattention" (henceforth, RI). Given

the slow adjustment in consumption to the wealth shocks due to RI, the quantity of the risk of the risky portfolio should be determined by its ultimate (long-term) consumption risk instead of its contemporaneous risk. Parker (2003) and Parker and Jullard (2005) presented convincing empirical evidence to argue that the ultimate consumption risk is a better measure for the riskiness of the risky portfolio than the contemporaneous risk because in the data consumption takes many periods to adjust to the innovations to risky assets. Therefore, if the ultimate consumption risk is greater than the contemporaneous risk under RI, investors with finite capacity would choose to invest less in the risky asset. In this case, it is irrational for investors to invest a large fraction of their wealth in the risky asset if they cannot devote enough channel capacity to monitoring their financial wealth because the innovations to their financial wealth generate large consumption risk in the long run.

In this paper we focus on two key aspects of labor income risk: the variance of labor income and the correlation between labor income and the equity return. We first consider idiosyncratic labor income risk that is uncorrelated with the equity return. In this case the optimal share invested in the risky asset is similar to that obtained in Luo (2010), except that the elasticity of consumption with respect to wealth is less than 1.[1] Consequently, modeling labor income risk increases the optimal share invested in the risky asset. When the equity return is correlated with labor income, there is a income-hedging component of the optimal allocation to the risky asset. For example, if the labor income risk is positively correlated with the shock to the equity return, the equity is less desirable because it offers a bad hedge against negative labor income shocks. The main objective of this paper is to apply the RI idea in an intertemporal model of portfolio choice with non-tradable labor income and examine how finite information-processing ability of inattentive investors affects strategic asset allocation, consumption, and the equilibrium equity premium. As the first contribution of this paper, we explicitly solve for optimal consumption and portfolio rules in an RI version of the intertemporal model of portfolio choice with non-tradable labor income after considering long-run consumption risk. Specifically, we show that consumption adjusts slowly to the innovation to the equity return because investors with finite capacity have to take some time to digest new information about changes in the true state. Consequently, the long-run consumption risk is a better measure for risk facing inattentive investors than the contemporaneous consumption risk. Since long-run consumption risk is higher for investors with low capacity, they choose to invest less in the risky asset. Furthermore, we show that introducing RI also increases the relative importance of the income-hedging demand to the traditional speculation demand of the risky asset via increasing the long-run consumption risk. The intuition behind this result is that under RI

[1] Note that in the model without nontradable labor income (or the retired investors' portfolio choice problem described in Viceira, 2001), the elasticity is exactly 1.

the innovation to the equity return has *systematic* effects on consumption growth affected by both the equity return and labor income.

We then show that the long-term consumption risk and the two demands for the risky asset depend on the interactions of the the degree of RI, the discount factor, the correlation between the equity return and labor income risk, and the wealth-to-income ratio. In this infinite horizon setting, the discount factor can be used to characterize the effective investment horizon;[2] hence, the fact that the demand for the risky asset depends on the discount factor also means that the investment horizon matters for portfolio allocation. In addition, we show that the volatility of consumption growth is increasing with the degree of inattention and the correlation between the two risks. The intuition is that the positive correlation increases the total fundamental uncertainty facing inattentive investors, which leads to larger volatility of consumption growth. Finally, we show that RI amplifies the role of the incme-hedging motive and thus increases the implied equity premium because inattentive investors with non-tradable labor income and finite capacity face greater long-run consumption risk and thus require higher compensation in equilibrium.

Some recent papers incorporate explicit information processing constraints into a variety of theoretical models and explore how they affect individual decisions, optimal monetary policy, and equilibrium. For example, Peng (2004) explored the effects of information constraints on the equilibrium asset price dynamics and consumption behavior under the continuous-time CARA framework. Peng and Xiong (2005) discussed investors' attention and overconfidence. Huang and Liu (2007) showed that inattention can lead to infrequent rebalances and inattention to important news may make investors overinvest or under-invest.[3] Abel, Eberly, and Panageas (2009) explored how the interactions of inattention and transactions costs rationalize the infrequent adjustments. Luo (2008) examined how RI helps explain the excess smoothness and excess sensitivity puzzles in the consumption literature. Batchuluun, Luo, and Young (2008) showed that fully-nonlinear portfolio decisions are discrete in a simple two-period economy. Mondria (2010) examined how investors optimally choose the composition of their information subject to an information flow constraint. Van Nieuwerburgh and Veldkamp (2010) built a general model to show that information acquisition can rationalize investing in a diversified fund and a concentrated set of assets. For earlier works on optimal asset allocation and asset pricing under incomplete information, see Detemple (1986), Gennotte (1986), Brennan (1998),

[2] That is, the lower the time discount factor, the shorter the investment horizon.

[3] They used "rational inattention" to describe a situation in which information is updated *completely but infrequently*. This alternative formulation is more tractable but is different from Sims' RI idea – it permits agents to process infinite amounts of information when they choose to do so. However, the essence of RI proposed by Sims (2003) is that agents cannot use all *available* information because most of the time they are not paying that much attention to market signals and thus cannot digest all of the available information.

Xia (2001), among others.

This paper is organized as follows. Section 2 presents an intertemporal portfolio choice model with non-tradable labor income and RI. Section 3 examines the implications of non-tradable labor income and RI for optimal consumption, long-run risk, and strategic asset allocation. Section 4 discusses the implications for consumption dynamics and the equilibrium equity premium. Section 5 concludes.

2. A Portfolio Choice Model with Labor Income and Rational Inattention

In this section, we first present and review a standard intertemporal portfolio choice model with non-tradable labor income in the vein of Merton (1969) and Viceira (2001), and then discuss how to incorporate rational inattention (RI) into this otherwise standard model. Following the log-linear approximation method proposed by Campbell (1993), Viceira (2001), and Campbell and Viceira (2002), we explicitly solve an RI version of the portfolio choice model with non-tradable labor income after considering the long-run consumption risk facing investors. A major advantage of the log-linearization approach is that we can approximate the original nonlinear problem by a log linear-quadratic (LQ) framework when the coefficient of relative risk aversion (CRRA) is close to 1 and thus can justify the optimality of Gaussian posterior uncertainty under RI. We finally discuss the implications of RI and labor income risk for optimal consumption and investment decisions of inattentive investors and the equilibrium equity premium.

2.1. Specification and Solution of the Standard Asset Allocation Model with Labor Income

In this subsection we present a simple intertemporal model of portfolio choice with non-tradable labor income.[4] We will introduce RI into this otherwise standard model in the next subsection.

2.1.1. Assumptions on Non-tradable Labor Income

Following Viceira (2001) and Campbell and Viceira (2002), we assume that labor income, $\widetilde{Y}_t$, is uninsurable and non-tradable in the sense that investors cannot write claims against future uncertain labor income. Note that an individual investor's labor income can be regarded as a dividend on his implicit holdings of human wealth. Specifically, following Carroll (1997), Cocco, Gomes, and Maenhout (2005), and Chan and Viceira (2005), we assume that labor

[4] The model is based on Campbell (1993), Viceira (2001), and Campbell and Viceira (Chapter 6, 2002), and is widely adopted in the macroeconomics and finance literature.

income is subject to both permanent and transitory shocks, and the log income is a composite process of a random walk process with drift plus an iid shock:

$$\widetilde{y}_{t+1} = y_{t+1} + \left(\varepsilon_{t+1} - \frac{1}{2}\omega_{\varepsilon}^{2}\right), \tag{2.1}$$

$$y_{t+1} = y_t + \nu_{t+1} + g \tag{2.2}$$

where $\widetilde{y}_{t+1} = \ln\left(\widetilde{Y}_{t+1}\right)$, $y_{t+1} = \ln\left(Y_{t+1}\right)$, ν_{t+1} and ε_{t+1} are serially uncorrelated, normally distributed shocks with mean zero and variances ω_{ν}^{2} and ω_{ε}^{2}, respectively, and g is the deterministic growth rate.[5] The two income shocks can be contemporaneously correlated with $\operatorname{cov}\left(\varepsilon_{t+1}, \nu_{t+1}\right) = \omega_{\varepsilon\nu}$. Y_{t+1} is the permanent component of labor income in the sense that a shock to this component has a permanent effect on the level of income; the second component of labor income, $\varepsilon_{t+1} - \frac{1}{2}\omega_{\varepsilon}^{2}$, captures a multiplicative transitory income shock with mean 1. It is straightforward to show that $E\left[\frac{\widetilde{Y}_{t+1}}{Y_{t+1}}\right] = 1$. This specification of labor income is standard in the literature

2.1.2. Assumptions on the Investment Opportunity Set

For simplicity, in this paper we assume that the investment opportunity set is constant. There are two tradable financial assets in the model economy: Asset e is risky, with iid one-period log (continuously compounded) return r_{t+1}^{e}, while the other asset f is riskless, with constant log return given by r^{f}. We refer to asset e as the market portfolio of equities, and asset f as savings or checking accounts. Furthermore, we assume that r_{t+1}^{e} has expected return μ, where $\mu - r^{f}$ is the equity premium, and an unexpected component u_{t+1} with $\operatorname{var}\left[u_{t+1}\right] = \omega_{u}^{2}$.

Furthermore, for the risky asset (e) to be a hedging device against labor income fluctuations, we assume that the two innovations to labor income, ν_{t+1} and ε_{t+1}, can be contemporaneously correlated with the innovation to the equity return:

$$\operatorname{covar}_{t}(u_{t+1}, \nu_{t+1}) = \omega_{u\nu}, \tag{2.3}$$

$$\operatorname{covar}_{t}(u_{t+1}, \varepsilon_{t+1}) = \omega_{u\varepsilon}, \tag{2.4}$$

Note that $\omega_{u\nu} = 0$ if labor income risk is idiosyncratic.

[5] Many consumption and savings studies supported the assumption that individuals' labor income is subject to both permanent and transitory shocks. For example, Carroll (1997) showed that transitory income shocks are important to consumers' precautionary savings.

2.1.3. Investors' Preference and Budget Constraint

We assume that investors choose consumption and asset holdings to maximize the intertemporal time-separable utility, defined over consumption:

$$\max_{\{C_t,\alpha_t\}_{t=0}^{\infty}} E_0 \left\{ \sum_{t=0}^{\infty} \left[\beta^t u\left(C_t\right) \right] \right\}, \tag{2.5}$$

where $u\left(C_t\right) = \frac{C_t^{1-\gamma}-1}{1-\gamma}$ is the power utility function, C_t represents individual's consumption at time t, β is the discount factor, and γ is the coefficient of relative risk aversion. When $\gamma = 1$, the utility function becomes logarithmic, $\log C_t$.

The intertemporal budget constraint for the typical investor is

$$A_{t+1} = R_{t+1}^p \left(A_t + Y_t - C_t\right), \tag{2.6}$$

where A_{t+1} is the individual's financial wealth which is defined as the value of financial assets carried over from period t at the beginning of period $t+1$, $A_t + Y_t - C_t$ is savings, and R_{t+1}^p is the one-period gross return on savings given by

$$R_{t+1}^p = \alpha_t \left(R_{t+1}^e - R^f \right) + R^f, \tag{2.7}$$

where $R_{t+1}^e = \exp(r_{t+1}^e)$, $R^f = \exp(r^f)$, and $\alpha_t = \alpha$ is the proportion of savings invested in the risky asset.[6] As shown in Campbell (1993), the following approximate expression for the log return on wealth holds:

$$r_{t+1}^p = \alpha(r_{t+1}^e - r^f) + r^f + \frac{1}{2}\alpha(1-\alpha)\omega^2. \tag{2.8}$$

Given the above model specification, it is well known that this simple discrete-time model can not be solved analytically.[7] We then follow the log-linearization method proposed in Campbell (1993), Viceira (2001), and Campbell and Viceira (2002) to approximately solve the model in closed-form. Specifically, the original intertemporal budget constraint, (2.6), can be log-linearized around the long-run means of the log consumption-income ratio and the log wealth-income ratio, $c - y = E\left[c_t - y_t\right]$ and $a - y = E\left[a_t - y_t\right]$, as follows:

$$a_{t+1} - y_{t+1} = \rho + \rho_a \left(a_t - y_t\right) - \rho_c \left(c_t - y_t\right) + \rho_\varepsilon \varepsilon_{t+1} - \Delta y_{t+1} + r_{t+1}^p, \tag{2.9}$$

[6] Given iid equity returns and power utility function, the share invested in equities, α_t, is constant over time.

[7] Of course, it can be solved by using numerical methods adopted widely in the modern consumption and portfolio choice literature.

where lowercase letters denote variables in logs and ρ, ρ_a, and ρ_c are log-linearization constants:

$$\rho_a = \frac{\exp(a-y)}{1+\exp(a-y)-\exp(c-y)} > 1; \tag{2.10}$$

$$\rho_c = \frac{\exp(c-y)}{1+\exp(a-y)-\exp(c-y)} > 0; \tag{2.11}$$

$$\rho_\varepsilon = \frac{1}{1+\exp(a-y)-\exp(c-y)} > 0; \tag{2.12}$$

$$\rho = -\left(1-\rho_a+\rho_c\right)\ln\left(1-\rho_a+\rho_c\right) - \rho_a \ln\left(\rho_a\right) + \rho_c \ln\left(\rho_c\right). \tag{2.13}$$

(See Viceira (2001) for a detailed derivation of these log-linearization constants.)

Following the solution method used in Viceira (2001), Campbell and Viceira (2002), and Chan and Viceira (2005), the optimal consumption and portfolio rules for this simple intertemporal asset allocation model with labor income can be written as

$$c_t = y_t + b_0 + b_1\left(a_t - y_t\right), \tag{2.14}$$

$$\alpha = \frac{1}{b_1}\left(\frac{\mu - r^f + 0.5\omega^2}{\gamma\omega_u^2}\right) + \left(1-\frac{1}{b_1}\right)\frac{\omega_{u\nu}}{\omega_u^2}, \tag{2.15}$$

where

$$b_1 = \frac{\rho_a - 1}{\rho_c} \in (0,1),$$

$$b_0 = -\frac{1}{\rho_a - 1}\left[\left(\frac{1}{\gamma} - b_1\right)E\left[r_{t+1}^p\right] + \frac{1}{\gamma}\log\beta + \frac{1}{2\gamma}\Omega - \rho - \left(1-b_1\right)g\right],$$

$\omega_{u\nu} = \rho_{u\nu}\omega_u\omega_\nu$ is the covariance between the equity return and labor income risk, and Ω is an irrelevant constant term. The log-linearization parameter b_1 is the elasticity of consumption with respect to financial wealth, while $1-b_1$ is the elasticity of consumption with respect to labor income. Note that b_1 is equal to 1 in the model in which labor income is tradable. It is clear from (2.14) and (2.15) that b_1 not only affects the curvature of the consumption function but also plays a crucial role in determining the optimal allocation in the risky asset. Optimal portfolio choice (2.15) clearly shows that optimal share invested in the risky asset has two components. The first component is the optimal asset allocation when labor income risk is idiosyncratic, i.e., uncorrelated with the risky asset; and the second is an income hedging demand component. Note that the first component in (2.15) is greater than the optimal portfolio choice obtained from the standard portfolio choice model without non-tradable labor income. The intuition behind this result is that in the presence of nontradabel labor income consumption becomes less sensitive to financial shock as $b_1 < 1$. Furthermore, when labor income and the return to the risky asset is correlated, the demand for the risky asset depends

not only on its expected excess return relative to its volatility, but also on its ability to hedge consumption against labor income fluctuations. For example, if the two shocks are negatively correlated, then the risky asset can be used to hedge against bad income shock, and thus increases the optimal share invested in the risky asset. Note that this hedging demand is proportional to the elasticity of consumption to labor income, $1 - 1/b_1$.

2.2. A Reduction of the State Space

Our first step to incorporate is to perform a reduction of the state space. Multivariate versions of the RI model require a constraint we term 'no subsidization.' With more than one state variable, agents need to allocate their attention differently across these variables and thus reduce their uncertainty at different rates; RI requires that the uncertainty regarding one variable cannot be increased in order to reduce uncertainty regarding another, as this would permit reductions that exceed the channel capacity. This constraint is nonlinear when the dimension is larger than one, even in the LQ model, so multivariate versions of the RI model are not any more tractable than the nonlinear models mentioned above. Using the same procedure adopted in Luo (2008), the above standard asset allocation model can be reduced to a model with a unique state variable – a linear combination of financial wealth and human wealth – that also has iid innovations. Specifically, define a new state variable s_t as follows

$$s_t = a_t + \lambda y_t,$$

where

$$\lambda = \frac{1 - \rho_a + \rho_c}{\rho_a - 1}, \tag{2.16}$$

which can also be written as $1/b_1 - 1$. The log-linearized budget constraint, (2.9), and the consumption function, (2.14), can be rewritten as

$$s_{t+1} = \rho + \rho_a s_t - \rho_c c_t + \rho_\varepsilon \varepsilon_{t+1} + \lambda \nu_{t+1} + r^p_{t+1} - g \tag{2.17}$$

and

$$c_t = b_1 s_t + b_0, \tag{2.18}$$

respectively. Note that here $b_1 \in (0, 1)$ measures the elasticity of consumption with respect to total wealth including both financial wealth and human wealth. We can now introduce RI into this univariate model.

2.3. Incorporating Rational Inattention into the Standard Model

Following Sims (2003), we introduce rational inattention (RI) into the otherwise standard intertemporal portfolio choice model by assuming consumers/investors face information-processing constraints and have only finite Shannon channel capacity to observe the state of the world. As in Sims (2003), we use the concept of entropy from information theory to characterize the uncertainty about a random variable; the reduction in entropy is thus a natural measure of information flow. Formally, entropy is defined as the expectation of the negative of the log of the density function, $-E\left[\log\left(f\left(X\right)\right)\right]$ (see Cover and Thomas 1991 for details). For example, the entropy of a discrete distribution with equal weight on two points is simply $E\left[\log\left(f\left(X\right)\right)\right] = -0.5\log_2\left(0.5\right) - 0.5\log_2\left(0.5\right) = 1$, and the unit of information transmitted is called one "bit". In this case, an agent can remove all uncertainty about X if the capacity devoted to monitoring X is $\kappa = 1$ bit.

With finite capacity $\kappa \in (0, \infty)$, the true state s following a continuous distribution cannot be observed without errors and thus the information set at time $t+1$, $\mathcal{I}_{t+1}$, is generated by the entire history of noisy signal $\left\{s_j^*\right\}_{j=0}^{t+1}$. Following the signal extraction and rational inattention literature, we assume in this paper that the noisy signal takes the additive form: $s_{t+1}^* = s_{t+1} + \xi_{t+1}$, where ξ_{t+1} is the endogenous noise caused by finite capacity. We further assume that ξ_{t+1} is an iid idiosyncratic shock and is independent of the fundamental shock. Note that the reason that the RI-induced noise is idiosyncratic is that the endogenous noise arises from the consumer's own internal information-processing constraint (i.e., finite information-processing channel). The investors with finite capacity will choose a new signal $s_{t+1}^* \in \mathcal{I}_{t+1} = \left\{s_1^*, s_2^*, \cdots, s_{t+1}^*\right\}$ that reduces the uncertainty of the state variable a_{t+1} as much as possible. Formally, this idea can be described by the following information constraint

$$\mathcal{H}\left(s_{t+1}|\mathcal{I}_t\right) - \mathcal{H}\left(s_{t+1}|\mathcal{I}_{t+1}\right) = \kappa, \tag{2.19}$$

where κ is the investor's information channel capacity, $\mathcal{H}\left(s_{t+1}|\mathcal{I}_t\right)$ denotes the entropy of the state *prior to* observing the new signal at $t+1$, and $\mathcal{H}\left(s_{t+1}|\mathcal{I}_{t+1}\right)$ is the entropy *after* observing the new signal. κ imposes an upper bound on the amount of information – that is, the change in the entropy – that can be transmitted in any given period. Finally, we assume that the noise ξ_{t+1} is Gaussian.[8] Finally, following the literature, suppose that the ex ante s_{t+1} is a Gaussian random variable.

Following Sims (2003, 2006), we assume that the typical investor maximizes his lifetime utility subject to the budget constraint, (2.17), as well as the information-processing constraint.

[8] As shown in Sims (2003), within the linear-quadratic-Gaussian setting, Gaussian noise is optimal.

The dynamic optimization problem of the investor can be written as[9]

$$\widehat{v} = \max_{\{c_t, \mathcal{D}_t\}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[c_t + \frac{1}{2}(1-\gamma)c_t^2 \right] \right\} \tag{2.20}$$

subject to

$$s_{t+1} = \rho + \rho_a s_t - \rho_c c_t + \rho_\varepsilon \varepsilon_{t+1} + \lambda \nu_{t+1} + r_{t+1}^p - g, \tag{2.21}$$

$$s_{t+1} | \mathcal{I}_{t+1} \sim \mathcal{D}_{t+1}, \tag{2.22}$$

$$s_t | \mathcal{I}_t \sim \mathcal{D}_t, \tag{2.23}$$

given $s_0 | \mathcal{I}_0 \sim N(\widehat{s}_0, \Sigma_0)$, and the information-processing constraint, (2.19), i.e., the rate of information flow at $t+1$ implicit in the specification of the distributions, $\mathcal{D}_t$ and $\mathcal{D}_{t+1}$ be less than channel capacity. The expectation is formed under the assumption that $\{c_t\}_{t=0}^{\infty}$ are chosen under the information processing constraints. For simplicity, here we assume that all individuals in the model economy have the same channel capacity; hence the average capacity in the economy is equal to individual capacity. Note that in the RI model the *perceived* state variable is not the traditional state variable (e.g., the wealth level s_t in this model), but the so-called *information state*: the distribution of the true state variable, s_t, conditional on the information set available at time t, $\mathcal{I}_t$.

As shown in Sims (2003, 2006), the linear-quadratic (LQ) specification can rationalize Gaussian *posterior* uncertainty as optimal, while the non-LQ setup easily generates optimal non-Gaussian posterior uncertainty.[10] Therefore, in our approximate LQ setting, ex-post Gaussian uncertainty is optimal, that is, $\mathcal{D}_{t+1}$ is normal:

$$s_{t+1} | \mathcal{I}_{t+1} \sim N(\widehat{s}_{t+1}, \Sigma_{t+1}),$$

where $\widehat{s}_{t+1} = E[s_{t+1} | \mathcal{I}_{t+1}]$ and $\Sigma_{t+1} = \text{var}(s_{t+1} | \mathcal{I}_{t+1})$ are the conditional mean and variance of a_{t+1}, respectively. The idiosyncratic error ξ_{t+1} and the noisy signal $s_{t+1}^* = s_{t+1} + \xi_{t+1}$ are both chosen to be Gaussian such that

$$\frac{1}{2}(\log \Psi_t - \log \Sigma_{t+1}) = \kappa, \tag{2.24}$$

[9] This objective function is a log-LQ approximation of the original power utility function.

[10] For example, Sims (2005) showed in a simple two-period saving problem that when the utility function is non-quadratic, the optimal ex post uncertainty follows non-Gaussian distribution. Fully nonlinear versions of the RI problem have either very short horizons or very computation demanding – the state of the world is the distribution of true states and this distribution is not well-behaved (it is not generally a member of a known class of distributions and tends to have 'holes,' making it difficult to characterize with a small number of parameters).

where $\Sigma_{t+1} = \text{var}\left[s_{t+1}|\mathcal{I}_{t+1}\right]$ and $\Psi_t = \text{var}\left(s_{t+1}|\mathcal{I}_t\right)$ are the posterior and the prior variance of s_{t+1}, respectively. This means that given a finite capacity κ per time unit, the optimizing consumer would choose a signal that reduces the entropy by $\frac{1}{2}(\log \Psi_t - \log \Sigma_{t+1})$.[11] Note that in the univariate state case this information constraint completes the characterization of the optimization problem.

Given the budget constraint, (2.17), taking conditional variances on both sides yields:

$$\text{var}_t\left(s_{t+1}\right) = \rho_a^2 \Sigma_t + \text{var}_t\left(r_{t+1}^p\right). \tag{2.25}$$

Substituting (2.25) into (2.24) then gives $\kappa = \frac{1}{2}\left[\log\left(\text{var}_t\left(r_{t+1}^p\right) + \rho_a^2\Sigma_t\right) - \log \Sigma_{t+1}\right]$, which has a steady state $\Sigma = \frac{\text{var}_t\left(r_{t+1}^p\right)}{\exp(2\kappa)-\rho_a^2}$, where $\text{var}_t\left(r_{t+1}^p\right) = \alpha^2\omega^2$.

We can now apply the separation principle in this log-LQ model to obtain the following modified consumption function by replacing a_t with $\widehat{a}_t$ in (2.14):

$$c_t = b_0 + b_1 \widehat{s}_t, \tag{2.26}$$

and the perceived state $\widehat{s}_t$ is characterized by the following Kalman filter equation:

$$\widehat{s}_{t+1} = (1-\theta)\left(\rho + \rho_a \widehat{s}_t - \rho_c c_t + \overline{r}_{t+1}^p - g\right) + \theta s_{t+1}^*, \tag{2.27}$$

where $\overline{r}_{t+1}^p = E\left(r_{t+1}^p\right) = \alpha^*(r_{t+1}^e - r^f) + r^f + \frac{1}{2}\alpha^*(1-\alpha^*)\omega^2$, $\theta = 1 - 1/\exp(2\kappa)$ is the optimal weight on the new observation (i.e., the Kalman gain), $s_{t+1}^* = s_{t+1} + \xi_{t+1}$ is the observed signal, and ξ_{t+1} are the iid noise with $\text{var}\left(\xi_{t+1}\right) = \Sigma/\theta$. Substituting (2.26) into (2.28) yields the following simplified Kalman filtering equation:

$$\widehat{s}_{t+1} = (1-\theta)\widehat{s}_t + \theta s_{t+1}^* + \Upsilon, \tag{2.28}$$

where Υ is an irrelevant constant.

Hence, combining equations (2.21), (2.26), with (2.28) gives the expression for individual consumption growth:[12]

$$\Delta c_{t+1} = \theta b_1 \left\{ \frac{\zeta_{t+1}}{1 - ((1-\theta)\rho_a)\cdot L} + \left[\xi_{t+1} - \frac{\xi_t}{1 - ((1-\theta)\rho_a)\cdot L}\right]\right\}, \tag{2.29}$$

[11] Note that given Σ_t, choosing Σ_{t+1} is equivalent with choosing the noise $\text{var}\left[\xi_t\right]$ since the usual updating formula for the variance of a Gaussian distribution is

$$\Sigma_{t+1} = \Psi_t - \Psi_t\left(\Psi_t + \text{var}\left[\xi_t\right]\right)^{-1}\Psi_t,$$

where Ψ_t is a function of Σ_t.

[12] Note that this MA(∞) expression requires that $(1-\theta)\rho_a < 1$, which is equivalent to $\kappa > \frac{1}{2}\log\left(\rho_a\right)$.

where $\zeta_{t+1} = \rho_\varepsilon \varepsilon_{t+1} + \lambda \nu_{t+1} + \alpha u_{t+1}$ is a linear combination of three innovations and can be regarded as the shock to total wealth, and L is the lag operator. (See Appendix 5.1 for the derivation.)

Note that none of the above expressions for consumption, the perceived state, and the change in consumption are the final solutions because the optimal fraction of savings invested in the stock market, α, is still *undetermined*. The reason is that we need to use the Euler equation to determine the optimal allocation in risky assets. However, as we will discuss in the next subsection, the standard Euler equation does not hold in the RI model because consumption under information-processing constraints adjusts *gradually and incompletely.* Therefore, we will discuss how to determine the optimal portfolio allocation after considering the long-term Euler equation and the ultimate consumption risk.

2.4. Long-run Consumption Risk under RI

Parker (2003) and Parker and Julliard (2005) provide convincing empirical evidence to argue that the long-run risk is a better measure of the true risk of the stock market if consumption reacts with a delay to changes in wealth because the contemporaneous covariance of consumption and wealth *understates* the risk of equity. Bansal and Yaron (2004) also document that consumption and dividend growth rates contain a long-run component. An adverse change in the long-run component lowers asset prices and thus makes holding equity very risky for investors. Hence, we need to use the long-term consumption risk to measure the risk of the equity in the RI model because the RI model predicts that consumption reacts gradually and with delay to the innovations to the equity.

In this subsection, we first define the long-term consumption risk in the RI model and then derive the optimal portfolio rule. Substituting the optimal portfolio rule into the consumption function and the changes in consumption gives us a complete solution to this simple optimal consumption and portfolio choice model with RI. Following Parker's work, we define the long-run consumption risk as the covariance of asset returns and consumption growth over the period of the return and many following periods. Because the RI model predicts that consumption reacts to the innovations to asset returns gradually and incompletely, it can rationalize the conclusion in Parker's papers that consumption risk should be long term instead of contemporaneous.[13]

Specifically, when agents behave optimally but only have finite channel capacity, we have

[13] Note that under full-information RE, the contemporaneous consumption risk is the same as the ultimate consumption risk because consumption adjusts to wealth shocks instantly and completely.

the following equality for the risky asset e and the risk-free asset f:[14]

$$E_t\left[R^e_{t+1}C^{-\gamma}_{t+1+S}\right] = E_t\left[R^f C^{-\gamma}_{t+1+S}\right],$$

which can be transformed to the following stationary form:

$$E_t\left[R^e_{t+1}(C_{t+1+S}/C_t)^{-\gamma}\right] = E_t\left[R^f(C_{t+1+S}/C_t)^{-\gamma}\right], \tag{2.30}$$

where the expectation $E_t[\cdot]$ is conditional on the entire history of the economy up to t and S is the horizon of many periods in the future over which consumption response under RI is studied.[15] (See Appendix 5.2 for the derivation.) Given the expression for consumption dynamics, it is clear that S is infinitely large because consumption takes infinite periods to react to the exogenous shock under information constraints. The standard equality, $E_t\left[R^e_{t+1}C^{-\gamma}_{t+1}\right] = E_t\left[R^f C^{-\gamma}_{t+1}\right]$, does not hold here because consumption reacts slowly with respect to the innovations to equity returns and thus cannot finish adjusting immediately and completely.

Log-linearizing equation (2.30) yields

$$E_t\left[r^e_{t+1}\right] - r^f + \frac{1}{2}\omega^2 = \gamma \lim_{S\to\infty} \text{covar}_t\left[c_{t+1+S} - c_t, r^e_{t+1}\right], \tag{2.31}$$

which means that the expected asset return can be written as:

$$E_t\left[r^e_{t+1}\right] - r^f + \frac{1}{2}\omega^2 = \lim_{S\to\infty}\sum_{j=0}^{S} \text{covar}_t\left[\Delta c_{t+j+1}, r^e_{t+1}\right], \tag{2.32}$$

where we have used $\gamma \simeq 1$, $c_{t+1+S} - c_t = \sum_{j=0}^{S}\Delta c_{t+1+j}$, and Δc_{t+1+j} is given by (2.29). Specifically, the long-run impact of the innovation in the equity return on consumption growth can be rewritten as

$$\lim_{S\to\infty}\left(\sum_{j=0}^{S}\text{covar}_t\left[\Delta c_{t+1+j}, r^e_{t+1}\right]\right) = b_1\left(\varsigma\alpha\omega^2 + \lambda\omega_{u\nu} + \theta\rho_\varepsilon\omega_{u\varepsilon}\right), \tag{2.33}$$

[14] The equality can be obtained by using $S+1$ period consumption growth to price a multiperiod return formed by investing in equity for one period and then transforming to the risk-free asset for the next S periods. Hence, the following multiperiod moment condition holds

$$C_t^{-\gamma} = E_t\left[\beta^{S+1}C^{-\gamma}_{t+S+1}R^e_{t+1}(R^f)^S\right].$$

[15] This measure has some appealing features; see Parker (2003) and Parker and Jullard (2005) for detailed discussions.

where ς is the ultimate consumption risk, $\lambda = (1 - \rho_a + \rho_c) / (\rho_a - 1)$, and $\omega_{u\nu} = \rho_{u\nu}\omega\omega_\nu$ is the covariance between the equity return and labor income risk:

$$\varsigma = \frac{\theta}{1 - (1 - \theta)\,\rho_a} > 1 \tag{2.34}$$

when the restriction $1 - (1 - \theta)\,\rho_a > 0$ holds. (See Appendix 5.3 for the derivation.) Note that this mild parameter restriction must be imposed to guarantee convergence. (See the MA(∞) process of consumption growth, (2.29).) By simple calculation we obtain $\frac{\partial \varsigma}{\partial \theta} < 0$ and $\frac{\partial \varsigma}{\partial \rho_a} > 0$ because $\rho_a > 1$, $\theta \in (0, 1)$. This ultimate consumption risk should be a better measure of the riskiness of the risky asset than the contemporaneous risk, as consumption adjusts gradually to the shocks to asset returns in the presence of RI.

3. Optimal Consumption and Strategic Asset Allocation under RI

3.1. Optimal Solution

Combining equations (2.32), (2.33), with (2.26) in Section 2 gives us optimal consumption and portfolio rules for inattentive investors with labor income risk. The following proposition gives a complete characterization of the model's solution:

Proposition 1. *Consider the optimal consumption and portfolio choice problem (2.19)-(2.23). Suppose that γ is close to 1, we then obtain that the optimal share invested in the equity is*

$$\alpha^* = \frac{1}{\varsigma}\left[\frac{1}{b_1}\frac{\pi}{\omega_u^2} + \left(1 - \frac{1}{b_1}\right)\frac{\varsigma\omega_{uv}}{\omega_u^2} - \rho_\varepsilon \frac{\varsigma\omega_{u\varepsilon}}{\omega_u^2}\right], \tag{3.1}$$

where $\pi = \mu - r^f + 0.5\omega^2$ and $\frac{1}{\varsigma} = \frac{1-(1-\theta)\rho_a}{\theta} < 1$. Furthermore, the consumption function is

$$c_t^* = b_0 + b_1 \widehat{s}_t, \tag{3.2}$$

the actual state variable evolves according to

$$s_{t+1} = \rho + \rho_a s_t - \rho_c c_t + \alpha^*(r_{t+1}^e - r^f) + r^f + \frac{1}{2}\alpha^*(1 - \alpha^*)\omega^2 - g + \lambda\nu_{t+1}, \tag{3.3}$$

and the estimated state $\widehat{a}_t$ is characterized by the following Kalman filtering equation

$$\widehat{s}_{t+1} = (1 - \theta)\left(\rho + \rho_a \widehat{s}_t - \rho_c c_t - g + \overline{r}_{t+1}^p\right) + \theta s_{t+1}^*. \tag{3.4}$$

where $\theta = 1 - 1/\exp(2\kappa)$ is the optimal weight on observation, ξ_t are the iid *idiosyncratic noise with $\omega_\xi^2 = \mathrm{var}\left(\xi_{t+1}\right) = \Sigma/\theta$, and $\Sigma = \frac{\omega_\zeta^2}{\exp(2\kappa) - \rho_a^2}$ is the steady state conditional variance.*

Finally, the change in individual consumption is

$$\Delta c_{t+1}^{*} = \theta b_1 \left\{ \frac{\rho_\varepsilon \varepsilon_{t+1} + \lambda \nu_{t+1} + \alpha^* u_{t+1}}{1-(1-\theta)\,\rho_a \cdot L} + \left[\xi_{t+1} - \frac{\xi_t}{1-(1-\theta)\,\rho_a \cdot L} \right] \right\}, \tag{3.5}$$

where $\zeta_{t+1} = \rho_\varepsilon \varepsilon_{t+1} + \lambda \nu_{t+1} + \alpha u_{t+1}$.

Proof. *See Appendix 5.3 for the derivation.* ■

It is clear from this proposition that optimal consumption and portfolio rules are *interdependent* in the presence of RI and labor income risk. Note that in the model without non-tradable labor income, Expression (3.1) reduces to the portfolio rule obtain in Luo (2009) and reduces to the Merton solution when the information-processing capacity increases to infinity (i.e., $\theta = 1$). Without labor income risk, the optimal asset allocation is solely determined by speculative motives, that is, the allocation is proportional to the expected excess return of the risky asset, and is inversely relative to the variance of the equity return and to the elasticity of consumption to perceived wealth, b_1.[16]

3.2. Implications for Strategic Asset Allocation

Expression (3.1) clearly shows that just like the full-information RE solution presented in Section 2.1, the strategic asset allocation under RI also has two components: the speculative demand and the income hedging demand. Note that the former characterizes the optimal allocation when labor income risk is uncorrelated with the risky asset ($\omega_{uv} = 0$). When $\omega_{uv} \neq 0$, the desirability of the risky asset depends not only on its expected excess return relative to its variance, but also on its ability to hedge consumption against bad realizations of labor income. In Viceira (2001) and Campbell and Viceira (2002), they analyze the optimal portfolio choices of long-horizon investors with undiversifiable labor income risk and showed that a positive correlation between labor income innovations and unexpected asset returns reduces the investor's willingness to hold the risky asset because the risky asset provides a poor hedge against unexpected declines in labor income. It is worth noting that under information-processing constraints both components are affected by long-run consumption risk due to gradual adjustments in consumption to wealth shocks. Specifically, given that $\rho_a > 1$ and $\theta \in (0, 1)$, RI affects the optimal allocation in the risky asset via the following two channels:

1. Reducing both the speculative demand and the income-hedging demand by the long-run consumption risk, ς. Note that $\varsigma > 1$ measures the long-run (accumulated) impacts of financial shocks on consumption.

[16] Note that here we have applied the assumption that the coefficient of relative risk aversion $\gamma \simeq 1$.

2. In addition, as shown in the second term in the bracket of (3.1), RI can increase the income hedging demand by ς because u_t is correlated with the two income shocks, ν_t and ε_t, and consumption reacts to the shock to total wealth $\zeta_t = \alpha u_t + \lambda \nu_t$ gradually and indefinitely.

To make it more clear, we rewrite (3.1) as:

$$\alpha^* = \frac{1}{\varsigma b_1}\frac{\pi}{\omega_u^2} + \left(1 - \frac{1}{b_1}\right)\frac{\omega_{uv}}{\omega_u^2} - \rho_\varepsilon \frac{\omega_{u\varepsilon}}{\omega_u^2}. \tag{3.6}$$

This expression clearly shows that RI increases the relative importance of the income-hedging demand to the speculative demand via the long-run consumption risk, i.e., under RI, the ratio of the income hedging demand to the speculative demand increases by ς. In addition, the second term in (3.6) also shows that RI has no impact on the absolute value of the income hedging demand. The reason is simple: under RI the innovation to the equity return has *systematic* effects on consumption growth affected by both the equity return and labor income. (See Expression (2.33).)

We now illustrate the theoretical findings presented above using numerical examples. We will use empirically plausible parameter values of stochastic processes of asset returns and labor income to illustrate these examples. Following Viceira (2001) and Campbell and Viceira (2002), the values for the parameters describing the investment opportunity set are based on the historical estimates of the average equity return, the short-term real interest rate, and the variance of excess equity returns in the U.S. stock market. Specifically, we set $R^f = 1.02$, $E_t\left[R^e_{t+1}/R^f\right] = 1.06$, and $\omega = 0.2$. The parameters of the labor income process are set as $E_t\left[Y_{t+1}/Y_t\right] = 1.03$ and $\omega_\nu = 0.1$. In addition, we set the time discount factor $\beta = 0.85$ and $\rho_{u\nu} = 0.35$.[17] Figure 1 illustrates the effects of RI on the long-run consumption risk, ς, for different values of the wealth-to-income ratio (W/Y). It clearly shows that the long-run risk is decreasing with the degree of attention measured by the Kalman gain θ. For example, when the investor with $W/Y = 15$ reduces his capacity devoted to monitoring the state evolution from $\theta = 50\%$ to 10%, the relative importance of the income-hedging demand to the speculation demand will increase by about 3.7 times.[18] It also shows that the long-run risk is increasing with the wealth-to-income ratio. This prediction could be an explanation for the investment behavior of the young and old. Financial advisors typically recommend that people shift investments away from the risky asset to the risk-free asset as they age. Our model predicts

[17] In this paper we set the relatively low values for β for two reasons. First, it helps in finding the fixed point of the log-linearization coefficients in numerical computations. Second, it could be due to the death probability that we do not model explicitly.

[18] $\theta = 10\%$ means that only 10 percent of the uncertainty is removed in each period upon receiving a new signal about the aggregate shock to the equity return.

that the old will face greater long-run consumption risk and thus invest less in the risky asset because the old have higher wealth-to-income ratios than the young.

In the standard full information RE portfolio choice model, the investment horizon is *irrelevant* for investors who have power utility, have only financial wealth, and face constant investment opportunities. As argued in Campbell and Viceira (2002), we may vary the effective investment horizon by varying the discount factor (β) that determines the relative weights investors put on the near future versus the distant future. The investors with large β would place a relatively high weight on the distant future, while those with small β place more weight on the near future. Hence, Expressions (3.6) and (2.10) mean that the investment horizon measured by β does matter for optimal asset allocation because it affects the log-linearization coefficient ρ_a and then the long-run consumption risk ς. The intuition is that those inattentive investors with larger β face smaller long-term consumption risk; consequently they choose to invest more wealth in the risky asset. Figure 2 illustrates how the investment horizon affects optimal asset allocation for given θ. It clearly shows that ς is decreasing with β. For example, when $\theta = 15\%$, $W/Y = 15$, and $\rho_{u\nu} = 0.35$, the long-run consumption risk facing the investor with $\beta = 0.85$ is 9, whereas it is 4 for the investor with $\beta = 0.86$. This prediction can thus be an alternative explanation for the investment behavior of the young and old. Some theoretical models have been proposed to evaluate this justification; for example, Jagannathan and Kocherlakota (1996), Viceira (2001), and Campbell and Viceira (2002). In the existing literature, the investment horizon and age are generally closely related. In other words, the old have low *effective* time discount factor because they face high death probability in every period. Specifically, the old people with low β face large ultimate consumption risk due to slow adjustment; consequently, it would be better for them to invest less in the stock market.

It is worth noting that although the correlation between labor income shocks and unexpected risky asset returns, $\rho_{u\nu}$, has no direct interaction with RI (θ) in (3.6), it does affect the long-run consumption risk and interact with RI via the log-linearization coefficient ρ_a. Figure 3 illustrates how the correlation affects long-run consumption risk for given θ when $\beta = 0.86$ and $W/Y = 15$. It clearly shows that ς is increasing with $\rho_{u\nu}$.

3.3. Implications for Consumption Dynamics

Equation (3.5) shows that individual consumption under RI reacts not only to fundamental shocks (u_{t+1}) but also to the endogenous noise $\left(\xi_{t+1}\right)$ induced by finite capacity. The endogenous noise can be regarded as a type of "consumption shock"or "demand shock". In the intertemporal consumption literature, some transitory consumption shocks are often used to make the model fit the data better. Under RI, the idiosyncratic noise due to RI provides a theory for these transitory consumption movements. Furthermore, Equation (3.5) also makes

it clear that consumption growth adjusts slowly and incompletely to the innovations to asset returns but reacts quickly to the idiosyncratic noise.

It is worth noting that the main difference between Gabaix and Laibson (1999)'s $6D$ *infrequent-adjustment* model and our RI model is that in their model, investors adjust their consumption plans *infrequently but completely* once they choose to adjust, whereas investors with finite capacity adjust their plans *frequently but incompletely* in every period. In addition, in the $6D$ model, the optimal fraction of savings invested in the risky asset is assumed to be *fixed* at the standard Merton solution, whereas optimal portfolio choice under RI reflects the larger long-term consumption risk caused by slow adjustments.

Using (3.5), we can obtain the stochastic properties of the joint dynamics of consumption and the equity return. The following proposition summarizes the major stochastic properties of consumption and the equity return.

Proposition 2. *Given finite capacity κ (i.e., θ) and optimal portfolio choice α^*, the volatility of consumption growth is*

$$\operatorname{var}\left[\Delta c_t^*\right] = \frac{\theta b_1^2}{1-(1-\theta)\rho_a^2}\omega_\zeta^2, \tag{3.7}$$

the relative volatility of consumption growth to the equity return is

$$\mu = \frac{\operatorname{sd}\left[\Delta c_t^*\right]}{\operatorname{sd}\left[u_t\right]} = \sqrt{\frac{\theta b_1^2}{1-(1-\theta)\rho_a^2}}\frac{\omega_\zeta}{\omega}, \tag{3.8}$$

the first-order autocorrelation of consumption growth is

$$\rho_{\Delta c(1)} = \operatorname{corr}\left[\Delta c_t^*, \Delta c_{t+1}^*\right] = 0, \tag{3.9}$$

and the contemporaneous correlation between consumption growth and the equity return is

$$\operatorname{corr}\left[\Delta c_{t+1}^*, u_{t+1}\right] = \frac{\theta\left(\alpha^*\omega^2 + \lambda\omega_{u\nu}\right)}{\sqrt{\theta/\left[1-(1-\theta)\rho_a^2\right]}\left(\omega\omega_\zeta\right)}, \tag{3.10}$$

where $\omega_\zeta = \sqrt{\alpha^{*2}\omega^2 + \lambda^2\omega_\nu^2 + 2\alpha^*\lambda\rho_{u\nu}\omega\omega_\nu}$.

Proof. See Appendix 5.4. ■

Expression (3.8) shows that RI affects the relative volatility of consumption growth to the equity return via two channels: (i) $\frac{\theta b_1^2}{1-(1-\theta)\rho_a^2}$ and (ii) α^* in the expression for ω_ζ^2. Holding the optimal share invested in the risky asset α^* fixed, RI increases the relative volatility of consumption growth via the first channel because $\partial\left(\frac{\theta b_1^2}{1-(1-\theta)\rho_a^2}\right)/\partial\theta < 0$. (3.5) indicates that RI has two effects on the volatility of Δc: (1) the gradual response to a fundamental shock

and (2) the presence of the RI-induced noise shocks. The former effect reduces consumption volatility, whereas the latter one increases it; the net effect is that RI increases the volatility of consumption growth holding α^* fixed. Furthermore, as shown above, RI reduces α^* as it increases the long-run consumption risk as shown in (3.6), which tends to reduce the volatility of consumption growth as households switch to safer portfolios. Figure 4 illustrates how RI affects the relative volatility of consumption to the equity return for different values of β.[19] It clearly shows that the relative volatility of consumption growth (μ) is decreasing with both the degree of attention (i.e., θ) and the discount factor in the presence of non-tradable labor income. Figure 5 clearly shows that μ is increasing with the correlation between consumption growth and the equity return if the degree of inattention is low enough (in this case, the condition is that $\theta \leq 0.34$).[20] The intuition is as follows. The correlation increases the total fundamental uncertainty facing the investor with low capacity measured by ω_ζ^2, which leads to larger volatility of consumption growth for given the uncertainty of the stock market.

Expression (3.9) means that there is no persistence in consumption growth under RI. The intuition of this result is as follows. Both MA(∞) terms in (3.5) affect consumption persistence under RI. Specifically, in the absence of the endogenous noises, the gradual response to the shock to the equity return due to RI leads to positive persistence in consumption growth: $\rho_{\Delta c(1)} = \theta\rho_a \frac{(1-\theta)-(1-\theta)^2\rho_a^2}{1-(1-\theta)^2\rho_a^2} \in [0,1)$. (See Appendix 5.4 for the derivation.) The presence of the noises generate negative persistence in consumption growth, exactly offsetting the positive effect of the gradual response to the fundamental shock under RI.

Expression (3.10) shows that RI reduces the contemporaneous correlation between consumption growth and the equity return because $\partial \operatorname{corr}\left(\Delta c_{t+1}^*, u_{t+1}\right)/\partial\theta > 0$. That is, RI weakens the link between consumption growth and the equity return. Figure 6 illustrates the effects of RI on the correlation for different values of β when $W/Y = 15$ and $\rho_{u\nu} = 0.35$. It clearly shows that the correlation between consumption growth and the equity return is increasing with the degree of attention (θ). Figure 7 clearly shows that $\operatorname{corr}\left(\Delta c_{t+1}^*, u_{t+1}\right)$ is positively correlated to the correlation between labor income and the equity return for given degree of inattention.

3.4. Implications for the Equilibrium Equity Premium

Using the portfolio choice model discussed above and there is a representative agent in the economy, the following pricing equation links aggregate consumption growth and the equity

[19] Here we also set $W/Y = 15$ and $\rho_{u\nu} = 0.35$ as in the last subsection.
[20] Here we set $\beta = 0.86$.

premium holds when $\gamma \simeq 1$:[21]

$$\pi = \alpha^* \varsigma b_1 \omega_u^2 + \varsigma (1 - b_1) \omega_{uv} \tag{3.11}$$

Suppose, as in the consumption-based CAPM literature, that the risk-free asset is an inside bond, so that in equilibrium the net supply of the risk free asset is 0 and then the share of the risky asset in financial wealth (α^*) is 100%. Accordingly, (3.11) becomes

$$\pi = \varsigma \left[b_1 \omega_u^2 + (1 - b_1) \omega_{uv} \right] \tag{3.12}$$

Hence, RI implies higher equity premium because the ultimate consumption risk $\varsigma = \frac{\theta}{1-(1-\theta)\rho_a} > 1$. The intuition behind this result is that investors facing higher ultimate consumption risk caused by finite capacity choose to invest less in the stock market and thus require higher risk compensation in equilibrium. Note that in the full-information case (3.11) reduces to $\pi = b_1 \omega_u^2 + (1 - b_1) \omega_{uv}$, which means that the equilibrium equity premium is determined by both the speculation demand and the income-hedging demand.

In addition, we can see from Expression (3.11) that RI has the same impacts on the speculation demand and the hedging demand. Specifically, the magnitude of the hedging demand, $(1 - b_1) \omega_{uv}$, can also be increased by ς in the presence of information constraints. For example, when we set $\beta = 0.85$, $W/Y = 15$, and $\rho_{u\nu} = 0.35$, it is straightforward to calculate that π can be increased by 4 times.

4. Conclusion

In this paper we have studied the strategic asset allocation of a household with non-tradable labor income and limited information-processing capacity (rational inattention). Rational inattention leads to slow adjustments in consumption and greater long-run consumption risk; consequently, it will reduce the demand for risky assets and increases the relative importance of the income-hedging demand to the traditional speculation demand. In addition, we showed that the correlation between labor income and the equity return increases both the relative volatility of consumption growth to the equity return and the contemporaneous correlation between consumption growth and the equity return. Finally, we show that RI increases the equilibrium equity premium by affecting both the speculation and income-hedging demands.

[21] For simplicity, here we assume that there is no transitory income risk such that

$$\alpha^* = \frac{1}{\varsigma b_1} \frac{\pi}{\omega_u^2} + \left(1 - \frac{1}{b_1}\right) \frac{\omega_{uv}}{\omega_u^2}.$$

In the future, it would be promising to examine how introducing endogenous labor supply can have significant effects on strategic asset allocation and consumption dynamics for inattentive investors.

5. Appendix

5.1. Deriving the Expression of Consumption Growth with Labor Income

Combining Equations (2.21), (2.26), with (2.28) yields

$$s_{t+1} - \widehat{s}_{t+1} = (1-\theta)\rho_a \left(s_t - \widehat{s}_t\right) + (1-\theta)\zeta_{t+1} - \theta\xi_{t+1},$$

where $\zeta_{t+1} = \rho_\varepsilon \varepsilon_{t+1} + \lambda\nu_{t+1} + \alpha u_{t+1}$. Using (2.21), (2.26), and (2.28), we also have

$$\Delta\widehat{s}_{t+1} = \theta\rho_a \left(s_t - \widehat{s}_t\right) + \theta(\zeta_{t+1} + \xi_{t+1}),$$

where

$$s_t - \widehat{s}_t = \frac{(1-\theta)\zeta_t - \theta\xi_t}{1-(1-\theta)\rho_a \cdot L}.$$

Hence, using (2.26), consumption growth can be written as (2.29) in the text.

5.2. Deriving the Long-term Euler Equation, (2.30)

When wealth is allocated efficiently across the two financial assets, the marginal investment in any asset yields the same expected increase in future utility:

$$E_t\left[u'\left(c_{t+1}\right)\left(R^e_{t+1} - R^f\right)\right] = 0. \tag{5.1}$$

Using the Euler equation for the risk free asset between $t+1$ and $t+1+S$, we have

$$u'\left(c_{t+1}\right) = E_{t+1}\left[(\beta R_f)^S u'\left(c_{t+1+S}\right)\right],$$

which can be substituted into (5.1) to get the long-term Euler equation:

$$\begin{aligned} & E_t\left[E_{t+1}\left[(\beta R_f)^S u'\left(c_{t+1+S}\right)\right]\left(R^e_{t+1} - R^f\right)\right] \\ = \; & E_t\left[(\beta R_f)^S u'\left(c_{t+1+S}\right)\left(R^e_{t+1} - R^f\right)\right] = 0, \end{aligned}$$

which implies

$$E_t\left[u'\left(c_{t+1+S}\right)\left(R^e_{t+1} - R^f\right)\right] = 0,$$

which can be transformed to Equation (2.30) in the text by dividing $u'(c_t)$ on both sides. Note that the expectation $E_t[\cdot]$ is conditional on the entire history of the economy up to t, and S is infinite over which consumption response under RI is studied.

5.3. Deriving the Optimal Allocation in the Risky Asset

Substituting (2.29) into $\sum_{j=0}^{S} \text{covar}_t\left[\Delta c_{t+1+j}, r_{t+1}^e\right]$ gives

$$\begin{aligned}
\text{covar}_t\left[\Delta c_{t+1+j}, r_{t+1}^e\right] &= \text{covar}_t\left[\theta b_1\left\{\frac{\zeta_{t+1}}{1-((1-\theta)\rho_a)\cdot L} + \left[\xi_{t+1} - \frac{\xi_t}{1-((1-\theta)\rho_a)\cdot L}\right]\right\}, u_{t+1}\right] \\
&= \text{covar}_t\left[\theta b_1 \frac{\rho_\varepsilon \varepsilon_{t+1} + \lambda \nu_{t+1} + \alpha u_{t+1}}{1-((1-\theta)\rho_a)\cdot L}, u_{t+1}\right] \\
&= \theta b_1\left(\alpha\omega^2 + \lambda\omega_{u\nu} + \rho_\varepsilon \omega_{u\varepsilon}\right).
\end{aligned}$$

Therefore,

$$\begin{aligned}
\lim_{S\to\infty}\left(\sum_{j=0}^{S} \text{covar}_t\left[\Delta c_{t+1+j}, r_{t+1}^e\right]\right) &= \frac{\theta b_1\left(\alpha\omega^2 + \lambda\omega_{u\nu} + \rho_\varepsilon\omega_{u\varepsilon}\right)}{1-(1-\theta)\rho_a} \\
&= \varsigma b_1\left(\alpha\omega^2 + \lambda\omega_{u\nu} + \rho_\varepsilon\omega_{u\varepsilon}\right),
\end{aligned}$$

where the ultimate consumption risk $\varsigma = \frac{\theta}{1-(1-\theta)\rho_a}$. Since

$$\pi = E_t\left[r_{t+1}^e\right] - r^f + \frac{1}{2}\omega^2 = \lim_{S\to\infty}\sum_{j=0}^{S} \text{covar}_t\left[\Delta c_{t+j+1}, r_{t+1}^e\right],$$

we can easily the expression for α, (3.1), in the text.

5.4. Deriving the Stochastic Properties of Consumption Dynamics

Taking unconditional variance on both sides of (3.5) yields

$$\begin{aligned}
\text{var}\left[\Delta c_t^*\right] &= \theta^2 b_1^2\left\{\frac{\omega_\zeta^2}{1-((1-\theta)\rho_a)^2} + \left[1 + \frac{(\theta\rho_a)^2}{1-((1-\theta)\rho_a)^2}\right]\omega_\xi^2\right\} \\
&= \theta^2 b_1^2\left[\frac{1}{1-((1-\theta)\rho_a)^2} + \frac{1+(2\theta-1)\rho_a^2}{1-((1-\theta)\rho_a)^2}\frac{1}{(1/(1-\theta)-\rho_a^2)\theta}\right]\omega_\zeta^2 \\
&= \frac{\theta b_1^2}{1-(1-\theta)\rho_a^2}\omega_\zeta^2
\end{aligned}$$

Using (3.5), we can compute the first-order autocovariance of consumption growth:

$$
\begin{aligned}
&\operatorname{cov}\left(\Delta c_t^*, \Delta c_{t+1}^*\right) \\
&= (\theta b_1)^2 \operatorname{cov}\left(\frac{\zeta_t}{1-(1-\theta)\rho_a \cdot L} + \left[\xi_t - \frac{\theta\rho_a\xi_{t-1}}{1-(1-\theta)\rho_a \cdot L}\right], \frac{\zeta_{t+1}}{1-(1-\theta)\rho_a \cdot L} + \left[\xi_{t+1} - \frac{\theta\rho_a\xi_t}{1-(1-\theta)\rho_a \cdot L}\right]\right) \\
&= (\theta b_1)^2 \operatorname{cov}\left(\frac{\zeta_t}{1-(1-\theta)\rho_a \cdot L} + \left[\xi_t - \frac{\theta\rho_a\xi_{t-1}}{1-(1-\theta)\rho_a \cdot L}\right], \frac{(1-\theta)\rho_a\zeta_t}{1-(1-\theta)\rho_a \cdot L} - \frac{\theta\rho_a\xi_t}{1-(1-\theta)\rho_a \cdot L}\right) \\
&= (\theta b_1)^2 (1-\theta)\rho_a \operatorname{cov}\left(\frac{\zeta_t}{1-(1-\theta)\rho_a \cdot L}, \frac{\zeta_t}{1-(1-\theta)\rho_a \cdot L}\right) \\
&+ (\theta b_1)^2 \operatorname{cov}\left(\xi_t - \frac{\theta\rho_a\xi_{t-1}}{1-(1-\theta)\rho_a \cdot L}, -\frac{\theta\rho_a\xi_t}{1-(1-\theta)\rho_a \cdot L}\right) \\
&= (\theta b_1)^2 (1-\theta)\rho_a \operatorname{var}\left(\frac{\zeta_t}{1-(1-\theta)\rho_a \cdot L}\right) + (\theta b_1)^2 \operatorname{cov}\left(\xi_t - \frac{\theta\rho_a\xi_{t-1}}{1-(1-\theta)\rho_a \cdot L}, -\frac{\theta\rho_a\xi_t}{1-(1-\theta)\rho_a \cdot L}\right) \\
&= (\theta b_1)^2 (1-\theta)\rho_a \operatorname{var}\left(\frac{\zeta_t}{1-(1-\theta)\rho_a \cdot L}\right) + (\theta b_1)^2 \operatorname{cov}\left(\xi_t, -\theta\rho_a\xi_t\right) \\
&+ (\theta b_1)^2 \operatorname{cov}\left(-\frac{\theta\rho_a\xi_{t-1}}{1-(1-\theta)\rho_a \cdot L}, -\frac{(\theta(1-\theta)\rho_a^2)\xi_{t-1}}{1-(1-\theta)\rho_a \cdot L}\right) \\
&= \frac{(\theta b_1)^2 (1-\theta)\rho_a\omega_\zeta^2}{1-((1-\theta)\rho_a)^2} - \frac{(\theta b_1)^2 \theta\rho_a\omega_\zeta^2}{(1/(1-\theta)-\rho_a^2)\theta} + \frac{(\theta b_1)^2 (\theta\rho_a)(\theta(1-\theta)\rho_a^2)}{1-(1-\theta)^2\rho_a^2}\frac{\omega_\zeta^2}{(1/(1-\theta)-\rho_a^2)\theta} \\
&= 0.
\end{aligned}
$$

We thus have

$$
\operatorname{corr}\left(\Delta c_t^*, \Delta c_{t+1}^*\right) = \frac{\operatorname{cov}\left(\Delta c_t^*, \Delta c_{t+1}^*\right)}{\sqrt{\operatorname{var}\left(\Delta c_t^*\right)}\sqrt{\operatorname{var}\left(\Delta c_{t+1}^*\right)}} = 0.
$$

Note that in the absence of the endogenous noise shocks, we have

$$
\begin{aligned}
\rho_{\Delta c} &= \operatorname{corr}\left(\Delta c_t^*, \Delta c_{t+1}^*\right) = (1-\theta)\rho_a \frac{\theta^2 b_1^2\omega_\zeta^2}{1-(1-\theta)^2\rho_a^2}\left(\frac{\theta b_1^2\omega_\zeta^2}{1-(1-\theta)\rho_a^2}\right)^{-1} \\
&= (1-\theta)\rho_a \frac{\theta^2}{1-(1-\theta)^2\rho_a^2}\left(\frac{\theta}{1-(1-\theta)\rho_a^2}\right)^{-1} \\
&= \theta\rho_a \frac{(1-\theta)-(1-\theta)^2\rho_a^2}{1-(1-\theta)^2\rho_a^2} \in [0,1),
\end{aligned}
$$

because

$$\begin{aligned}
&\operatorname{cov}\left(\Delta c_t^*, \Delta c_{t+1}^*\right) = \operatorname{cov}\left(\frac{\theta b_1 \zeta_t}{1-(1-\theta)\rho_a \cdot L}, \frac{\theta b_1 \zeta_{t+1}}{1-(1-\theta)\rho_a \cdot L}\right) \\
&= (1-\theta)\rho_a (\theta b_1)^2 \operatorname{cov}\left(\frac{\zeta_t}{1-(1-\theta)\rho_a \cdot L}, \frac{\zeta_t}{1-(1-\theta)\rho_a \cdot L}\right) \\
&= \frac{(1-\theta)\rho_a (\theta b_1)^2 \omega_\zeta^2}{1-(1-\theta)^2 \rho_a^2}.
\end{aligned}$$

Finally, using (3.5), it is straightforward to show that

$$\begin{aligned}
\operatorname{corr}\left(\Delta c_{t+1}^*, u_{t+1}\right) &= \frac{\operatorname{cov}\left(\Delta c_{t+1}^*, u_{t+1}\right)}{\operatorname{sd}\left(\Delta c_{t+1}^*\right)\operatorname{sd}\left(u_{t+1}\right)} \\
&= \frac{\theta\left(\alpha^* \omega^2 + \lambda \omega_{u\nu}\right)}{\sqrt{\theta/\left[1-(1-\theta)\rho_a^2\right]}\left(\omega \omega_\zeta\right)} \\
&= \frac{\theta\left[\alpha^* + \lambda \rho_{u\nu}\left(\omega_\nu/\omega\right)\right]}{\sqrt{\theta/\left[1-(1-\theta)\rho_a^2\right]}\sqrt{\alpha^{*2} + \lambda^2\left(\omega_\nu/\omega\right)^2 + 2\alpha\lambda\rho_{u\nu}\left(\omega_\nu/\omega\right)^2}},
\end{aligned}$$

where $\omega_\zeta^2 = \alpha^* \omega^2 + \lambda^2 \omega_\nu^2 + 2\alpha\lambda\omega_{u\nu}$ and $\operatorname{cov}\left(\Delta c_{t+1}^*, u_{t+1}\right) = \theta b_1\left(\alpha^* \omega^2 + \lambda \omega_{u\nu}\right)$.

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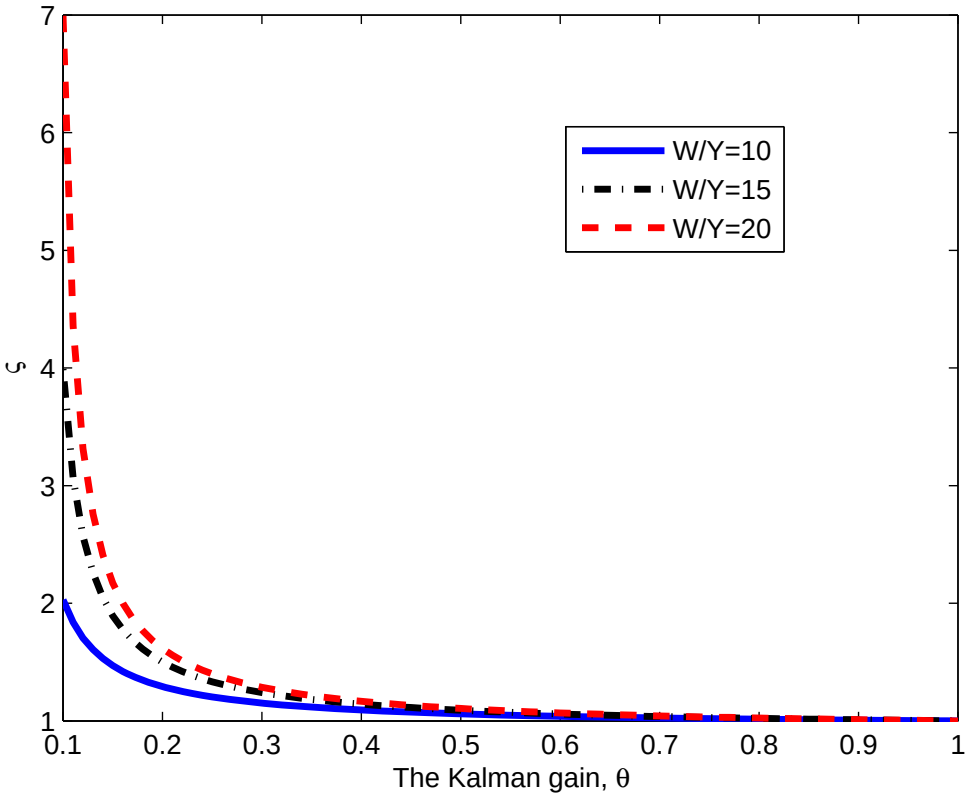

Figure 1: The Effects of RI on Long-run Consumption Risk

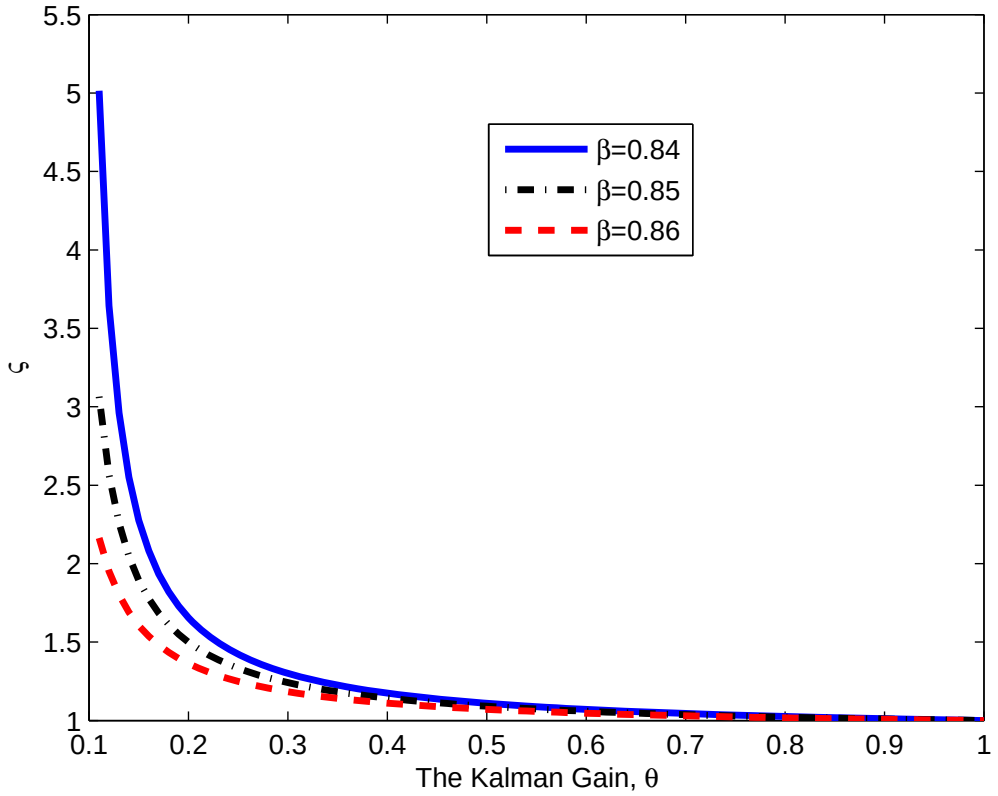

Figure 2: The Effects of Investment Horizon and RI on Long-run Risk

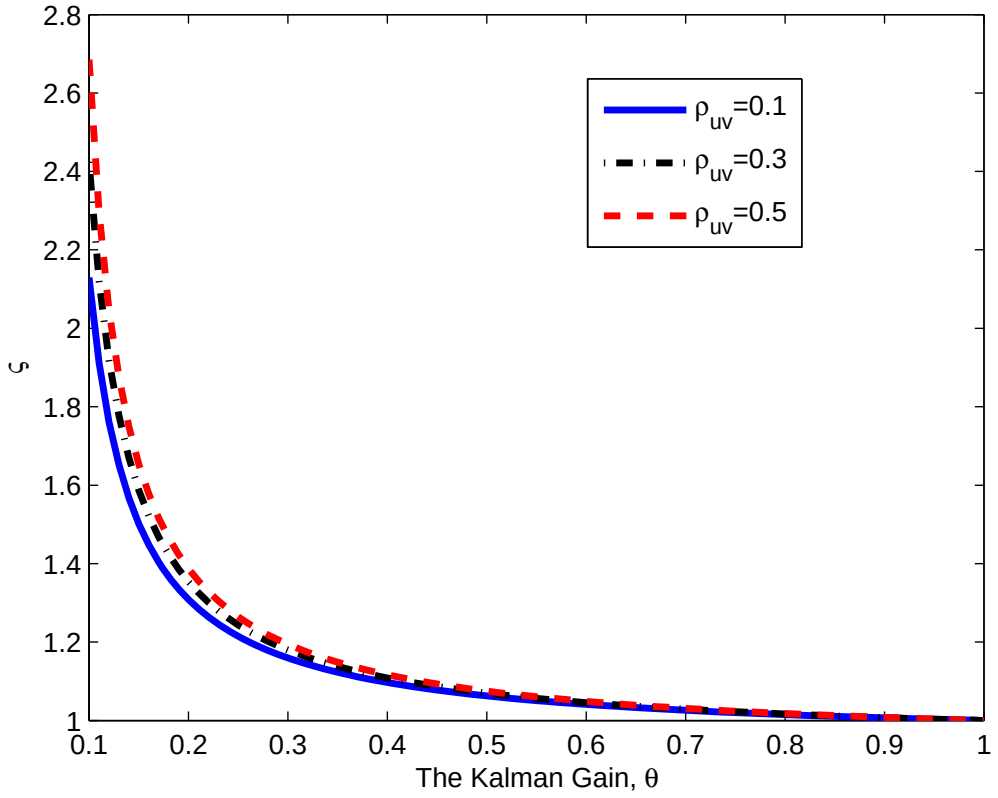

Figure 3: The Effects of Correlated Labor Income on Long-run Risk

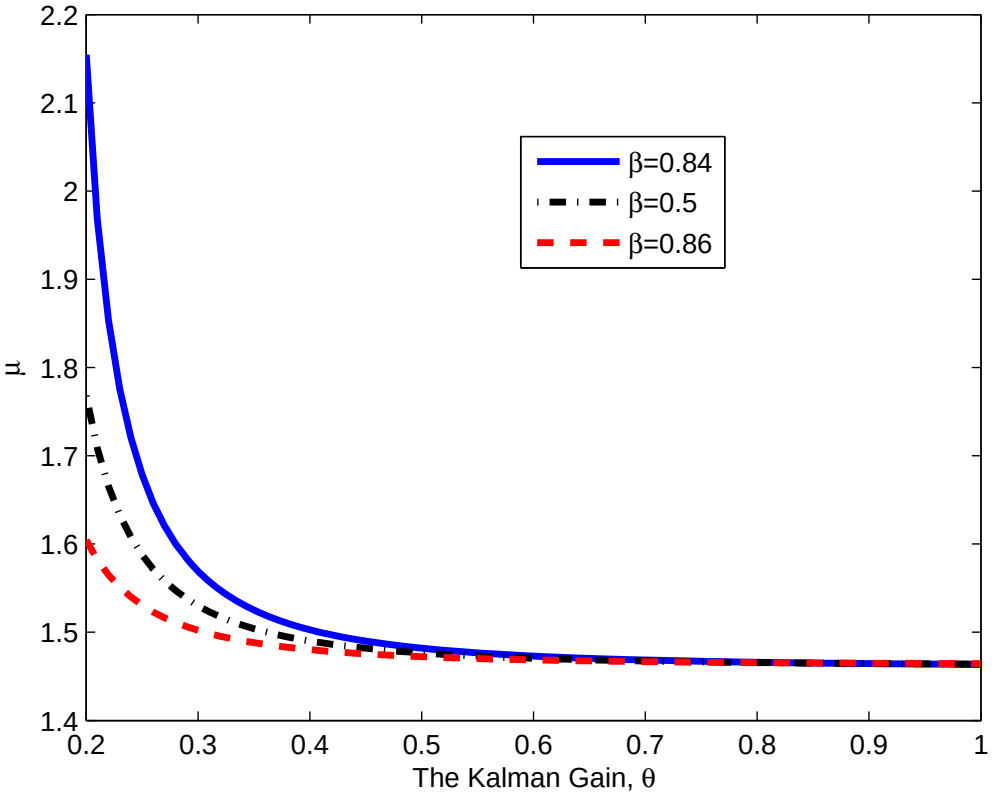

Figure 4: The Effects of RI on the Relative Volatility of Consumption Growth

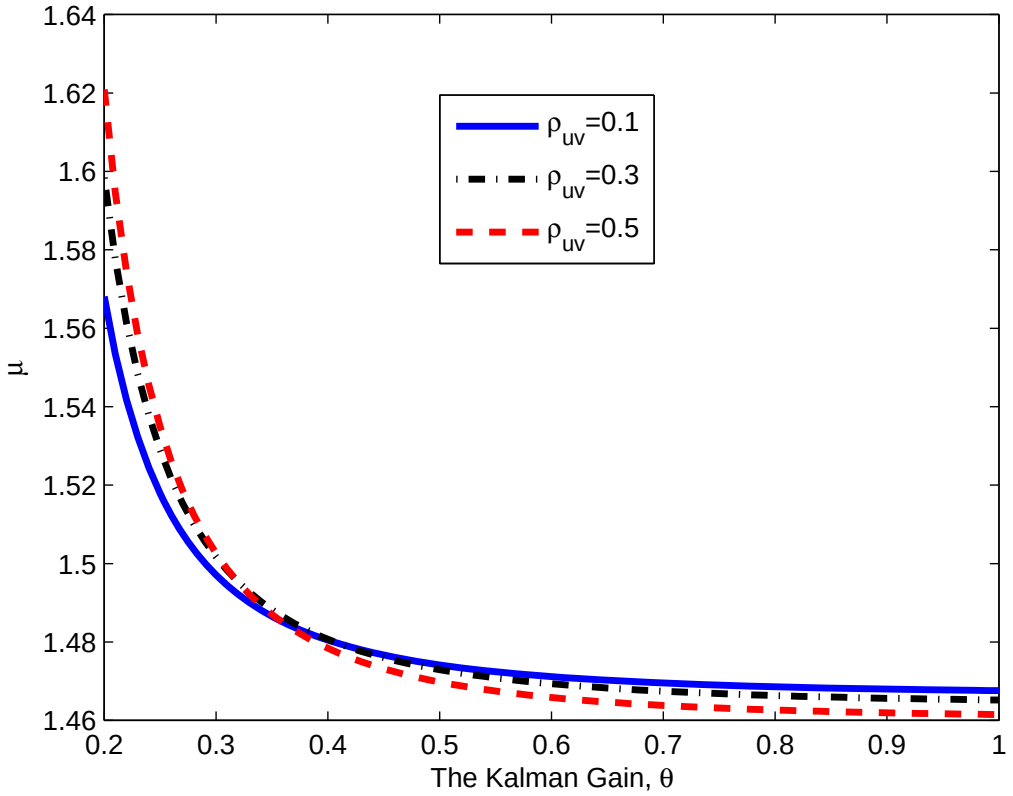

Figure 5: The Effects of RI on the Relative Volatility of Consumption Growth

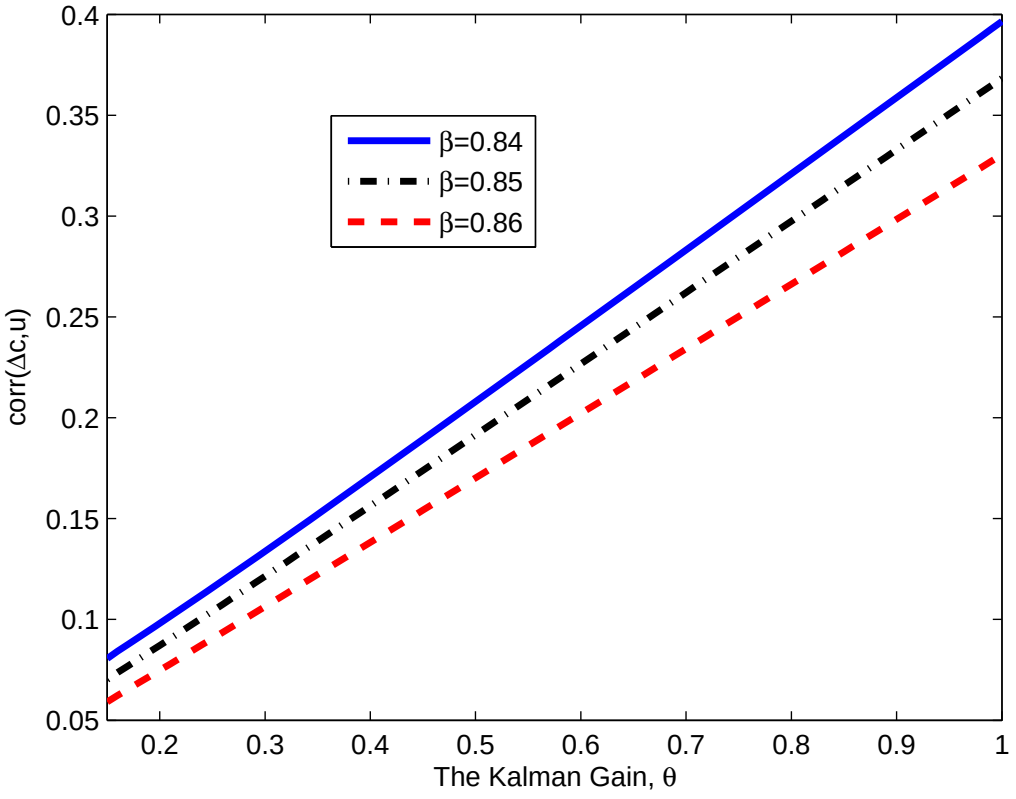

Figure 6: The Effects of RI on the Correlation between Consumption Growth and Equity Returns

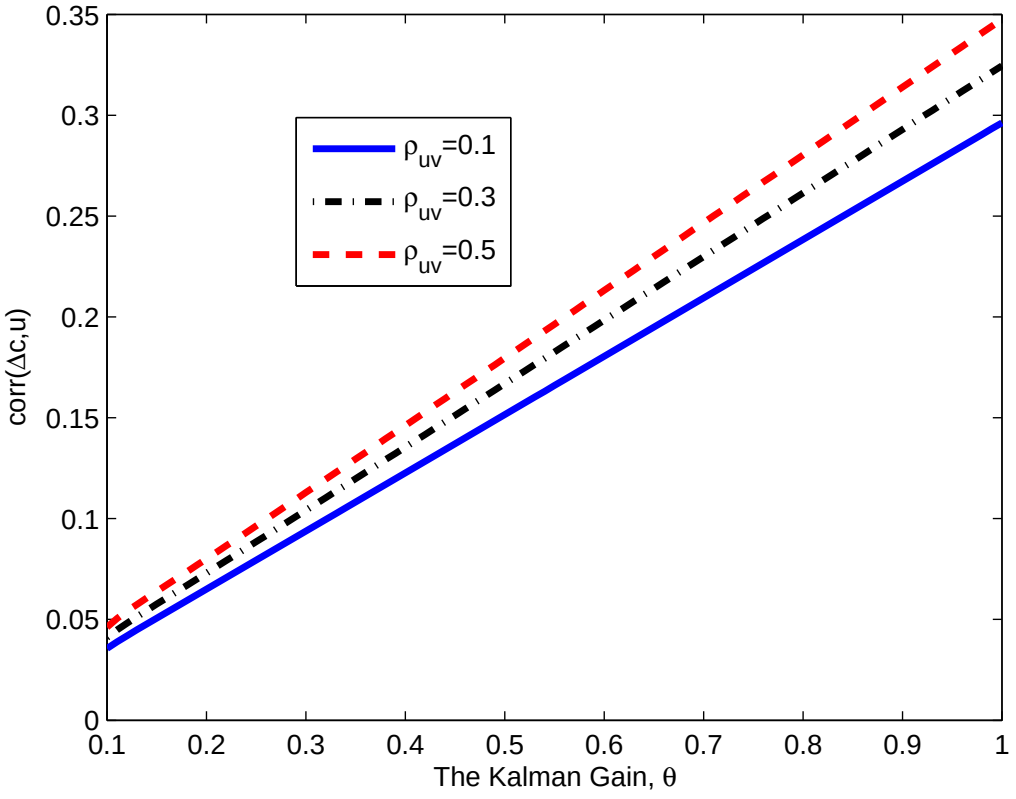

Figure 7: The Effects of RI on the Correlation between Consumption Growth and Equity Returns