Online Appendix for "Rational Inattention and the Dynamics of Consumption and Wealth in General Equilibrium" (Not for Publication)

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1 Online Appendix A: Optimal Form of the Noisy Signal

When $\kappa = \infty$, the true state (permanent income), s_t, is governed by the following unit root process:

$$s_t = s_{t-1} + \zeta_t. \tag{1}$$

We then introduce RI into this FI-RE problem by assuming that the consumer cannot observe the true state perfectly. Following Sims (2003) and Mackowiak and Wiederholt (2009), the consumer need to solve the following optimizing problem to infer the property of the RI-induced noise:

$$\min_{\{b,c\}} E\left[(s_t - \widehat{s}_t)^2\right], \tag{2}$$

subject to

$$s_t = \sum_{l=0}^{\infty} \zeta_{t-l}, \tag{3}$$

$$\widehat{s}_t = \sum_{l=0}^{\infty} b_l \zeta_{t-l} + \sum_{l=0}^{\infty} c_l \xi_{t-l}, \tag{4}$$

$$I\left(\{s_t\}, \{\widehat{s}_t\}\right) \leq \kappa, \tag{5}$$

where the sequences, $\{b_l\}_{l=0}^{\infty}$ and $\{c_l\}_{l=0}^{\infty}$, are absolutely summable and ζ_t and ξ_t are independent Gaussian white noise processes. The consumer chooses the $\widehat{s}_t$ process to track s_t. The last constraint restricts the information flow between s_t and $\widehat{s}_t$. We can show that given (1), the following process is a solution to Problem (2)-(5),:

$$\widehat{s}_t^* = \sum_{l=0}^{\infty} \left[1 - (1-\theta)^{1+l}\right] \zeta_{t-l} + \sum_{l=0}^{\infty} \sqrt{\frac{\theta(1-\theta)}{1-(1-\theta)^2}} (1-\theta)^l \xi_{t-l}, \tag{6}$$

where $\theta = 1 - \exp(-2\kappa)$ is the Kalman gain. The main idea of the proof is that we first establish a lower bound for the mean square error at the optimal solution, and then show that $\widehat{s}_t^*$ attains this bound and satisfies the information flow constraint. The following is the detailed proof.

We can quantify the information flow between two stochastic processes as the average per period amount of information that one process contains about the other process. Let $(s_1, \cdots, s_T)$ and $(\widehat{s}_1, \cdots, \widehat{s}_T)$ denote the first T elements of the processes, $\{s_t\}$ and $\{\widehat{s}_t\}$, respectively. The amount of information that $\widehat{s}^T = (\widehat{s}_1, \cdots, \widehat{s}_T)$ contains about $s^T = (s_1, \cdots, s_T)$ can be written as:

$$I\left(s^T, \widehat{s}^T\right) = H\left(s^T\right) - H\left(s^T | \widehat{s}^T\right).$$

The information flow between the two processes is defined as follows:

$$I\left(\{s_t\}, \{\widehat{s}_t\}\right) = \lim_{T\to\infty} \frac{1}{T} I\left(s_1, \cdots, s_T; \widehat{s}_1, \cdots, \widehat{s}_T\right).$$

Let $s^{t-1} = (s_1, \cdots, s_{t-1})$. Applying the chain rule for entropy yields:

$$H\left(\widehat{s}^T\right) = H\left(\widehat{s}_1\right) + \sum_{t=2}^{T} H\left(\widehat{s}_t|\widehat{s}^{t-1}\right) \text{ and } H\left(\widehat{s}^T|s^T\right) = H\left(\widehat{s}_1|s^T\right) + \sum_{t=2}^{T} H\left(\widehat{s}_t|\widehat{s}^{t-1}, s^T\right),$$

which means that

$$H\left(\widehat{s}_t|\widehat{s}^{t-1}, s_t\right) \geq H\left(\widehat{s}_t|\widehat{s}^{t-1}, s^T\right).$$

Combining these results yields

$$I\left(s^T, \widehat{s}^T\right) \geq H\left(\widehat{s}_1\right) - H\left(\widehat{s}_1|s^T\right) + \sum_{t=2}^{T}\left[H\left(\widehat{s}_t|\widehat{s}^{t-1}\right) - H\left(\widehat{s}_t|\widehat{s}^{t-1}, s_t\right)\right]$$

Furthermore, using the chain rule for entropy, we have

$$\begin{aligned} H\left(\widehat{s}_t|\widehat{s}^{t-1}\right) - H\left(\widehat{s}_t|\widehat{s}^{t-1}, s_t\right) &= H\left(\widehat{s}_t, \widehat{s}^{t-1}\right) - H\left(\widehat{s}^{t-1}\right) - \left[H\left(\widehat{s}_t, \widehat{s}^{t-1}, s_t\right) - H\left(\widehat{s}^{t-1}, s_t\right)\right] \\ &= H\left(\widehat{s}^{t-1}, s_t\right) - H\left(\widehat{s}^{t-1}\right) - \left[H\left(\widehat{s}_t, \widehat{s}^{t-1}, s_t\right) - H\left(\widehat{s}_t, \widehat{s}^{t-1}\right)\right] \\ &= H\left(s_t|\widehat{s}^{t-1}\right) - H\left(s_t|\widehat{s}_t, \widehat{s}^{t-1}\right). \end{aligned}$$

Using the fact that s^T and $\widehat{s}^T$ have a multivariate normal distribution yields

$$H\left(s_t|\widehat{s}^{t-1}\right) - H\left(s_t|\widehat{s}^{t}\right) = \frac{1}{2}\ln\left(\frac{\text{var}\left(s_t|\widehat{s}^{t-1}\right)}{\text{var}\left(s_t|\widehat{s}^{t}\right)}\right).$$

Taking conditional variance on both sides of (1) yields: $\text{var}\left(s_t|\widehat{s}^{t-1}\right) = \text{var}\left(s_{t-1}|\widehat{s}^{t-1}\right) + \omega_\zeta^2$. Using the facts that $\text{var}\left(s_t|\widehat{s}^t\right) = E\left[\left(s_t - \widehat{s}_t\right)^2\right]$ and $E\left[\left(s_{t-1} - \widehat{s}_{t-1}\right)^2\right]$, we have

$$\begin{aligned} I\left(s^T, \widehat{s}^T\right) &\geq H\left(\widehat{s}_1\right) - H\left(\widehat{s}_1|s^T\right) + \sum_{t=2}^{T}\left[H\left(s_t|\widehat{s}^{t-1}\right) - H\left(s_t|\widehat{s}_t, \widehat{s}^{t-1}\right)\right] \\ &= H\left(\widehat{s}_1\right) - H\left(\widehat{s}_1|s^T\right) + \frac{1}{2}\sum_{t=2}^{T}\ln\left(1 + \frac{\omega_\zeta^2}{E\left[\left(s_t - \widehat{s}_t\right)^2\right]}\right). \end{aligned}$$

Dividing the inequality by T on both sides and letting T go to infinity yields

$$I\left(\{s_t\}, \{\widehat{s}_t\}\right) \geq \frac{1}{2}\ln\left(1 + \frac{\omega_\zeta^2}{E\left[\left(s_t - \widehat{s}_t\right)^2\right]}\right), \tag{7}$$

which means that at a solution, $E\left[\left(s_t - \widehat{s}_t\right)^2\right] \geq \frac{\omega_\zeta^2}{\exp(2\kappa)-1}$.

Second, $\widehat{s}_t^*$ has the property

$$s_t - \widehat{s}_t^* = \frac{1}{\exp(2\kappa)}\left(s_{t-1} - \widehat{s}_{t-1}^*\right) + \left(\frac{1}{\exp(2\kappa)}\zeta_t - \sqrt{\frac{\exp(2\kappa) - 1}{\exp(2\kappa)\left(\exp(2\kappa) - 1\right)}}\xi_t\right) \tag{8}$$

It implies that

$$E\left[(s_t-\widehat{s}_t)^2\right]=\frac{\omega_\zeta^2}{\exp(2\kappa)-1}.$$

Given that $s_t=s_{t-1}+\zeta_t$, we have

$$\widehat{s}_t=\frac{1}{\exp(2\kappa)}\widehat{s}_{t-1}+\left(1-\frac{1}{\exp(2\kappa)}\right)s_t+\sqrt{\frac{\exp(2\kappa)-1}{\exp(2\kappa)\left(\exp(2\kappa)-1\right)}}\xi_t, \tag{9}$$

which means that

$$\text{var}\left(\widehat{s}_t^*|\widehat{s}_1^*,\cdots,\widehat{s}_{t-1}^*;s^T\right)=\text{var}\left(\widehat{s}_t^*|\widehat{s}_1^*,\cdots,\widehat{s}_{t-1}^*;s_t\right), \text{ or } H\left(\widehat{s}_t^*|\widehat{s}_1^*,\cdots,\widehat{s}_{t-1}^*;s^T\right)=H\left(\widehat{s}_t^*|\widehat{s}_1^*,\cdots,\widehat{s}_{t-1}^*;s_t\right)$$

Therefore, for the process $\widehat{s}_t^*$, all the weak inequalities in the first half of the proof hold with equality, and

$$I\left(\{s_t\},\{\widehat{s}_t\}\right)=\frac{1}{2}\ln\left(1+\frac{\omega_\zeta^2}{E\left[(s_t-\widehat{s}_t)^2\right]}\right)=\kappa. \tag{10}$$

Substituting $s_t=s_{t-1}+\zeta_t$ into (9) and using (8) and the fact that $\theta=1-\exp(-2\kappa)$, we can get (6).

Finally, we show that the noisy signal,

$$\widetilde{s}_t=s_t+\frac{\exp(\kappa)}{\exp(2\kappa)-1}\zeta_t, \tag{11}$$

has the property that $\widehat{s}_t^*=E\left[s_t|\widetilde{s}^t\right]$. Using the formula for updating a linear projection, we have

$$E\left[s_t|\widetilde{s}^t\right]=E\left[s_t|\widetilde{s}^{t-1}\right]+\theta\left(\widetilde{s}_t-E\left[\widetilde{s}_t|\widetilde{s}^{t-1}\right]\right),$$

where $\theta=\frac{E\left[\left(s_t-E\left[s_t|\widetilde{s}^{t-1}\right]\right)\left(\widetilde{s}_t-E\left[\widetilde{s}_t|\widetilde{s}^{t-1}\right]\right)\right]}{E\left[\left(\widetilde{s}_t-E\left[\widetilde{s}_t|\widetilde{s}^{t-1}\right]\right)^2\right]}$ is the Kalman gain. Using the expression for the noisy signal, (11), to replace $\widetilde{s}_t$, we have

$$E\left[s_t|\widetilde{s}^t\right]=(1-\theta)E\left[s_{t-1}|\widetilde{s}^{t-1}\right]+\theta\left(s_t+\frac{\exp(\kappa)}{\exp(2\kappa)-1}\zeta_t\right),$$

where $\theta=\frac{E\left[\left(s_{t-1}-E\left[s_{t-1}|\widetilde{s}^{t-1}\right]\right)^2\right]+\omega_\zeta^2}{E\left[\left(s_{t-1}-E\left[s_{t-1}|\widetilde{s}^{t-1}\right]\right)^2\right]+\omega_\zeta^2+\frac{\exp(2\kappa)}{(\exp(2\kappa)-1)^2}\omega_\zeta^2}$. Finally, after plugging in the guess $E\left[s_t|\widetilde{s}^t\right]=\widehat{s}_t^*$ and $E\left[\left(s_t-E\left[s_t|\widetilde{s}^t\right]\right)^2\right]=\frac{\omega_\zeta^2}{\exp(2\kappa)-1}$, we obtain that

$$\widehat{s}_t^*=(1-\theta)\widehat{s}_{t-1}^*+\theta\left(s_t+\frac{\exp(\kappa)}{\exp(2\kappa)-1}\zeta_t\right),$$

which is just (6).

2 Online Appendix B: Optimality of Ex Post Gaussianity under RI

Following Sims (2003, 2010), we first define the expected loss function due to limited information-processing capacity as

$$L_t = E_t \left[v_0 \left(s_t \right) - \widehat{v} \left(x_t \right) \right], \tag{12}$$

where s_t is the unobservable state variable, x_t is the best estimate of the true state, $\widehat{v}(x_t) = -\frac{\psi(1+r)}{r} \exp\left(-\frac{r}{\psi} x_t\right)$ is the value function under RI and $v_0(s_t) = -\frac{\psi(1+r)}{r} \exp\left(-\frac{r}{\psi} s_t\right)$ is the corresponding value function when $\kappa = \infty$. It is straightforward to show that:

$$\begin{aligned}
& \min E_t \left[v_0 \left(s_t \right) - \widehat{v} \left(x_t \right) \right] \\
& = \min - \frac{\psi \left(1 + r \right)}{r} E_t \left[\exp \left(- \frac{r}{\psi} s_t \right) - \exp \left(- \frac{r}{\psi} x_t \right) \right] \\
& \simeq \min - \frac{\psi \left(1 + r \right)}{r} E_t \left[- \frac{r}{\psi} \exp \left(- \frac{r}{\psi} x_t \right) \left(s_t - x_t \right) + \frac{1}{2} \left(\frac{r}{\psi} \right)^2 \exp \left(- \frac{r}{\psi} x_t \right) \left(s_t - x_t \right)^2 \right] \\
& \Longleftrightarrow \min - \frac{\psi \left(1 + r \right)}{2r} E_t \left[\left(\frac{r}{\psi} \right)^2 \exp \left(- \frac{r}{\psi} x_t \right) \left(s_t - x_t \right)^2 \right] \\
& \Longleftrightarrow \min - \frac{r \left(1 + r \right)}{2\psi} \exp \left(- \frac{r}{\psi} x_t \right) E_t \left[\left(s_t - x_t \right)^2 \right].
\end{aligned}$$

Since x_t is non-stationary process, we normalize this objective function by dividing it by the value function under RI:

$$\min \frac{E_t \left[v_0 \left(s_t \right) - \widehat{v} \left(x_t \right) \right]}{\widehat{v} \left(x_t \right)} \Longleftrightarrow \min \frac{1}{2} \left(\frac{r}{\psi} \right)^2 E_t \left[\left(s_t - x_t \right)^2 \right] \Longleftrightarrow \min \frac{1}{2} \left(\frac{r}{\psi} \right)^2 \Sigma_t, \tag{13}$$

where Σ_t is the conditional variance at t. The normalization method is also used in Maenhout (2004) and Liu, Pan, and Wang (2005) in which they used this normalization method to assure the homothecity or scale invariance of the optimal consumption and portfolio decision problem when investors are concerned about model misspecification. Note that the agent's filtering problem, (13), is invariant to the scale of perceived total resources x_t after normalization, which we use so that the optimal choice of attention does not disappear as the value of total wealth increases.

The (approximate) loss function under CARA derived above is essentially the same as that obtained in the LQG RI model proposed in Sims (2003). Since the only difference in these two settings is just in the constant coefficient in the loss function, the CARA specification does not affect the optimality of ex post Gaussianity in Sims' LQG setting after we approximate the value functions we obtained in the CARA-Gaussian setting.

Furthermore, we use the following procedure to justify the quadratic approximation, (13):

1. We first conjecture that the quadratic approximation is an accurate approximation for the original exponential loss function, and the higher-order moments in the expansion of the CARA loss function are trivial.

2. Given that the loss function is quadratic, we can obtain the optimality of the ex post Gaussian variables and Gaussian noises and noisy signals (i.e., both x and s are Gaussian).

3. For Gaussian variables, s_t and x_t, the higher-order moments (the third and fourth moments) of the loss function can be written as

$$\frac{(r/\psi)^2}{6}\exp\left(\frac{-rb_0-rx_t}{\psi}\right)E_t\left[(s_t-x_t)^3\right]-\frac{(r/\psi)^3}{24}\exp\left(\frac{-rb_0-rx_t}{\psi}\right)E_t\left[(s_t-x_t)^4\right]$$
$$=-\frac{(r/\psi)^3}{24}\exp\left(\frac{-rb_0-rx_t}{\psi}\right)\Sigma^2\times 3,$$

where we use the facts that for Gaussian variables, we have

$$E_t\left[(s_t-x_t)^j\right]=\begin{cases}\sigma^j\,(j-1)!!, \text{ when } j \text{ is even,}\\ 0, \text{ when } j \text{ is odd.}\end{cases},$$

where $x_t = E_t[s_t]$ and σ is the standard deviation of x.

4. It is straightforward to calculate that the ratio of the sum of the third and fourth moments to the second moment is:

$$ratio=\frac{-\frac{(r/\psi)^3}{24}\exp\left(\frac{-rb_0-rx_t}{\psi}\right)\Sigma^2\times 3}{-\frac{r}{2\psi}\exp\left(\frac{-rb_0-rx_t}{\psi}\right)\Sigma}=\frac{(r\alpha)^2\,\Sigma}{4}=0.0287^2\times 15.77=0.013,$$

where we use the fact that $\Sigma = 15.77$ when $\theta = 10\%$. We can then verify that our guess that the quadratic approximation is accurate is correct.

3 Online Appendix C: Details on the Data and Sample Selection

Following Blundell, Pistaferri, and Preston (2008), we define the household income as total household income (including wage, financial, and transfer income of head, wife, and all others in household) minus financial income (defined as the sum of annual dividend income, interest income, rental income, trust fund income, and income from royalties for the head of the household only) minus the tax liability of non-financial income. This tax liability is defined as the total federal tax liability multiplied by the non-financial share of total income. Tax liabilities after 1992 are not reported in the PSID and so we estimate them using the TAXSIM program from the

National Bureau of Economic Research. Our final household income measure can be expressed as:

$$\text{income measure} = (\text{total HH income} - \text{financial income}) - \text{taxes} \times \frac{\text{total HH income} - \text{financial income}}{\text{total HH income}}.$$

Our household sample selection closely follows that of Blundell, Pistaferri, and Preston (2008) as well.[1] We exclude households in the PSID poverty and Latino subsamples. We exclude households in years of family composition change, change in marital status, or female headship, as well as in years where the head or wife is under 30 or over 65. Households in years with missing education, region, income, and imputed consumption responses are also excluded. We also exclude households in years where they report a negative income or a food consumption level in the top or bottom 5 percent of all reported values in that year. Income and consumption values are then deflated by the CPI to constant $1982-1984$ dollars. Our final panel contains $7,111$ unique households with $58,034$ yearly income responses and $48,990$ imputed nondurable consumption values.[2] With this constructed panel of household income and consumption, we next drop households in years where year-over-year food consumption changes are more than 20 percent or less than -20 percent.

In constructing the sample with both income and wealth information, we normalize each reported wealth and income value to the mean of the income the year reported, and then exclude outliers of this distribution at the top and bottom 1 percent. We then take the standard deviations of the change in normalized value from the previous report for both wealth and income to calculate our ratio. Our final panel for wealth and income has $23,630$ observations across 6232 households. This panel is somewhat smaller than our panel of consumption and income due to the limited number of years that wealth measures are reported.

Finally, when estimating the income process, we focus on the sample period to the years $1980-1996$, due to the PSID survey changing to a biennial schedule after 1996. To further restrict the sample to exclude households with dramatic year-over-year income changes, we eliminate household incomes with year-over-year level changes in the top and bottom 5 percent of the distribution in each year.

[1] They create a new panel series of consumption that combines information from PSID and CEX, focusing on the period when some of the largest changes in income inequality occurred. For other explanations for observed consumption and income inequality, see Krueger and Perri (2006) and Attanasio and Pavoni (2011).

[2] There are more household incomes than imputed consumption values because food consumption - the main input variable in Guvenen and Smith's nondurable demand function - is not reported in the PSID for the years 1987 and 1988. Dividing the total income responses by unique households yields an average of 7-8 years of responses per household. These years are not necessarily consecutive as our sample selection procedure allows households to be excluded in certain years but return to the sample if they later meet the criteria once again.

4 Online Appendix D: Solving the Habit Formation Model

Following Alessie and Lusardi (1997), we model internal habit formation by assuming that the period utility is defined on $c_t - \gamma c_{t-1}$. For each stochastic consumption stream, $\{c_t\}_{t=0}^{\infty}$, the utility stream of the consumer with recursive preference, $\{u_t\}_{t=0}^{\infty}$, can be recursively defined as follows:

$$u_t = u\left(c_t - \gamma c_{t-1}\right) + \beta u\left(\mathcal{CE}_t\left(u^{-1}\left(u_{t+1}\right)\right)\right),$$

where $u(c) = -\psi \exp(-c/\psi)$, $v(c) = -\exp(-\alpha c)/\alpha$, and $\mathcal{CE}_t(x_{t+1}) = v^{-1}(E_t[v(x_{t+1})])$ is the certainty equivalent of x_{t+1} conditional on the period-t information. The agent solves in period t the following optimization problem:

$$u\left(J\left(s_t\right)\right) = \max_{c_t} u\left(c_t - \gamma c_{t-1}\right) + \beta u\left(\mathcal{CE}_t\left(J\left(s_{t+1}\right)\right)\right),$$

subject to $s_{t+1} = (1+r)\,s_t - c_t + \zeta_{t+1}$.

We first conjecture that the typical consumer has the following value function: $J(s_t) = A_1 s_t + A_2 c_{t-1} + B$, where A and B are undetermined coefficients. Substituting the guessed value function into the Bellman equation yields:

$$-\psi \exp\left(-\frac{1}{\psi}J\left(s_t\right)\right) = \max_{c_t}\left\{-\psi \exp\left(-\frac{1}{\psi}\left(c_t - \gamma c_{t-1}\right)\right) - \beta\psi \exp\left(-\frac{1}{\psi}J\left(\left(1+r\right)s_t - c_t - \frac{1}{2}\alpha A\omega_{\zeta}^2\right)\right)\right\},$$

where we use the facts that $E_t[s_{t+1}] = (1+r)\,s_t - c_t$ and $\text{var}_t[s_{t+1}] = \omega_{\zeta}^2$. The FOC for c_t is

$$\exp\left(-\frac{1}{\psi}\left(c_t - \gamma c_{t-1}\right)\right) = \beta A \exp\left(-J\left(\left(1+r\right)s_t - c_t - \frac{1}{2}\alpha A\omega_{\zeta}^2\right)\right)$$

The Envelop theorem is:

$$\exp\left(-\frac{1}{\psi}J\left(s_t\right)\right) = \beta R \exp\left(-J\left(\left(1+r\right)s_t - c_t - \frac{1}{2}\alpha A\omega_{\zeta}^2\right)\right).$$

Combining these two conditions yields

$$c_t = \gamma c_{t-1} + J\left(s_t\right) - \psi \ln\left(\frac{A_1 - A_2}{R}\right) = \left(\gamma + A_2\right)c_{t-1} + A_1\widehat{s}_t + B - \psi \ln\left(\frac{A_1 - A_2}{R}\right) \quad (14)$$

Substituting the consumption function into the Bellman equation and matching the terms, we have

$$A_1 = r\left(1 - \frac{\gamma}{1+r}\right),\; A_2 = -\frac{r}{1+r}\gamma,\; B = \frac{1}{r}\left[\psi \ln\left(\frac{1+\rho}{1+r}\right) - \frac{1}{2}\alpha A_1^2\omega_{\zeta}^2\right] + \psi \ln\left(\frac{r}{R}\right).$$

Substituting them into (14) yields the consumption function in the main text:

$$c_t = \frac{\gamma}{1+r}c_{t-1} + r\left(1 - \frac{\gamma}{1+r}\right)s_t + \frac{\psi}{r}\ln\left(\frac{1+\rho}{1+r}\right) - \frac{1}{2}\alpha r\left(1 - \frac{\gamma}{1+r}\right)^2\omega_{\zeta}^2. \quad (15)$$

Substituting (15) into $d_t \equiv ra_t + y_t - c_t$, we can solve for the corresponding saving function as follows:

$$d_t = (1-\phi_1)\,\phi\,(y_t - \overline{y}) + \frac{r\gamma}{1+r}\frac{\zeta_t}{1-\gamma\cdot L} - \frac{1}{(1-\gamma)}\left[\frac{\psi}{r}\ln\left(\frac{1+\rho}{1+r}\right) - \frac{1}{2}r\alpha\left(\frac{1+r-\gamma}{1+r}\right)^2\omega_\zeta^2\right]. \tag{16}$$

In general equilibrium with the equilibrium interest rate r^*, we have

$$\frac{\psi}{r}\ln\left(\frac{1+\rho}{1+r^*}\right) = \frac{1}{2}\alpha r^*\widetilde{\Gamma}(\gamma, r^*)\,w_\zeta^2, \tag{17}$$

where $\widetilde{\Gamma}(\gamma, r^*) = \left(\frac{1+r^*-\gamma}{1+r^*}\right)^2 < 1$. The equilibrium consumption function (15) turns out to

$$c_t^* = \frac{r^*(1+r^*-\gamma)}{1+r^*}s_t + \frac{\gamma}{1+r^*}c_{t-1}^*, \tag{18}$$

for $t \geq 0$, c_{-1}, a_0, y_0, and s_0 are given.

To derive the relative volatility of financial wealth (a_t) to labor income (y_t) in general equilibrium, we first write the expression for individual saving as follows:

$$\begin{aligned}\Delta a_{t+1}^* = d_t^* &= (1-\phi_1)\,\phi\,(y_t - \overline{y}) + \frac{r^*\gamma}{1+r^*}\frac{\zeta_t}{1-\gamma\cdot L} \\ &= \sum_{j=0}^{\infty}\left[(1-\phi_1)\,\phi\phi_1^j + \frac{r^*\gamma}{1+r^*}\phi\gamma^j\right]w_{t-j}.\end{aligned} \tag{19}$$

Taking unconditional variance on both sides yields:

$$\operatorname{var}\left(\Delta a_{t+1}^*\right) = \operatorname{var}(w_{t+1})\sum_{j=0}^{\infty}\left[(1-\phi_1)\,\phi\phi_1^j + \frac{r^*\gamma}{1+r^*}\phi\gamma^j\right]^2, \tag{20}$$

which implies the expression for the relative volatility of individual wealth growth to income growth in the main text.

In general equilibrium, the difference in total wealth can be written as

$$\begin{aligned}\Delta s_{t+1}^* &= r^* s_t^* - c_t^* + \zeta_{t+1} \\ &= r^*\left[a_t^* + \phi\left(y_t + \frac{\phi_0}{r^*}\right)\right] - c_t^* + \zeta_{t+1} \\ &= d_t^* + (r^*\phi - 1)\,y_t + \phi\phi_0 + \zeta_{t+1}, \\ &= \sum_{j=0}^{\infty}\left[(1-\phi_1)\,\phi\phi_1^j + \frac{r^*\gamma\phi}{1+r^*}\gamma^j + (r^*\phi - 1)\,\phi_1^j\right]w_{t-j} + \phi w_{t+1}.\end{aligned} \tag{21}$$

In the next step, we write the expression for the change in consumption as

$$\begin{aligned}\Delta c_{t+1}^{*} &= \frac{\gamma}{(1+r^{*})}\Delta c_{t}^{*} + \frac{r^{*}(1+r^{*}-\gamma)}{1+r^{*}}\Delta s_{t+1}^{*} \\ &= \frac{\gamma}{(1+r^{*})}\Delta c_{t}^{*} + \frac{r^{*}(1+r^{*}-\gamma)}{1+r^{*}}\left\{\sum_{j=0}^{\infty}\left[\begin{array}{c}(1-\phi_1)\phi\phi_1^{j}+ \\ \frac{r^{*}\gamma}{1+r^{*}}\phi\gamma^{j}+(r^{*}\phi-1)\phi_1^{j}\end{array}\right]w_{t-j}+\phi w_{t+1}\right\}.\end{aligned} \tag{22}$$

Taking unconditional variance on both sides yields:

$$\begin{aligned}\operatorname{var}\left(\Delta c_{t+1}^{*}\right) = &\left(\frac{\gamma}{1+r^{*}}\right)^{2}\operatorname{var}\left(\Delta c_{t}^{*}\right)+ \\ &\left(\frac{r^{*}(1+r^{*}-\gamma)}{1+r^{*}}\right)^{2}\left\{\sum_{j=0}^{\infty}\left[\begin{array}{c}(1-\phi_1)\phi\phi_1^{j}+ \\ (r^{*}\phi-1)\phi_1^{j}+\frac{r^{*}\gamma}{1+r^{*}}\phi\gamma^{j}\end{array}\right]^{2}+\phi^{2}\right\}\operatorname{var}\left(w_{t+1}\right),\end{aligned}$$

which implies the expression for the relative volatility of individual consumption growth to income growth in the main text.

5 Online Appendix E: The Model with Borrowing Constraints

In the interest of keeping the paper self-contained, in this appendix we detail the exact version of Huggett (1993) we use to discuss the effects of borrowing constraints. Households differ according to labor earnings y and assets a, as in the main text, but $a \geq \underline{a}$ is imposed. Given a constant interest rate r, the dynamic program of the household is

$$V(a,y) = \max_{c\geq 0, a'\geq 0}\left\{\frac{c^{1-\sigma}-1}{1-\sigma}+\beta E\left[V\left(a',y'\right)|y\right]\right\}$$

subject to

$$c + a' \leq (1+r)\,a + y.$$

Let $\Gamma(a,y)$ denote the stationary distribution of households over the idiosyncratic states; market clearing then requires

$$\begin{aligned}A &= \int_A\int_Y a\Gamma(a,y) \\ C &= \int_A\int_Y c(a,y)\,\Gamma(a,y)\end{aligned}$$

and in a stationary equilibrium

$$A = 0.$$

We choose an unremarkable set of parameters: $\sigma = 1.5$, $\beta = 0.99$, $\underline{a} = -2$, and a process for idiosyncratic earnings given by

$$\begin{aligned}\log\left(y'\right) &= 0.95\log(y) + 0.1e' \\ e &\sim N(0,1);\end{aligned}$$

we approximate this process using Rouwenhorst's method with 7 points. Our results are insensitive to reasonable changes in these parameters, in particular to changes in the borrowing constraint (tighter constraints lead to even more counterfactual implications). The model is solved using a straightforward algorithm: (1) Given r, we can solve the household problem using dynamic programming with Piecewise Cubic Hermite splines as an approximation of V and a nonlinear optimization method to do the maximization (we use Feasible Sequential Quadratic Programming or FSQP – see Zhou, Tits, and Lawrence 1997 for details); (3) Given the policy functions from step (2), we follow the iterative method from Young (2010) to obtain Γ; (4) We evaluate whether $A = 0$ and update using Brent's method until this equation is satisfied. All code is written in Fortran95 and available upon request (with the exception of the FSQP routine which is not in the public domain and cannot be distributed; interested academic readers can obtain the free SLICOT Library which contains FSQP from `http://www.icm.tu-bs.de/NICONET/`).

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