Online Appendix for "Elastic Attention, Risk Sharing, and International Comovements" (Not for Publication)

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1 Online Appendix A: Comparison with Other Hypotheses

Habit formation has been modelled directly as a structure of preferences in which psychological factors make consumers prefer gradual adjustment in consumption; consequently, consumption volatility is more painful than it would be in the absence of habits. The key difference between habit formation and RI without noises is that slow adjustment in consumption under habit formation is optimal because consumers are assumed to prefer to smooth not only consumption but also consumption growth, while slow consumption adjustment under RI is optimal because capacity constraints make consumers take more time to acquire and process information. Therefore, habit formation by itself cannot help resolve the cross-country consumption correlation puzzle. This is consistent with the conclusion obtained in a habit formation SOE model proposed by Fuhrer and Klein (2006).

The slow adjustment mechanism can also be generated by assuming that consumers cannot

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distinguish the two components of income specified as follows:[1]

$$y_t = y_t^p + y_t^i, \tag{1}$$

$$y_t^p = y_{t-1}^p + \varepsilon_t, \tag{2}$$

$$y_t^i = \overline{y} + \epsilon_t, \tag{3}$$

where y_t^p and y_t^i are the permanent and transitory income components, respectively, and ε_t and ϵ_t are orthogonal iid shocks with mean 0 and variance ω^2 and ω_ϵ^2, respectively. Specifically, following Pischke (1991), given that the change in income is

$$\Delta y_{t+1} = \varepsilon_{t+1} + \epsilon_{t+1} - \epsilon_t, \tag{4}$$

the best forecast is to recognize that Δy_{t+1} is a moving-average process of order one:

$$\Delta y_{t+1} = \nu_{t+1} - \alpha \nu_t, \tag{5}$$

where the innovation, ν_t, with mean 0 and variance ω_ν^2, is not a fundamental driving process – it contains information on current and lagged permanent and transitory income shocks. Equating the variances and autocorrelation coefficients of the original and derived processes, (4) and (5), we have

$$\omega_\nu^2 = \frac{\omega_\epsilon^2}{\alpha} \text{ and } \alpha = -\frac{1 - \sqrt{1 - 4\varrho^2}}{2\varrho},$$

where $\varrho = -\omega_\epsilon^2 / \left(\omega^2 + 2\omega_\epsilon^2\right)$ and $\alpha \in [0, 1]$ will be large if the variance of the transitory shock ω_ϵ^2 is large relative to the variance of the permanent shock ω^2 and will converge to 0 as ω_ϵ^2 approaches to 0. Following the same procedure in the main text of the paper, we can solve for the expression for the change in aggregate consumption as follows:

$$\Delta c_t = \frac{R - \alpha}{R} \frac{\varepsilon_{t+1}}{1 - \alpha \cdot L}. \tag{6}$$

where the slow adjustment mechanism is captured by the factor $1/(1 - \alpha \cdot L)$. Under incomplete information, the presence of the transitory shock plays a role in strengthening the inertial responses to the aggregate income shock because α is a function of the variance of the transitory shock. If α is a large value, the effect will be initially small but highly persistent. However, given that $\omega^2 \ggg \omega_\epsilon^2$ in our estimation using the U.S. data, we can easily calculate that α is close to 0. In other words, given the estimated income process, the propagation mechanism in the incomplete information model is extremely weak, and the expression for the changes in consumption is almost identical to the one we obtained in our benchmark model.

[1] Boz, Daude, and Durdu (2011) incorporate this type of incomplete information into a SOE-RBC model and examine how it affects business cycle dynamics in emerging markets.

The inattentiveness and infrequent adjustment model is proposed in Reis (2006). Specifically, Reis (2006) assumes that during the intervals of inattentiveness, consumption dynamics are determined by the standard determinant consumer's optimizing problem and consumption is determined by the standard stochastic consumer problem at the adjustment dates. Reis then finds that aggregate consumption growth between two consecutive periods, t and $t+1$, in the model economy can be written as

$$\Delta c_{t+1} = \text{constant } + \Psi(0) e_{t+1} + \Psi(1) e_t + \cdots + \Psi(I) e_{t+1-I}, \tag{7}$$

where $\Psi(s) \geq \Psi(s+1) \geq 0$ for $s = 1, 2, \cdots, I$, and $\{e_t\}$ are mutually uncorrelated "news" unpredictable one period ahead. Expression (7) reveals that aggregate consumption exhibits slow adjustment because "news" diffuses across all individuals slowly. This conclusion is therefore also consistent with that obtained in our RI model without noises.

2 Online Appendix B: Endogeneous Capital Accumulation

2.1 Introducing Capital Accumulation

In this section, we discuss how elastic attention affects cross-country consumption correlation in a SOE model when we consider endogenous capital accumulation. Specifically, we follow Glick and Rogoff (1995), Gruber (2002) and Luo et al. (2014) to model the firm sector, and assume that the production function is

$$y_t = a_t k_t^{\alpha} - \frac{g}{2} \frac{i_t^2}{k_t}, \tag{8}$$

where k_t is the capital stock, i_t is gross investment, $\frac{g}{2}\frac{i_t^2}{k_t}$ measures the loss of output due to adjustment costs ($g > 0$ is a constant), and a_t is a multiplicative country-specific productivity shock that follows

$$a_{t+1} = (1-\rho)\overline{a} + \rho a_t + \epsilon_{t+1}, \tag{9}$$

where $\rho \in [0,1]$ is the persistence coefficient, $\overline{a}$ is the mean of the country-specific productivity shock, and ϵ_{t+1} is an iid Gaussian innovation with mean 0 and variance ω^2. For simplicity, here we assume that the firm has perfect state observations.

The objective of the firm is to choose capital and investment to maximize the following profit function

$$\max E_0 \left[\sum_{t=0}^{\infty} \left(\frac{1}{R}\right)^t \left(a_t k_t^{\alpha} - \frac{g}{2}\frac{i_t^2}{k_t} - i_t \right) \right].$$

subject to the capital accumulation equation

$$k_{t+1} = (1-\delta) k_t + i_t,$$

for $t \geq 0$. Following the same procedure used in Glick and Rogoff (1995), we can solve for the optimal capital accumulation and investment rules as follows:

$$k_t = \lambda_1 k_{t-1} + \frac{\alpha \overline{k}^{\alpha}}{g\lambda_2} \sum_{j=t}^{\infty} \left(\frac{1}{\lambda_2}\right)^{j-t} E_{t-1}\left[a_j\right] + \Omega, \tag{10}$$

$$i_t = \lambda_1 i_{t-1} + \lambda_i \Delta a_t, \tag{11}$$

where $\Omega = \frac{\alpha \overline{a} \overline{k}^{\alpha}}{g(1-\lambda_2 \cdot L)}$ is an irrelevant constant term, L is the lag operator, $\lambda_i = \frac{\rho \alpha \overline{k}^{\alpha}}{g\lambda_2(1-\rho)}$, and the eigenvalues, $\lambda_1 \in (0,1)$ and $\lambda_2 > 1$, satisfy $\lambda_1 + \lambda_2 = 1 + R - (\alpha - 1)\, \alpha \overline{a} \overline{k}^{\alpha-1}/g$ and $\lambda_1 \lambda_2 = R$. Here we assume that the firm is owned by the household. Given the inelastic labor supply assumption in this model, we are able to model consumption-saving and investment decisions separately at first and then combine the decision rules.[2]. We can now use the expressions of the innovation to total wealth to derive the innovation to consumers' perceived income as follows:[3]

$$\zeta_{t+1} \equiv \frac{1}{R} \sum_{j=t+1}^{\infty} \left(\frac{1}{R}\right)^{j-(t+1)} \left(E_{t+1} - E_t\right)\left[y_j\right] = \Xi \epsilon_{t+1},$$

where

$$\Xi = \frac{1}{R-\rho}\left[1 + \frac{\alpha\rho\left(R+\delta\right)}{g\left(R-\lambda_1\right)\left(\lambda_2-\rho\right)}\right]. \tag{12}$$

The above expression shows a linear relationship between the innovation to total wealth and the innovation to the aggregate productivity.

For the SOW, we have a similar expression:

$$\zeta_{t+1}^* \equiv \Xi^* \epsilon_{t+1}^*,$$

where $\Xi^* = \frac{1}{R-\rho^*}\left[1 + \frac{\alpha\rho^*(R+\delta^*)}{g^*(R-\lambda_1^*)(\lambda_2^*-\rho^*)}\right]$ and

$$a_{t+1}^* = \left(1-\rho^*\right)\overline{a}^* + \rho^* a_t^* + \epsilon_{t+1}^*, \tag{13}$$

where $\rho^* \in [0,1]$ is the persistence coefficient, $\overline{a}^*$ is the mean of the country-specific productivity shock, and ϵ_{t+1}^* is an iid Gaussian innovation with mean 0 and variance ω^{*2}. As in our benchmark model, we also assume that there is a positive contemporaneous correlation between ϵ_{t+1} and ϵ_{t+1}^*:

$$\operatorname{corr}\left(\epsilon_{t+1}, \epsilon_{t+1}^*\right) = \phi.$$

Given the productivity processes, the productivity correlation is

$$\operatorname{corr}\left(a_{t+1}, a_{t+1}^*\right) = \Pi_a \phi, \tag{14}$$

where $\Pi_a = \frac{\sqrt{(1-\rho^2)(1-\rho^{*2})}}{1-\rho\rho^*}$. Note that $\Pi_a = 1$ when $\rho = \rho^*$ and $\Pi_a < 1$ when $\rho \neq \rho^*$.

[2] See Glick and Rogoff (1995) and Gruber (2002) for detailed discussions on this specification.

[3] The derivation of this result is available from the corresponding author upon request.

2.2 Theoretical Implications for Cross-Country Consumption Correlations

Combined with the FI-RE model proposed in Section 2 of the paper, the change in consumption in the home country and the ROW can be written as

$$\Delta c_t = (R-1)\,\Xi\epsilon_t \text{ and } \Delta c_t^* = (R-1)\,\Xi^*\epsilon_t^*,$$

respectively. Using these two expressions, the international consumption correlation can thus be written as

$$\operatorname{corr}(\Delta c_t, \Delta c_t^*) = \operatorname{corr}(\epsilon_t, \epsilon_t^*) = \frac{1}{\Pi_a}\operatorname{corr}(a_t, a_t^*)\,. \tag{15}$$

Note that if the estimated productivity persistence parameters, ρ and ρ^*, are different and less than 1, $\Pi_a < 1$ and $\operatorname{corr}(c_t, c_t^*) > \operatorname{corr}(a_t, a_t^*)$. This prediction contradicts the empirical evidence, just as in Section **??**.

As shown in the benchmark model, the change in individual consumption can be expressed as

$$\Delta c_t^{RI} = \theta(R-1)\left[\frac{\zeta_t}{1-(1-\theta)R\cdot L} + \left(\xi_t - \frac{\theta R\xi_{t-1}}{1-(1-\theta)R\cdot L}\right)\right]. \tag{16}$$

Assume that $\xi = \overline{\xi}_t + \xi_t^i$, where $\overline{\xi}_t = E^i[\xi_t]$ is the common noise and define $\mu \equiv \frac{\operatorname{sd}(\overline{\xi}_t)}{\operatorname{sd}(\xi_t)} \in [0,1]$ to measure the relative importance of the common components of the error generated by ζ_t. After aggregating all consumers, idiosyncratic components are cancelled out. We then have the following expression for the change in aggregate consumption in terms of the productivity shocks:

$$\Delta c_t = (R-1)\left[\frac{\theta\zeta_t}{1-(1-\theta)R\cdot L} + \theta\left(\overline{\xi}_t - \frac{\theta R\overline{\xi}_{t-1}}{1-(1-\theta)R\cdot L}\right)\right]. \tag{17}$$

In the case of $\mu = 0$, all the endogenous noises are cancelled out

$$\Delta c_t^{RI} = \theta(R-1)\frac{\zeta_t}{1-(1-\theta)R\cdot L} = \theta(R-1)\frac{\Xi\epsilon_t}{1-(1-\theta)R\cdot L}. \tag{18}$$

For the ROW, we have a similar expression. It is clear from these expressions that consumption adjusts gradually to productivity shocks. For different countries, the level of their fundamental uncertainty (i.e., the variance of the productivity shock) can affect optimal consumption decisions. Different levels of the volatility of productivity shocks lead to different levels of optimal attention, θ, and thus lead to heterogenous consumption adjustments in response to the productivity shocks.[4] Just like in the benchmark model, this mechanism helps explain why the consumption correlation is lower than the income correlation under optimal attention.

The consumption correlation between the home country and the ROW in the presence of capital accumulation can be written as

$$\operatorname{corr}(\Delta c_t^{RI}, \Delta c_t^{*RI}) = \Pi\operatorname{corr}(\epsilon_t, \epsilon_t^*) = \frac{\Pi}{\Pi_a}\operatorname{corr}(a_t, a_t^*)\,, \tag{19}$$

[4] Note that the higher θ is, the more new information a country can process.

where

$$\Pi = \frac{1}{[1-(1-\theta)(1-\theta^*)R^2]\sigma(\theta,\mu)\sigma(\theta^*,\mu)},$$

$$\sigma(\theta,\mu) = \sqrt{\frac{1}{1-[(1-\theta)R]^2} + \mu^2\left\{\frac{1}{(1-(1-\theta)\,R^2)\theta} - \frac{1}{1-[(1-\theta)R]^2}\right\}},$$

$$\sigma(\theta^*,\mu) = \sqrt{\frac{1}{1-[(1-\theta^*)R]^2} + \mu^2\left\{\frac{1}{(1-(1-\theta^*)\,R^2)\theta^*} - \frac{1}{1-[(1-\theta^*)R]^2}\right\}}.$$

It is clear that in the special case when $\mu = 0$,

$$\Pi = \frac{\sqrt{(1-(1-\theta)^2R^2)(1-(1-\theta^*)^2R^2)}}{1-(1-\theta)(1-\theta^*)R^2} \leq 1.$$

In addition, when $\theta = \theta^*$, $\Pi = 1$, which gives the same result as in the benchmark model. As θ and θ^* drift apart, the consumption correlation becomes smaller relative to the income correlation.

References

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