Online Appendix for "Money, Growth and Welfare in a Schumpeterian Model with the Spirit of Capitalism" (Not for Publication)*

Qichun He† Yulei Luo‡ Jun Nie§ Heng-fu Zou¶

October 6, 2020

1. Online Appendix A: Inelastic Labor Supply and CIA on R&D Only

The inelastic labor supply is captured by $\eta = 0$. For simplicity, we assume $\beta = 1$. Repeating similar steps, we have the free labor mobility condition as

$$(\gamma - 1) l_{x,t} = (1 + i) \left(l_{r,t} + \rho/\varphi - \frac{\theta c_t}{\varphi a_t} \right). \tag{1}$$

The CIA constraint binds: $b_t = m_t$. The output market clearing condition gives $c_t = y_t/L$. Given $e_t = v_t/L$, the consumption wealth ratio is

$$\frac{c_t}{a_t} = \frac{y_t}{v_t + b_t L} = \frac{\gamma w_t L_{x,t}}{(1+i) w_t L_{r,t}/\lambda + w_t L_{r,t}} = \frac{\gamma l_{x,t}}{(1+i)/\varphi + l_{r,t}}. \tag{2}$$

Plugging the labor market clearing condition $l_{r,t} + l_{x,t} = 1$ and (2) into (1), we get a univariate quadratic equation for manufacturing labor l_x:

$$\Phi_1 l_x^2 - \Phi_2 l_x + \Phi_3 = 0, \tag{3}$$

where $\Phi_1 = (\gamma + i)\varphi$, $\Phi_2 = (1+i)(\varphi + \rho) + (\gamma + i)(1 + i + \varphi) + (1+i)\theta\gamma$ and $\Phi_3 = (1+i)\left(1 + \frac{\rho}{\varphi}\right)(1 + i + \varphi$

*The views expressed here are the opinions of the authors only and do not necessarily represent those of the Federal Reserve Bank of Kansas City or the Federal Reserve System. All remaining errors are our responsibility.

†He: China Economics and Management Academy, Central University of Finance and Economics, Beijing, China. Email: qichunhe@gmail.com.

‡Luo: Faculty of Business and Economics, University of Hong Kong, Hong Kong. Email: yulei.luo@gmail.com.

§Nie: Research Department, Federal Reserve Bank of Kansas City, U.S.A. E-mail: jun.nie@kc.frb.org.

¶Zou: China Economics and Management Academy, Central University of Finance and Economics, Beijing, China. Email: hzoucema@gmail.com.

We can rewrite (3) as

$$\left(l_x - \frac{(1+i)\left(1+\frac{\rho}{\varphi}\right)}{\gamma+i}\right)\left(l_x - \frac{\varphi+1+i}{\varphi}\right) = \frac{(1+i)\,\theta\gamma}{\varphi\,(\gamma+i)} l_x. \tag{4}$$

Proposition 1. *Under inelastic labor supply, CIA on R&D only and* $\frac{(1+i)\left(1+\frac{\rho}{\varphi}\right)}{\gamma+i} \leq 1$, *the effect of a higher nominal interest rate on growth depends on the degree of the SOC* (θ), *whereas it is negative without the SOC. When the growth-enhancing effect of the SOC dominates (more likely with a larger* θ*), a higher nominal interest rate increases growth; when the growth-reducing effect from the CIA on R&D dominates, a higher nominal interest rate reduces growth.*

Proof. Using (4), when $\theta = 0$, the solutions are two positive numbers, one is smaller than one ($\frac{(1+i)\left(1+\frac{\rho}{\varphi}\right)}{\gamma+i} \leq 1$) and the other is larger than one ($\frac{\varphi+1+i}{\varphi} > 1$). Because manufacturing labor cannot be larger than 1, the only admissible solution is the smaller positive root $l_x|_{\theta=0} = \frac{(1+i)\left(1+\frac{\rho}{\varphi}\right)}{\gamma+i}$ (point A in Figure 1). When $\theta = 0$, $\frac{\partial l_x|_{\theta=0}}{\partial i} > 0$. That is, when the nominal interest rate increases, point A moves to the right along the horizontal axis to B, yielding a larger manufacturing labor l_x and thereby a lower R&D labor l_r (i.e., lower long-run growth).

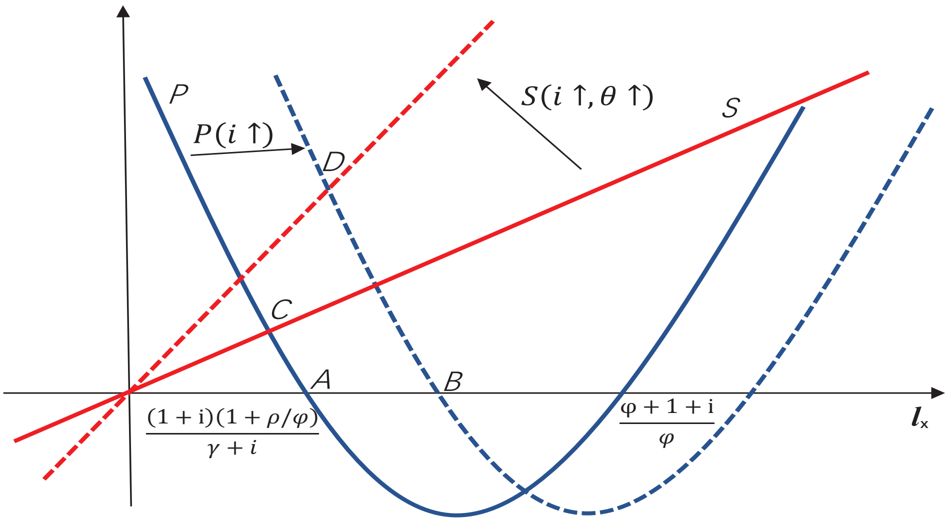
When $\theta > 0$, the solutions to (4) are the intersections of the parabola on the LHS of (4) and the positively-sloped straightline passing through the origin of coordinates (the RHS of (4)): the smaller positive root becomes smaller and the larger positive root becomes larger. The admissible solution is the smaller positive root (the equilibrium is point C in Figure 1).

When the nominal interest rate increases, the parabola on the LHS of (4) shifts to the right. Meanwhile, the straightline on the RHS of (4) rotates counter-clockwise. The new equilibrium will be point D. Obviously, whether point D is to the left or right of point C depends on the size of θ. Therefore, whether a higher nominal interest rate increases/decreases manufacturing labor l_x depends on the size of θ. ■

The intuition can be explained as follows. Under inelastic labor supply, the *consumption-leisure choice effect* that reduces total labor supply is absent. That is, the L curve remains unchanged. Both the SOC and the CIA on R&D affect growth through the free labor mobility condition (i.e., through shifting the M curve).

On the one hand, an increase in the nominal interest rate raises the borrowing cost of entrepreneurs, shifting labor away from R&D to manufacturing ($\frac{\partial\left(l_x|_{\theta=0}=\frac{(1+i)\left(1+\frac{\rho}{\varphi}\right)}{\gamma+i}\right)}{\partial i} > 0$ in (4)). Although we consider the special case of $\beta = 1$, we can prove that this *negative labor reallocation effect* due to the CIA on R&D increases with the strength of the CIA on R&D (i.e., β) when $\beta < 1$, given

Figure 1. Equilibrium Labor Allocation under CIA on R&D and Inelastic Labor Supply



the same increase in the nominal interest rate. Graphically, a larger β leads to a larger rightward shift in the M curve, thereby decreasing R&D labor l_r and increasing manufacturing labor l_x, all else equal.

On the other hand, the increased manufacturing labor l_x and decreased R&D labor l_r tend to increase the consumption wealth ratio c_t/a_t (note that although a_t is the state variable of households, it can be changed by firms/entrepreneurs). With the SOC, the savings decision will depend on $\frac{\theta c_t}{a_t} + r_t - \rho$ under CIA on R&D. An increase in c_t/a_t raises the marginal benefit of saving through the direct preference for wealth. This would push people to substitute consumption with savings. More savings lowers the real interest rate (i.e., the borrowing cost of entrepreneurs) and thereby increases the return to entrepreneurship, shifting labor away from manufacturing to R&D. This *positive labor reallocation effect* due to the SOC becomes larger with a higher degree of the SOC θ (the RHS of (4)), given the same increase in the nominal interest rate. Graphically, a higher θ leads to a larger leftward shift in the M curve, thereby increasing R&D labor l_r and decreasing manufacturing labor l_x, all else equal.

Therefore, the total effect of the nominal interest rate on R&D labor (and thereby long-run growth) depends on the relative magnitudes of the degree of the SOC θ versus the strength of the CIA on R&D (i.e., β). In other words, whether the M curve shifts depends on the relative sizes of θ versus β.

Given the complexity of the value function, we will examine the welfare implications of SOC quantitatively after calibrating the key model parameters using the US data.

2. Online Appendix B: Inelastic Labor Supply and CIA on Consumption Only

In this section, inelastic labor supply is captured by $\eta = 0$, which yields $l_{r,t} + l_{x,t} = 1$. The CIA constraint becomes $c_t \leq m_t$.

Proposition 2. *Under inelastic labor supply and CIA on consumption, the steady state growth increases with the nominal interest rate i as long as $\theta > 0$ and $\frac{1+\rho/\varphi}{\gamma} \leq 1$. Moreover, the higher the degree of the SOC (i.e., a larger θ), the larger the effect of the nominal interest rate on growth, all else equal.*

Proof. Using $l_{r,t} + l_{x,t} = 1$, imposing $\beta = 0$ and repeating similar steps, we end up with a univariate quadratic equation for manufacturing labor l_x:

$$\gamma^2 \varphi l_x^2 + \Lambda_2^1 l_x - \Lambda_3 = 0, \tag{5}$$

where $\Lambda_2^1 = \gamma \left[1 - (\varphi + \rho) + \theta (1 + i)\right]$ and $\Lambda_3 = 1 + \rho/\varphi$. Because $\gamma^2 \varphi > 0$ and $\Lambda_3 > 0$, (5) always has one negative root and one positive root. Because manufacturing labor cannot be negative, the only admissible solution is the positive root.

When $\theta = 0$ (i.e., there is no SOC), using (5), we have the standard Schumpeterian model with the CIA constraint on consumption. l_x is determined by

$$(\gamma l_x - \Lambda_3)(\gamma \varphi l_x + 1) = 0, \tag{6}$$

where the two roots are $l_x|_{\theta=0} = \frac{\Lambda_3}{\gamma} = \frac{1+\rho/\varphi}{\gamma}$ and $l_x|_{\theta=0} = -\frac{1}{\gamma\varphi}$. The only admissible solution is the positive root $l_x|_{\theta=0} = \frac{\Lambda_3}{\gamma} = \frac{1+\rho/\varphi}{\gamma}$. Therefore, we assume $\frac{1+\rho/\varphi}{\gamma} \leq 1$ to ensure $l_x \leq 1$.

When the nominal interest rate increases, the y-intercept of the quadratic function in (5) $-\Lambda_3$ remains constant, so does the coefficient of the quadratic term $\gamma^2 \varphi$ (i.e., the shape of the parabola remains unchanged). The axis of symmetry of the quadratic function in (5) is $-\frac{\Lambda_2^1}{2\gamma^2\varphi}$, which shifts to the left as the nominal interest rate increases. Taken together, the graph of the quadratic function in (5) (i.e., the parabola) remains unchanged and wholly shifts to the left as the nominal interest rate increases. Therefore, the positive root of the quadratic equation (i.e., manufacturing labor l_x) decreases. As a result, R&D labor l_r and thereby long-run growth would increase as the nominal interest rate i increases. One can observe from (5) that the increase in θ and that in i tend to impact l_x similarly, which proves the second part of the Proposition. ■

When $\theta = 0$ (i.e., there is no SOC), l_r and l_x (thereby growth and welfare) are independent of the nominal interest rate under inelastic labor supply and CIA on consumption.

The intuition is as follows. Under inelastic labor supply, the effect of the nominal interest rate on labor supply through the consumption-leisure choice is absent. Without the SOC, the free labor mobility condition is not affected by the nominal interest rate under the CIA constraint on consumption. Therefore, without the SOC, both the R&D labor l_r and manufacturing labor l_x (thereby growth and welfare) are not functions of the nominal interest rate (i.e., money is superneutral). By contrast, the existence of the SOC will yield the same labor reallocation effect that shifts labor away from manufacturing to R&D when the nominal interest rate increases. As a result, long-run growth increases and a higher degree of the SOC increases the effect of the nominal interest rate on growth.

Under inelastic labor supply and CIA on consumption, Corollary 1 still holds. Concerning welfare, imposing $\eta = 0$ yields

$$U = \frac{1}{\rho}\left[\ln\left(l_{x,0}\right) + \theta \ln\left(\frac{1}{\varphi} + \gamma l_{x,0}\right) + \frac{(1+\theta)\, g}{\rho} + \Omega\right], \tag{7}$$

where Ω is a constant.

Proposition 3. *Under inelastic labor supply and CIA on consumption, the steady state welfare increases with the nominal interest rate i when $\theta \in (0, \underline{\theta})$ and $\frac{\rho}{\varphi \ln\gamma} < \frac{1+\rho/\varphi}{\gamma} \leq 1$, and it decreases with i when $\theta \in \left(\overline{\theta}, \infty\right)$ and $\rho > \frac{\ln\gamma}{\gamma}$.*

Proof. Taking derivative of U in (7) with respect to i, using $\partial l_{x,0}/\partial i = -\partial l_{r,0}/\partial i$ under inelastic labor supply and $\partial g/\partial i = \varphi \ln\gamma \partial l_{r,0}/\partial i$, we have

$$\frac{\partial U}{\partial i} = \frac{\partial U}{\partial l_{x,0}}\frac{\partial l_{x,0}}{\partial i} + \frac{\partial U}{\partial g}\frac{\partial g}{\partial i} = \frac{1}{\rho}\left[\frac{1}{l_{x,0}} + \frac{\varphi\gamma\theta}{1+\varphi\gamma l_{x,0}} - \frac{(1+\theta)\,\varphi\ln\gamma}{\rho}\right]\underset{(-)}{\frac{\partial l_{x,0}}{\partial i}}, \tag{8}$$

where $\partial l_{x,0}/\partial i < 0$ when $\theta > 0$, according to the previous Proposition. Therefore, $sign\left(\frac{\partial U}{\partial i}\right) > 0$ if $\left[\frac{1}{l_{x,0}} + \frac{\varphi\gamma\theta}{1+\varphi\gamma l_{x,0}} - \frac{(1+\theta)\varphi\ln\gamma}{\rho}\right] < 0$. We have

$$\left[\frac{1}{l_{x,0}} + \frac{\varphi\gamma\theta}{1+\varphi\gamma l_{x,0}} - \frac{(1+\theta)\,\varphi\ln\gamma}{\rho}\right] < \left[\frac{1}{l_{x,0}} + \frac{\varphi\gamma\theta}{\varphi\gamma l_{x,0}} - \frac{(1+\theta)\,\varphi\ln\gamma}{\rho}\right] \tag{9}$$

$$= (1+\theta)\left[\frac{1}{l_{x,0}} - \frac{\varphi\ln\gamma}{\rho}\right]. \tag{10}$$

According to the previous Proposition, we have proved that $l_x|_{\theta=0} = \frac{1+\rho/\varphi}{\gamma} \leq 1$ for any i under inelastic labor supply. Therefore, we first need $\frac{1}{l_x|_{\theta=0}} < \frac{\varphi\ln\gamma}{\rho}$, which is equivalent to $\frac{\rho}{\varphi\ln\gamma} < \frac{1+\rho/\varphi}{\gamma}$. Moreover, the previous Proposition shows that $l_{x,0}$ is decreasing in both θ and i. Therefore, for a given i, when θ increases from zero, $l_{x,0}$ decreases from $\frac{1+\rho/\varphi}{\gamma}$. Therefore, as long as $\theta < \underline{\theta}$ ($\underline{\theta}$ is pinned down by $l_{x,0}\left(\underline{\theta}\right) = \frac{\rho}{\varphi\ln\gamma}$), we have $sign\left(\frac{\partial U}{\partial i}\right) > 0$. However, when θ continues to increase, it is possible that $\left[\frac{1}{l_{x,0}} + \frac{\varphi\gamma\theta}{1+\varphi\gamma l_{x,0}} - \frac{(1+\theta)\varphi\ln\gamma}{\rho}\right] > 0$. We can find the $\overline{\theta} > \underline{\theta}$ by solving

$\left[\frac{1}{l_{x,0}} + \frac{\varphi\gamma\theta}{1+\varphi\gamma l_{x,0}} - \frac{(1+\theta)\varphi\ln\gamma}{\rho}\right] = 0$ ($l_{x,0}$ is a function of θ). If $\rho > \frac{\ln\gamma}{\gamma}$, the middle term in the square bracket increases at a rate of $\varphi\gamma\theta$ as θ approaches infinity, whereas the third term increases at a rate of $\frac{\theta\varphi\ln\gamma}{\rho}$. The middle term increases faster if $\rho > \frac{\ln\gamma}{\gamma}$, and $sign\left(\frac{\partial U}{\partial i}\right) < 0$ when $\theta \in \left(\overline{\theta}, \infty\right)$. ■

The intuition is as follows. According to (7), an increase in the nominal interest rate has two opposing effects on the welfare: it reduces welfare by decreasing manufacturing labor $l_{x,0}$ (thereby initial consumption and wealth) and it increases welfare through raising the growth rate g. Therefore, there is a tradeoff between lower initial consumption and wealth and higher future consumption and wealth. When the nominal interest rate increases, the existence of the SOC (i.e., households enjoy direct utility from holding wealth) makes households change their consumption-portfolio choice by reducing initial consumption: substituting consumption with more savings in the form of equity shares. The value of innovations increases due to lower borrowing costs, which generates *the labor reallocation effect* that increases R&D labor and thereby growth. When θ is below a threshold, the welfare gain from higher future consumption and wealth dominates the welfare loss from lower initial consumption and wealth, and total welfare increases with the nominal interest rate. However, when θ is beyond a threshold, the welfare loss from lower initial consumption and wealth dominates the welfare gain from higher future consumption and wealth, and total welfare decreases with the nominal interest rate.

The reason to impose $\frac{\rho}{\varphi\ln\gamma} < \frac{1+\rho/\varphi}{\gamma} \leq 1$ is as follows. For instance, when $\gamma = 1$, we have $sign\left(\frac{\partial U}{\partial i}\right) \leq 0$. That is, in an economy without growth (when the step-size of innovation is 1), welfare cannot be increasing in the nominal interest rate even when $\theta > 0$. This is understandable because the trade-off between lower initial consumption and wealth and higher future consumption and wealth is possible only when there is growth. When $\rho > \frac{\ln\gamma}{\gamma}$ (a larger ρ means households are more impatient and discount future consumption more heavily), it is more likely that the welfare loss from lower initial consumption/wealth dominates the welfare gain from higher future consumption/wealth.

Under inelastic labor supply and CIA on consumption, we show that higher nominal interest rates always increase growth when $\theta > 0$. By contrast, our results also indicate that higher nominal interest rates increase welfare when $\theta > 0$ and θ is below a threshold; beyond a threshold, the effect of a higher nominal interest rate on welfare depends on structural parameters. Therefore, concerning optimal monetary policy, the Friedman rule (Friedman, 1969) of keeping the nominal interest rate at zero is suboptimal when $\theta > 0$ and θ is below a threshold; the Friedman may be optimal when θ is above a threshold (this is possible in theory, but in real world situations, θ may not be that large). We summarize this point in the following Corollary.

Corollary 2 *Under inelastic labor supply and CIA on consumption, the Friedman rule is suboptimal when $\theta > 0$ and θ is below a threshold; depending on structural parameters, the Friedman may be optimal* *when θ is above the threshold.*

3. Online Appendix C: Money Supply and Nominal Interest Rate

In this online appendix, we show that under our calibrated parameter values, an increase in money supply also leads to an increase in the nominal interest rate based on the following equation:

$$\frac{\dot{M}_t}{M_t} = i_t - \rho + \theta\left(1 + i_t\right)\frac{c_t}{a_t}. \tag{11}$$

The existence of the SOC slightly complicates the relationship between the nominal interest rate i_t and the money supply growth rate $\dot{M}_t/M_t$ by introducing an extra term on the right side of (11). However, as Figure 2 shows, the positive relationship is well maintained under the calibrate parameter values and different values of β (the tightness of the CIA constraint on R&D).

Figure 2. *Relationship between Money Supply and Nominal Interest Rate*