Rational Inattention and the Dynamics of Consumption and Wealth in General Equilibrium*

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Abstract

We propose a recursive utility version of a basic Huggett (1993) model to study the implications of rational inattention (or RI, Sims 2003, 2010) for the cross-sectional dispersion of consumption and wealth (relative to income) in general equilibrium. We find that incorporating RI can significantly improve the model's predictions in both dimensions in general equilibrium. In addition, we find that intertemporal substitution plays an important role in determining the two key dispersion moments via affecting the degree of optimal attention in equilibrium. Finally, we show that alternative models that rely on habit formation, incomplete information about current income, or borrowing constraints are not consistent with the facts we document.

Keywords: Rational Inattention; General Equilibrium; Consumption and Wealth Dispersion.

JEL Classification Numbers: C61; D83; E21.

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1. Introduction

Our interest in this paper is to evaluate the general equilibrium implications of limited information-processing capacity (rational inattention or RI) for the joint cross-sectional dispersions of consumption, wealth, and income. In intertemporal consumption-savings problems, *prudent* households save today for three reasons: (i) they anticipate future declines in income (saving for a rainy day), (ii) they face uninsurable risks (precautionary savings), and (iii) they are patient relative to the interest rate. For example, the "permanent income hypothesis" (PIH) of Friedman (1957) emphasizes the motive (i) in which consumption is solely determined by permanent income (the annuity value of total wealth) and follows a random walk (see Hall 1978).[1]

A growing recent literature inspired by Sims (2003) shows that RI plays an important role in influencing the consumption and saving dynamics and has also gained some empirical support.[2] Specifically, Sims (2003), Luo (2008), and Luo and Young (2010) introduce RI into the basic partial equilibrium PIH environment; RI implies that agents process signals slowly and therefore appear to respond sluggishly to innovations in permanent income. This sluggish response appears to deliver changes in consumption in response to anticipated income changes, and as a result also delivers smaller responses to permanent income changes; that is, the model delivers both excess sensitivity and excess smoothness in the consumption behavior we observed in the US data. Some empirical studies found that incomplete information about the state plays an important role in affecting individual agents' optimal decisions. For example, Coibion and Gorodnichenko (2015) and Andrade and Le Bihan (2013) find pervasive evidence consistent with Sims (2003)'s rational inattention theory using the U.S. and European surveys of professional forecasters and other agents, respectively.[3]

However, the above RI-PIH models are partial equilibrium, taking as given a constant exogenous risk-free rate. By construction, the models shut off the feedback loop from the equilibrium interest rate to the dynamics of consumption and wealth. Furthermore, these RI models do not separate risk aversion from intertemporal substitution in determining consumption dynamics, while many papers on recursive utility (RU) preferences highlight the importance of the separation between risk aversion and intertemporal substitution in affecting optimal consumption-portfolio rule and asset pricing. (See, for example, Epstein and Zin 1989, Campbell 2003, Vissing-Jørgensen and Attanasio 2003, Guvenen 2006.) This paper therefore fills the gap by providing a tractable

[1] This statement holds, for example, if households have quadratic utility and have access to a single risk-free bond with a constant return. If utility is not quadratic, the random walk nature of consumption is only approximately true, but the PIH still holds.

[2] See Veldkamp (2011) for a textbook treatment on how to build and test economic models with information choice and frictions including rational inattention.

[3] Hong, Torous, and Valkanov (2007) find evidence for rational inattention in the financial markets. Specifically, they find that investors in the stock market react gradually to information contained in industry returns about their fundamentals and that information diffuses only gradually across markets.

RU framework to explore the general equilibrium implications of rational inattention for the joint cross-sectional dispersion of consumption, wealth and income. We investigate both the theoretical mechanism (how RI influences the equilibrium interest rate and the dispersion of consumption and wealth to income) and the empirical performance (how RI could lead the model to fit the data better compared to alternative hypotheses).

Specifically, we propose an equilibrium precautionary saving model in which inattentive consumers have recursive utility with exponential risk and intertemporal attitudes and face uninsurable labor income. We disentangle two distinct aspects of preferences: the agent's elasticity of intertemporal substitution (EIS) with the coefficient of absolute risk aversion (CARA).[4] We solve our model in closed-form, and analytically inspect how risk aversion and intertemporal substitution interact with rational inattention and then affect the equilibrium interest rate as well as the cross-sectional dispersion of consumption and wealth (relative to income). We find that if attention is elastic (i.e., consumers can adjust the degree of attention in response to changes in the real world), the EIS plays an important role in affecting the equilibrium properties of the model economy.[5] In particular, we show that an increase in EIS affects the equilibrium interest rate through two channels: (i) it increases the relative importance of the impatience-induced dissaving effect (*the direct channel*) and (ii) it reduces the optimal attention level and thus increases the inattention-induced precautionary saving amount (*the indirect channel*). In addition, the optimal level of attention in our RU model is mainly determined by the EIS, and is independent of the degree of risk aversion. We also show that the general equilibrium exists and is unique in such an RI-RU economy.

We then compare the theory with alternative hypotheses and examine whether our RI model can better explain the data. We use the RI model to interpret the dispersion of consumption and wealth (relative to income) that we observe in the data. We find that RI substantially improves the predictions of the model for these relative dispersions. The full-information rational expectations model, for the income process we estimate from micro data, implies that these dispersions are much lower than in the data; the separation of risk aversion and intertemporal substitution made feasible by recursive utility does not help increase these dispersions. With RI, as households become less attentive, relative dispersions in consumption and wealth both rise. Interestingly, we find that the same value of the RI parameter roughly matches both moments, providing an overidentification test of the model. The general equilibrium effects turn out to be less significant when

[4] Constant-relative-risk-aversion (CRRA) utility functions are more common in macroeconomics, mainly due to balanced-growth requirements. CRRA utility would greatly complicate our analysis because the intertemporal consumption model with CRRA utility and stochastic labor income has no explicit solution and leads to non-linear consumption rules. Introducing RI would then be substantially more difficult and involve approximations of unknown quality.

[5] Coibion and Gorodnichenko (2015) find that information rigidities were falling from the late 1960s to the early 1980s as the volatility of macroeconomic variables was rising, while these rigidities had been consistently increasing since the start of the Great Moderation (1983 − 1984). They then argue that one should be careful when treating information rigidities at the macro level as a structural parameter because these rigidities vary over time in response to changes in macroeconomic conditions.

consumers are less information-constrained, acting to slightly reduce the dispersions in the model relative to a partial equilibrium exercise with a fixed interest rate.

Next, we ask whether our model provides a story for the observed changes in the cross-sectional distributions of consumption and income. The significant increase in household income inequality or dispersion for the U.S. in the 1980s and 1990s is a well-documented fact. Many studies have found that the dispersions of U.S. household earnings and incomes have a strong upward trend.[6] In addition, the literature has documented that the recent increase in income inequality in the U.S. has not been accompanied by a corresponding rise in consumption inequality over the 1980s and 1990s. In other words, over the sampling period, income and consumption inequality diverged and the relative volatility of consumption to income has decreased, as discussed in Krueger and Perri (2006) and Blundell, Pistaferri, and Preston (2008). With elastic attention we find that our model naturally predicts a decline in the relative volatility of consumption to income, in response to a rise in income volatility, and the magnitude is close to that found in our data; this experiment provides additional support for our model.

To further justify the validation of RI in explaining the data, we compare our RI model with several alternative models that are commonly used to study consumption dynamics in the literature. We first compare our RI model with models of sluggish movements in consumption, namely habit formation and incomplete information about current income. As the habit parameter increases, consumption changes become sluggish as households try to smooth changes in consumption rather than levels. Thus, there is a sense in which the two model frameworks look similar. We show that this similarity does not extend to the equilibrium cross-sectional dispersion moments we examine here; unlike RI, habit formation moves the relative dispersion of consumption to income away from its empirical counterpart, and the model's prediction of the wealth dispersion is well below its empirical counterpart. In addition, we find that stronger habit formation leads to higher, not lower, interest rates.

With incomplete information, households cannot distinguish between permanent and transitory innovations in current income; although similar to rational inattention on its face, there is a key difference – under RI, agents cannot observe the value of their assets in addition to their income, and therefore need to extract endogenous as well as exogenous components. It turns out that incomplete information economies behave very much like habit models – the consumption dispersion measure gets too small, and the wealth dispersion measure remains far too low.

Finally, we consider whether introducing borrowing constraints can deliver the observed dispersion measures. Specifically, we examine a standard Huggett (1993) model, where households face a common fixed borrowing constraint equal to twice the average income. In this model,

[6] See Katz and Autor (1999) for a survey of these empirical findings. Table 1 also reports the increases in the variance of income growth over the 1980s and 1990s.

households with below-median wealth have a relative volatility of consumption changes to income changes that is roughly twice as large as those with above-median wealth. Because households close to the borrowing constraint have very high marginal utilities of consumption, any increase in their income triggers a strong increase in consumption, which causes the relative volatility to be high. In the data, however, we find that our measure of relative volatility does not vary across the wealth distribution significantly, which is evidence against borrowing constraints playing a key role for the dynamics we emphasize.

This paper is organized as follows. Section 2 constructs a precautionary saving model with a continuum of inattentive consumers who have the recursive utility and face uninsurable labor income. Section 3 solves optimal consumption-saving rules under rational inattention and characterizes the unique general equilibrium of this economy. Section 4 examines how RI affects the interest rate and the joint dynamics of consumption, income, and wealth quantitatively. Section 5 compares the rational inattention model to the models with habit formation, incomplete information about current income, and borrowing constraints. In the appendices we provide the key proofs and derivations.

2. A Caballero-Huggett-Wang Economy with Rational Inattention

2.1. A Full-information Rational Expectations Model with Recursive Utility and Precautionary Savings

In this section, we first consider a full-information rational expectations (FI-RE) recursive utility model with labor income and precautionary savings. Although the expected utility model has many attractive features, it implies that the agent's elasticity of intertemporal substitution is the reciprocal of the coefficient of relative risk aversion. However, risk aversion (attitudes towards atemporal risks) and intertemporal substitution (attitudes towards shifts in consumption over time) capture two distinct aspects of preferences, and should not necessarily be linked so tightly. In this paper, we assume that agents in our model economy have recursive preferences of the Kreps-Porteus/Epstein-Zin type, and can disentangle the degree of risk aversion from the elasticity of intertemporal substitution. Specifically, for every stochastic consumption stream, $\{c_t\}_{t=0}^{\infty}$, the utility stream, $\{f(U_t)\}_{t=0}^{\infty}$, is recursively defined as follows:[7]

$$f(U_t) = f(c_t) + \frac{1}{1+\rho} f(\mathcal{CE}_t[U_{t+1}]) \tag{1}$$

where $\rho > 0$ is the agent's subjective discount rate, $f(x) = -\psi \exp(-x/\psi)$,

$$\mathcal{CE}_t[U_{t+1}] = g^{-1}(E_t[g(U_{t+1})]), \tag{2}$$

[7] Angeletos and Calvet (2006) adopt similar recursive utility preferences to study the effects of idiosyncratic production risks on business cycles and growth.

is the certainty equivalent of U_{t+1} conditional on the period t information, and $g(U_{t+1}) = -\exp(-\alpha U_{t+1})/\alpha$. In (1), $\psi > 0$ governs the elasticity of intertemporal substitution (EIS), while $\alpha > 0$ governs the coefficient of absolute risk aversion (CARA). If $\psi = 1/\alpha$, the functions f and g are the same and the recursive utility reduces to the standard time-separable expected utility function used in Caballero (1990) and Wang (2003). In addition, $\psi = 1/\alpha$ also implies that the consumer is indifferent about the time at which uncertainty is resolved.[8]

Following Caballero (1990) and Wang (2003), the flow budget constraint is written as

$$a_{t+1} = (1+r)\, a_t + y_t - c_t, \tag{3}$$

where r is a constant rate of interest and labor income, y_t, follows a stationary AR(1) process with Gaussian innovations

$$y_t = \phi_0 + \phi_1 y_{t-1} + w_t,\ t \geq 1,\ |\phi_1| < 1, \tag{4}$$

where $w_t \sim N\left(0, \sigma^2\right)$, $\phi_0 = (1-\phi_1)\,\overline{y}$, $\overline{y}$ is the mean of y_t, and the initial levels of labor income y_0 and asset a_0 are given.[9]

In order to facilitate the introduction of rational inattention, we follow Luo (2008) and Luo and Young (2010) and reduce the multivariate model to a univariate model with iid innovations to total wealth. Let total wealth, $s_t = a_t + h_t$, be defined as a new state variable. Here

$$h_t \equiv \frac{1}{1+r} E_t \left[\sum_{j=0}^{\infty} \left(\frac{1}{1+r} \right)^j y_{t+j} \right], \tag{5}$$

is human wealth (defined as the discounted expected present value of current and future labor income); evaluating the sum yields $h_t = \phi\,(y_t + \phi_0/r)$, where $\phi = 1/\left(1+r-\phi_1\right)$.[10] Using (3) and (4), it is straightforward to show that the evolution of the new state variable can be written as

$$s_{t+1} = (1+r)\, s_t - c_t + \zeta_{t+1}, \tag{6}$$

where the time $(t+1)$ innovation to total wealth can be written as

$$\zeta_{t+1} \equiv \frac{1}{1+r} \sum_{j=t+1}^{\infty} \left(\frac{1}{1+r} \right)^{j-(t+1)} (E_{t+1} - E_t)\left[y_j\right], \tag{7}$$

[8]Consumers prefer early resolution of uncertainty if $\alpha > 1/\psi$ and late resolution if $\alpha < 1/\psi$. Luo and Young (2016) show how the distaste for late resolution of uncertainty and rational inattention deliver a high equity premium and a low portfolio weight of risky assets.

[9]Assuming that the individual income shock includes two components, one permanent and the other transitory, does not change the main results in this paper. Here we follow Wang (2003) and adopt specification (4), in order to simplify the algebra. A detailed derivation of the model with the two-income shock specification is available from the corresponding author by request.

[10]See Appendix 7.1 for the derivation.

which can be reduced to $\zeta_{t+1} = \phi w_{t+1}$ if we use income process (4).[11]

2.2. Incorporating Rational Inattention

In this section, we follow Sims (2003) and incorporate rational inattention (RI) due to finite information-processing capacity into the above permanent income model with the CARA-Gaussian specification. Under RI, consumers have only finite Shannon channel capacity available to observe the state of the world. Following the literature, we first assume the noisy signal takes the additive form:[12] $s^*_{t+1} = s_{t+1} + e_{t+1}$, where e_{t+1} is the endogenous noise caused by finite capacity. We further assume that e_{t+1} is an iid idiosyncratic shock and is independent of the fundamental shocks hitting the economy. Agents with finite capacity will choose a new signal $s^*_{t+1} \in \mathcal{I}_{t+1} = \left\{s^*_1, s^*_2, \cdots, s^*_{t+1}\right\}$ that reduces the uncertainty about the variable s_{t+1} as much as possible. Formally, this idea can be characterized by the following information constraint:

$$\mathcal{H}\left(s_{t+1}|\mathcal{I}_t\right) - \mathcal{H}\left(s_{t+1}|\mathcal{I}_{t+1}\right) \leq \kappa, \tag{8}$$

where κ is the investor's information channel capacity, $\mathcal{H}\left(s_{t+1}|\ \mathcal{I}_t\right)$ denotes the entropy of the state prior to observing the new signal at $t+1$, and $\mathcal{H}\left(s_{t+1}|\ \mathcal{I}_{t+1}\right)$ is the entropy after observing the new signal. Finally, following the literature, we suppose that the prior distribution of s_{t+1} is Gaussian.

In a linear-quadratic-Gaussian (LQG) setting, Sims (2003) and Shafieepoorfard and Raginsky (2013) show that *ex post* Gaussian distributions, $s_t|\mathcal{I}_t \sim N\left(E\left[s_t|\mathcal{I}_t\right], \Sigma_t\right)$ with $\Sigma_t = E_t\left[\left(s_t - \widehat{s}_t\right)^2\right]$, are optimal. Here we first assume *ex post* Gaussian distributions of the true state and Gaussian noise but adopt negative exponential recursive utility preferences. Because both the optimality of *ex post* Gaussianity and the standard Kalman filter are based on the linear-quadratic-Gaussian (LQG) specification, the applications of these results in the RI models with negative exponential preferences are only approximately valid.[13] However, we can verify that the loss function due to RI is approximately quadratic and consequently the optimality of the *ex post* Gaussianity of the state approximately holds in the recursive utility model. (See Online Appendix B for the detailed proof.)

Since both *ex ante* and *ex post* distributions of the state are Gaussian, (8) reduces to

$$\ln\left(|\Psi_t|\right) - \ln\left(|\Sigma_{t+1}|\right) \leq 2\kappa, \tag{9}$$

[11]See Appendix 7.1 for the derivation.

[12]Mackowiak and Wiederholt (2009) show that this additive noisy signal form is optimal if the state being tracked is a stationary Gaussian AR(1) process. The proof for the optimal form of the noisy signal in their paper can be extended to our model in which the state being tracked is a random walk if the channel capacity, κ, is greater than a lower bound. Please see the proof in Online Appendix A for the details.

[13]See Mondria (2010), Van Nieuwerburgh and Veldkamp (2009, 2010), and Veldkamp (2011) for applications of CARA preferences in RI models.

where $\Sigma_{t+1} = \text{var}_{t+1}(s_{t+1})$ and $\Psi_t = \text{var}_t(s_{t+1}) = (1+r)^2 \Sigma_t + \text{var}_t(\zeta_{t+1})$ are the posterior and prior variances of the state variable, s_{t+1}, respectively. In our univariate model, (9) fully determines the value of the steady state conditional variance Σ:

$$\Sigma = \frac{\text{var}_t(\zeta_{t+1})}{\exp(2\kappa) - (1+r)^2}, \tag{10}$$

which means that Σ is entirely determined by the variance of the exogenous shock $(\text{var}_t(\zeta_{t+1}))$ and finite capacity (κ). To guarantee that the state is stabilizable and the unconditional variance converges, we need to make the following assumption:

Assumption 1

$$\kappa > \ln(1+r). \tag{11}$$

It is worth noting that this restriction is very weak if r is small; in general equilibrium r will be smaller than ρ, so for short time periods this condition is not restrictive at all.

The evolution of the estimated state $\widehat{s}_t$ is governed by the Kalman filtering equation

$$\widehat{s}_{t+1} = (1-\theta)\left((1+r)\widehat{s}_t - c_t\right) + \theta s^*_{t+1}, \tag{12}$$

where $\widehat{s}_t = E_t[s_t]$ is the conditional mean of the state, s_t and $\theta = 1 - 1/\exp(2\kappa)$ is the steady state Kalman gain; it measures the fraction of uncertainty removed by a new signal in each period, and is the only new parameter introduced by the rational inattention framework. Combining (6) with (12) yields

$$\widehat{s}_{t+1} = (1+r)\widehat{s}_t - c_t + \widehat{\zeta}_{t+1}, \tag{13}$$

where

$$\widehat{\zeta}_{t+1} = \theta(1+r)(s_t - \widehat{s}_t) + \theta(\zeta_{t+1} + e_{t+1}) \tag{14}$$

is the innovation to $\widehat{s}_{t+1}$ and is independent of all the other terms on the RHS of (13). $\widehat{\zeta}_{t+1}$ is an MA(∞) process with $E_t\left[\widehat{\zeta}_{t+1}\right] = 0$ and

$$\text{var}\left(\widehat{\zeta}_{t+1}\right) = \Gamma(\theta, r)\,\omega^2_\zeta, \tag{15}$$

where $\omega^2_\zeta = \text{var}_t(\zeta_{t+1})$,

$$\Gamma(\theta, r) = \frac{\theta}{1 - (1-\theta)(1+r)^2} > 1 \tag{16}$$

for $\theta < 1$, and

$$s_t - \widehat{s}_t = \frac{(1-\theta)\zeta_t}{1 - (1-\theta)(1+r)\cdot L} - \frac{\theta e_t}{1 - (1-\theta)(1+r)\cdot L} \tag{17}$$

is the estimation error with $E_t[s_t - \widehat{s}_t] = 0$ and $\text{var}(s_t - \widehat{s}_t) = \frac{1-\theta}{1-(1-\theta)(1+r)^2}\omega^2_\zeta$; L is the standard lag operator. To guarantee that the sum of these infinite series converges, we impose the restriction $\kappa > 0.5\ln(1+r)$, which is weaker than the assumption, $\kappa > \ln(1+r)$. From (16), it is clear that $\frac{\partial\Gamma}{\partial r} > 0$ and $\frac{\partial\Gamma}{\partial\theta} < 0$.

3. General Equilibrium under RI

3.1. Optimal Consumption and Savings Functions

The consumption function and the value function under RU and RI can be obtained by solving the Bellman equation:

$$f\left(J\left(\widehat{s}_t\right)\right) = \max_{c_t}\left\{f\left(c_t\right) + \frac{1}{1+\rho}f\left(\mathcal{CE}_t\left[J\left(\widehat{s}_{t+1}\right)\right]\right)\right\}, \tag{18}$$

subject to (13)-(17). The following proposition summarizes the main results from the above precautionary-savings model with RI:

Proposition 1. *For a given Kalman gain, θ, the value function is*

$$\widehat{v}\left(\widehat{s}_t\right) = -\frac{\psi}{r}\exp\left(-\frac{1}{\psi}\left\{r\widehat{s}_t - \psi\ln(1+r) + \left[\frac{\psi}{r}\ln\left(\frac{1+\rho}{1+r}\right) - \frac{1}{2}\alpha r\omega_{\widehat{\zeta}}^2\right]\right\}\right), \tag{19}$$

the consumption function is

$$c_t^* = r\widehat{s}_t + \frac{\psi}{r}\Psi(r) - \Pi(\theta, r), \tag{20}$$

and the saving function is

$$d_t^* = \widetilde{f}_t + r\left(s_t - \widehat{s}_t\right) + \Pi(\theta, r) - \frac{\psi}{r}\Psi(r), \tag{21}$$

where $\widetilde{f}_t \equiv (1-\phi_1)\phi(y_t - \overline{y})$ captures the consumer's demand for savings "for a rainy day", $\widehat{s}_t$ *is governed by (13), $s_t - \widehat{s}_t$ is an $MA(\infty)$ estimation error process given in (17), $\Pi(\theta, r) \equiv \alpha r\omega_{\widehat{\zeta}}^2/2 = r\alpha\Gamma(\theta, r)\omega_{\zeta}^2/2$* is the precautionary saving demand, *and $\Psi(r) \equiv \ln\left(\frac{1+\rho}{1+r}\right)$ captures the patience-induced saving.*

Proof. See Appendix 7.1 for the derivations. ■

Given the value function (19), we can show that the loss function due to RI is approximately quadratic and the optimality of the ex post Gaussianity of the state still approximately holds in the RU model, which justifies the Kalman filtering equation, (13). (See Online Appendix B for the detailed proof.)

As κ converges to ∞ (or θ converges to 1), $\widehat{s}_t$ and $\omega_{\widehat{\zeta}}^2$ reduce to $\widehat{s}_t$ and ω_{ζ}^2, respectively; consequently, our RI model reduces to the full-information rational expectations (FI-RE) model.[14] To

[14]If $\alpha = 1/\psi$ and $\theta = 1$, the consumption function with RI, (20), reduces to that of the Wang (2003) model.

explore the effects of limited attention on precautionary savings within the RU framework, we first define the precautionary saving premium due to limited attention as

$$P_{ri} \equiv \frac{1}{2}\left(\Gamma\left(\theta,r\right)-1\right)\alpha r\omega_{\zeta}^{2}, \tag{22}$$

where $\Gamma(\theta, r)$ is given in (16). It is clear that this premium is decreasing with the degree of attention θ, and is increasing with the coefficient of absolute risk aversion (α) and the persistence and volatility of the income shock (ϕ_1 and σ) for any given θ. Thus, the incomplete information that RI forces upon the households leads to an increase in saving.

To further explore the precautionary savings premium in (22), we isolate the effects of RI on individual consumption and saving by rewriting (20) as

$$c_t^* = r\widehat{s}_t + \left\{ \frac{\psi}{r}\Psi\left(r\right) - \frac{1}{r\alpha}\left[\begin{array}{c} \ln\left(E_t\left[\exp\left(-r\alpha\theta\left(1+r\right)\left(s_t-\widehat{s}_t\right)\right)\right]\right)+\frac{1}{2}\left(r\alpha\theta\omega_{\zeta}\right)^2+ \\ \frac{1}{2}\left(1-\theta\right)\Gamma\left(\theta,r\right)\left(r\alpha\omega_{\zeta}\right)^2 \end{array}\right]\right\}, \tag{23}$$

where $\Psi(r) = \ln\left(\frac{1+\rho}{1+r}\right)$ measures the relative importance of patience to the interest rate in determining optimal consumption (it is greater than 0 if $\rho > r$),

$$\frac{1}{\alpha r}\ln\left(E_t\left[\exp\left(-\alpha r\theta\left(1+r\right)\left(s_t-\widehat{s}_t\right)\right)\right]\right) = \frac{1}{2}r\alpha\theta\left(1-\theta\right)\Gamma\left(\theta,r\right)\left(1+r\right)^2\omega_{\zeta}^2$$

is the precautionary savings premium due to the time t estimation error, $\left(r\alpha\theta\omega_{\zeta}\right)^2/2$ is the precautionary savings premium driven by the exogenous fundamental income shocks $\{w_t\}$, and $(1-\theta)\Gamma(\theta)\left(r\alpha\omega_{\zeta}\right)^2/2$ captures the precautionary savings premium driven by the endogenous noise shocks, $\{e_t\}$.[15] From (20), for finite capacity ($\kappa < \infty$ or $\theta \in (0,1)$), the precautionary saving premium due to fundamental shocks is smaller than that in the full-information case, $\left(r\alpha\theta\omega_{\zeta}\right)^2/2 < \left(r\alpha\omega_{\zeta}\right)^2/2$, because of the incomplete adjustment of consumption to the fundamental shock; however, we have two new positive terms that increase the total savings more than the absolute value of the reduced savings: (i) the premium due to the estimation error and (ii) the premium due to the RI-induced endogenous noise.

Given the available information at time t and the fact that $E_t[s_t - \widehat{s}_t] = 0$, the conditional mean of (21) can be written as

$$\widetilde{d}_t = \widetilde{f}_t + \Pi\left(\theta,r\right) - \frac{\psi}{r}\Psi\left(r\right), \tag{24}$$

where $\widetilde{f}_t$, $\Pi(\theta, r)$, and $\Psi(r)$ are defined in Proposition 1.

[15]This result is derived by using Equation (17) and the iid property of the processes $\left\{\widehat{\zeta}_t\right\}$, $\{\zeta_t\}$, and $\{e_t\}$.

3.2. Existence and Uniqueness of General Equilibrium

As in Wang (2003), we assume that the economy is populated by a continuum of *ex ante* identical, but *ex post* heterogeneous agents, of total mass normalized to one, with each agent solving the optimal consumption and savings problem with RI specified in (18). Similar to Huggett (1993), we also make the following assumption:

Assumption 2 *The risk-free asset in our model is a pure-consumption loan and is in zero net supply. The initial cross-sectional distribution of permanent income is a stationary distribution* $\Phi(\cdot)$.

By the law of large numbers in Sun (2006), provided that the spaces of agents and the probability space are constructed appropriately, aggregate permanent income and the cross-sectional distribution of permanent income $\Phi(\cdot)$ are constant over time.

Proposition 2. *The total savings demand "for a rainy day" in the precautionary savings model with RI equals zero for any positive interest rate. That is,* $F_t(r) = \int_{y_t} \widetilde{f}_t(r)\, d\Phi(y_t) = 0$, *for* $r > 0$.

Proof. The proof uses the LLN and is the same as that in Wang (2003). ■

Proposition 2 states that the total savings "for a rainy day" is zero, at any positive interest rate. Therefore, from (21), for $r > 0$, the expression for total savings under RI in the economy at time t is

$$D(\theta, r) \equiv \Pi(\theta, r) - \frac{\psi}{r}\Psi(r). \tag{25}$$

Given (25), an equilibrium under RI is defined by an interest rate r^* satisfying

$$D(\theta, r^*) = 0. \tag{26}$$

The following proposition shows the existence of the equilibrium and the PIH holds in the RI general equilibrium.

Proposition 3. *There exists a unique equilibrium with an interest rate* $r^* \in (0, \rho)$ *in the precautionary-savings model with RI. In equilibrium, each agent's consumption is described by the PIH, in that*

$$c_t^* = r^* \widehat{s}_t, \tag{27}$$

where $\widehat{s}_t = E[s_t|\mathcal{I}_t]$ *is the perceived value of permanent income. The evolution equations of wealth and consumption are*

$$\Delta c_{t+1}^* = r^* \widehat{\zeta}_{t+1}, \tag{28}$$

$$\Delta a_{t+1}^* = \frac{1-\phi_1}{1+r^*-\phi_1}(y_t - \overline{y}) + r^*(s_t - \widehat{s}_t), \tag{29}$$

respectively, where $\widehat{\zeta}_{t+1}$ *is specified in (14) with* $E_t\left[\widehat{\zeta}_{t+1}\right]=0$, $\text{var}\left(\widehat{\zeta}_{t+1}\right)=\Gamma\left(\theta,r^*\right)\omega_\zeta^2$, *and* $\Gamma\left(\theta,r^*\right)=\frac{\theta}{1-(1-\theta)(1+r^*)^2}$. *In the general equilibrium, the value function under RI can be written as*

$$\widehat{v}\left(\widehat{s}_t\right)=-\frac{\psi\left(1+r\right)}{r}\exp\left(-\frac{r}{\psi}\widehat{s}_t\right), \tag{30}$$

Proof. If $r>\rho$, the two terms, $\Pi\left(\theta,r\right)$ and $-\psi\Psi\left(r\right)/r$, in the expression for total savings $D\left(\theta,r^*\right)$, are positive, which contradicts the equilibrium condition, $D\left(\theta,r^*\right)=0$. Since $\Pi\left(\theta,r\right)-\psi\Psi\left(r\right)/r<0$ (>0) when $r=0$ ($r=\rho$), the continuity of the expression for total savings implies that there exists at least one interest rate $r^*\in\left(0,\rho\right)$ such that $D\left(\theta,r^*\right)=0$. From (20), we can obtain the individual's optimal consumption rule under RI in general equilibrium as $c_t^*=r^*\widehat{s}_t$. Substituting (13) and (27), we can obtain (28). (27) into (3) yields (29). The proof of uniqueness is longer and relegated to Appendix 7.2. ■

The intuition behind Proposition 3 is similar to that in Wang (2003). With an individual's constant total precautionary savings demand $\Pi\left(\theta,r\right)$, for any $r>0$, the equilibrium interest rate r^* must be such that each individual's dissavings demand due to impatience is exactly balanced by their total precautionary-savings demand, $\Pi\left(\theta,r^*\right)=\psi\Psi\left(r\right)/r$. Figure 1 plots aggregate saving as a function of θ, given the parameters $\alpha=2$, $\psi=0.54$, $\phi_1=0.92$, $\sigma=0.175$, and $\rho=0.04$.[16] It is clear from the figure that the equilibrium interest rate increases with θ.

Regarding uniqueness, Toda (2017) demonstrates that the FI-RE model used here can have multiple stationary equilibria, provided the income process is sufficiently rich; the AR(1) process we use here does not satisfy the requirements for multiple equilibria, though. Our results suggest that RI does not deliver any new insights into the nature of multiple equilibria, so we do not investigate this issue further.

The magnitude of the EIS (ψ) is a key issue in macroeconomics and asset pricing. For example, Parker (2002) and Vissing-Jorgensen and Attanasio (2003) estimate the IES to be well in excess of one. Hall (1988) and Campbell (2003), on the other hand, estimate its value to be well below one. Here we choose $\psi=0.5$ for illustrative purposes and will examine how EIS affects the general equilibrium under RI in Section 4 when we do the quantitative analysis.[17] From the equilibrium condition (26), it is clear that a high value of ψ would amplify the relative importance of the dissaving effect $\Psi\left(r\right)$ for the equilibrium interest rate. The intuition behind this result is simple. When ψ is higher, consumption growth responds less to changes in the interest rate. To clear the market,

[16] In Section 4.1, we will provide more details about how we estimate the income process using the U.S. data. The main result here is robust to the choices of these parameter values.

[17] Guvenen (2006) finds that stockholders have a higher EIS (around 1.0) than non-stockholders (around 0.1). Crump, Eusepi, Tambalotti, and Topa (2015) find that the EIS is precisely and robustly estimated to be around 0.8 in the general population using the newly released FRBNY Survey of Consumer Expectations (SCE).

the consumer must be offered a higher equilibrium risk free rate in order to be induced to save more, making his consumption tomorrow even more in excess of what it is today (less smoothing).

Given (20) and (26), it is clear that even though the individual increases their total precautionary savings in response to information frictions for a given r, the level of aggregate savings equals zero. That is, RI does not affect the aggregate wealth in the economy, because the equilibrium interest rate is pushed down to counteract this precautionary savings increase.[18] With lower Shannon channel capacity, the equilibrium interest rate is lower.

From the equilibrium condition,

$$\frac{1}{2}r^{*}\alpha\Gamma\left(\theta,r^{*}\right)\omega_{\zeta}^{2}-\frac{\psi}{r^{*}}\ln\left(\frac{1+\rho}{1+r^{*}}\right)=0, \tag{31}$$

it is straightforward to show that

$$\frac{dr^{*}}{d\theta}=\frac{r^{*3}\left(2+r^{*}\right)}{\left[1-\left(1-\theta\right)\left(1+r^{*}\right)^{2}\right]^{2}}\left\{\frac{2r^{*}\theta\left[1-\left(1-\theta\right)\left(1+r^{*}\right)\right]}{\left[1-\left(1-\theta\right)\left(1+r^{*}\right)^{2}\right]^{2}}+\frac{2\psi}{\alpha\left(1+r^{*}\right)\omega_{\zeta}^{2}}\right\}^{-1}. \tag{32}$$

where $1-\left(1-\theta\right)\left(1+r^{*}\right)^{2}>0$. It is clear from this expression that r^{*} is decreasing in the degree of inattention $1-\theta$. The first row of Table 2 reports the general equilibrium interest rates for different values of θ.[19] We can see from the table that r^{*} decreases as the degree of inattention increases. For example, if θ is reduced from 1 to 0.1, r^{*} is reduced from 3.41 percent to 2.89 percent. In addition, it is clear that

$$\frac{dr^{*}}{d\alpha}<0 \text{ and } \frac{dr^{*}}{d\psi}>0.$$

That is, the equilibrium interest rate decreases with the degree of risk aversion and increases with the degree of intertemporal substitution. Here it is worth noting that although both the CARA model and the LQ model lead to the PIH in general equilibrium, both risk aversion and intertemporal substitution play roles in affecting the dynamics of consumption and wealth in the CARA model via the equilibrium interest rate channel.

One might ask what a reasonable value of θ is, and if there is any way to calibrate it outside a model. Unfortunately, there is no direct survey evidence on the value of channel capacity of ordinary households in the economics literature, and thus it is not straightforward to answer these questions; estimates of learning capacity exist, but they are not directly useful since we are interested in the capacity that will be devoted to economic activity (specifically, consumption and

[18] If we introduced an asset with elastic supply, such as the capital stock in Aiyagari (1994), the same effects would be present but the stock of capital would rise (and the change in the interest rate would be smaller as a result). How much smaller depends on the elasticity of output with respect to capital (the share parameter on capital in a Cobb-Douglas production function).

[19] Here we also set $\alpha=3$, $\phi_1=0.92$, $\sigma=0.175$, and $\rho=0.04$.

saving). In lieu of such evidence, we simply note that 0.1 is the value needed to match portfolio holdings in Luo (2010) and is therefore not obviously unreasonable (a caveat can be found in Luo and Young 2016, where a significantly larger number is obtained using recursive utility). Coibion and Gorodnichenko (2015) have the most "model independent" measure of θ, and they find $\theta = 0.5$ provides a good fit for a variety of forecast and survey data, and a variety of other papers obtain a number of different values depending on what facts they bring to bear. We will show below that $\theta = 0.1$ allows us to match some cross-sectional dispersion facts, but are cognizant that this parameter's value is quite uncertain.

3.3. Elastic Attention

Instead of using fixed channel capacity to model finite information-processing ability, one could assume that the marginal cost of information-processing (i.e., the shadow price of information-processing capacity) is fixed. That is, the Lagrange multiplier on (9) is constant.[20] In the univariate case, the objective of the agent with finite capacity in the filtering problem is to minimize the discounted expected mean square error (MSE),

$$L_t = E_t \left[v_0 \left(s_t \right) - v \left(x_t \right) \right], \tag{33}$$

where $v_0(\cdot)$ and $v(\cdot)$ are the value functions in the FI-RE model and the RI model, respectively, s_t is the unobservable state variable and x_t is the best estimate of the true state, subject to the information-processing constraint, or

$$\min_{\{\Sigma_t\}} \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} \Phi \Sigma_t + \lambda \ln \left(\frac{(1+r)^2 \Sigma_{t-1} + \omega_\zeta^2}{\Sigma_t} \right) \right] \right\},$$

where $\Phi = (r/\psi)^2$, Σ_t is the conditional variance at t, and λ is the Lagrange multiplier corresponding to (9).[21]

Solving this problem yields the optimal steady state conditional variance:

$$\Sigma(r, \lambda^*) = \frac{(1+r)^2 (1-\beta) \lambda^* - \Phi + \sqrt{\left[(1+r)^2 (1-\beta) \lambda^* - \Phi \right]^2 + 4\lambda^* (1+r)^2 \Phi}}{2\Phi (1+r)^2} \omega_\zeta^2, \tag{34}$$

where $\lambda^* = \lambda / \omega_\zeta^2$ is the normalized shadow price of information-processing capacity. It is straightforward to show that as λ goes to 0, $\Sigma = 0$; and as λ goes to ∞, $\Sigma = \infty$. Comparing (34) with (10),

[20] See Maćkowiak and Wiederholt (2015) and Matejka and McKay (2015) for applications of elastic attention in other macroeconomic settings.

[21] As in the fixed-capacity case, although we adopt the CARA-Gaussian setting, the loss function due to imperfect-state-observation is approximately quadratic. See Online Appendix B for the derivation of the objective function and discussion of the approximation quality.

it is clear that there is a one-to-one mapping between λ and θ when the two RI modeling strategies lead to the same steady state conditional variance:[22]

$$\theta\left(r,\lambda^{*}\right)=1-\frac{1}{1+r}\left\{1+\frac{2\Phi}{\left(1+r\right)^{2}\left(1-\beta\right)\lambda^{*}-\Phi+\sqrt{\left[\left(1+r\right)^{2}\left(1-\beta\right)\lambda^{*}-\Phi\right]^{2}+4\lambda^{*}\left(1+r\right)^{2}\Phi}}\right\}^{-1}. \tag{35}$$

It is clear from this expression that the higher the income uncertainty, the more capacity is devoted to monitoring the evolution of the state. In other words, using this RI modeling strategy, the consumer is allowed to adjust the optimal level of capacity in such a way that the marginal cost of information-processing for the problem at hand remains constant. Figure 2 clearly shows that the optimal level of attention is decreasing with the value of EIS for different values of the marginal information-processing cost.[23] That is, when consumers are more reluctant in substituting their consumption over time (low EIS), they choose to devote more capacity and attention to monitoring the evolution of the state. The intuition behind this result is that when inattentive consumers really like flat consumption profile, they devote more attention to monitoring the state in order to avoid making the consumption profile steep, which occurs with low attention due to "accidental savings".[24] In summary, we use the following two-stage optimization procedure to solve the joint optimal control-filtering model under RI:

1. Given that κ is constant capacity, guess that the ex post Gaussian distribution of the true state and additive iid Gaussian noises due to RI are still optimal when the agent has recursive utility. Given the optimality of ex post Gaussianity and Gaussian noises, we can apply the standard Kalman filter and dynamic programming to solve the RI model explicitly. Using the loss function derived from the value functions under RI and FI-RE, we can then verify that our guess about the optimality of ex post Gaussianity and Gaussian noise is correct.

2. Minimizing the same loss function due to the information-processing constraint obtained in Stage 1 and fixed marginal cost leads to optimal conditional variance (Σ^{*}) and endogenous attention (κ and θ), which verifies that the assumption of constant channel capacity we used in Stage 1 is correct.

Although there is a one-to-one mapping between the above two RI modeling strategies, they have distinct implications for the model's propagation mechanism if the economy is experiencing regime switching. With inelastic capacity, the propagation mechanism governed by the Kalman gain is fixed regardless of changes in fundamental uncertainty, whereas with elastic capacity the

[22]It is obvious that θ converges to its lower limit $\underline{\theta}=1-\exp\left(-2r\right)$ as λ goes to ∞; and it converges to 1 as λ goes to 0.

[23]Here we set $\beta=0.96$ and $r=0.025$.

[24]Given the relationship between λ and θ (or κ), in the following analysis we just use the value of θ to measure the degree of optimal attention.

propagation mechanism will change in response to changes in fundamental uncertainty. In a recent study, Coibion and Gorodnichenko (2015) used the SPF forecast survey data to test the degree of information rigidities governed by the Kalman gain and find that the information rigidities were decreasing with the volatility of the macroeconomic conditions. Specifically, they find that information rigidities were falling from the late 1960s to the start of the Great Moderation ($1983-1984$) and have declined since then, and argue that one should be wary of treating the degree of information rigidities as a structural parameter because it responds to changes in macroeconomic conditions. Cheremukhin, Popova, and Tutino (2015) provided experimental evidence to show that processing information with elastic capacity is more suitable for describing participants' behavior than a fixed information-processing capacity. In the following analysis, we show that elastic attention delivers an accurate prediction about the response of consumption dispersion to changes in income volatility.

Since κ and θ in the elastic attention case depend on both the equilibrium interest rate and labor income uncertainty, the equilibrium interest rate is now determined implicitly by the following function:

$$D\left(\theta\left(r^{*},\widetilde{\lambda}\right),r^{*}\right)\equiv\Pi\left(\theta\left(r^{*},\widetilde{\lambda}\right),r^{*}\right)-\frac{\psi}{r}\Psi\left(r^{*}\right). \tag{36}$$

Figure 3 illustrates how r^* varies with labor income uncertainty, σ, for fixed information-processing cost, λ – the aggregate saving function is increasing with the interest rate and the general equilibrium interest rate is decreasing with labor income uncertainty. We can see from Table 4 that if the economy becomes more volatile (i.e., larger σ), the Kalman gain (θ) increases while the equilibrium interest rate (r^*) decreases. This result is different from that obtained in the fixed capacity case in which θ and r^* move in the same direction. (See Table 2.) The main reason for this result is that income uncertainty affects the equilibrium interest rate via two channels: (i) The direct channel which leads to higher aggregate savings (i.e., the ω_ζ^2 term in (36)) and (ii) the indirect channel which leads to lower aggregate savings (i.e., the $\theta\left(r^*,\widetilde{\lambda}\right)$ term in (36)), and the direct channel dominates.[25] In the next section, after estimating the income process using the U.S. data, we will examine how changes in income uncertainty affects the level of optimal attention and the equilibrium interest rate, and the relative volatility of consumption and wealth to income.

4. Empirical and Quantitative Results

In this section we assess our GE-RI model's implications for the dynamics of consumption, income and wealth. To construct empirical counterparts that are comparable with the theoretical moments derived in the model, we construct a panel with individual consumption, income and wealth based on the Panel Study of Income Dynamics (PSID) and the Consumer Expenditure Survey (CEX), using the imputation approach from Blundell, Pistaferri, and Preston (2008) as extended by Guvenen

[25]We have been unable to prove that r^* is unique under elastic capacity, precisely because of the indirect channel.

and Smith (2014). Then, using the estimated income process, we show the GE-RI model significantly fits the data better than the full information model in terms of the consumption and wealth dynamics. Our closed-form solutions explicitly show the different channels through which RI drives the results.

4.1. Empirical Evidence

In order to measure the relative consumption dispersion in the data, $\mathrm{sd}(\Delta c)/\mathrm{sd}(\Delta y)$, we construct a panel data set which contains both consumption and income at the household level. The PSID does not include enough consumption expenditure data to create a full picture of household nondurable consumption. Such detailed expenditures are found, though, in the CEX from the Bureau of Labor Statistics. To create a panel of consumption to match the PSID income measures, we use an estimated demand function for imputing nondurable consumption following Guvenen and Smith (2014). The demand function uses demographic information and food consumption which can be found in both the CEX and PSID. Thus, we use this demand function of food consumption and demographic information (including age, family size, inflation measures, race, and education) to estimate nondurable consumption for PSID households, creating a consumption panel. To exclude extreme outliers, we then follow Floden and Linde (2001) and normalize both income and consumption measures as ratios of the mean of each year, and exclude households in the bottom and top 1 percent of the distribution of those ratios. (For details on how we select the sample, please see the Online Appendix C.) Figure 4 shows the relative dispersion of consumption to income between 1980 and 2000. The basic pattern confirms but extends the findings in Blundell, Pistaferri, and Preston (2008) – relative consumption dispersion declines in the 1980s, but this decline stops around 1990.

In order to calculate the relative volatility of wealth to income ratio, $\mathrm{sd}(\Delta a)/\mathrm{sd}(\Delta y)$, we use wealth information included in the PSID data. Notice that the PSID only reports household wealth variables every five years starting in 1983, and then every other year starting in 1998. To be consistent with the model, we define wealth as the sum of the net value of liquid assets (checking, savings, money market, etc.), vehicles, home equity, and other assets such as bonds, insurance policies, and trusts. All reported values are again deflated by the CPI to constant $1982-1984$ dollars. Figure 5 reports the results, which shows the relative volatility of wealth to income has been relatively stable in the sample period.

When estimating the income process, we follow Floden and Linde (2001) to normalize household income measures as ratios of the mean for that year and exclude the top and bottom 1 percent of the normalized income measure for the year. To eliminate possible heteroskedasticity in the income measures, we regress each on a series of demographic variables in a fixed-effect panel regression to remove variation caused by differences in age and education. We next subtract these fitted values from each measure to create a panel of income residuals. We then use this panel to estimate the household income process as specified by equation (4) by running panel regressions on lagged income. As the last row of Table 1 reports, the estimated values of ϕ_1 and σ are 0.919 and 0.175, respectively.

4.2. Empirical Implications for the Cross-Sectional Dispersion of Consumption, Wealth, and Income

The following proposition summarizes the main results for the implications of RI for the relative dispersion of consumption and wealth to income.[26]

Proposition 4. *Under RI, the relative volatility of individual consumption growth to income growth is*

$$\mu_{cy} \equiv \frac{sd\left(\Delta c_t^*\right)}{sd\left(\Delta y_t\right)} = \frac{r^*}{1+r^*-\phi_1}\sqrt{\frac{\left(1+\phi_1\right)\Gamma\left(\theta,r^*\right)}{2}}, \tag{37}$$

and the relative volatility of financial wealth to income is

$$\mu_{ay} \equiv \frac{sd\left(\Delta a_t^*\right)}{sd\left(\Delta y_t\right)} = \frac{1}{\sqrt{2}\left(1+r^*-\phi_1\right)}\sqrt{1-\phi_1+\frac{r^{*2}\left(1-\theta\right)\left(1+\phi_1\right)}{1-\left(1-\theta\right)\left(1+r^*\right)^2}+\frac{2r^*\left(1-\theta\right)\left(1-\phi_1^2\right)}{1-\phi_1(1-\theta)\left(1+r^*\right)}}. \tag{38}$$

Proof. See Appendix 7.1. ■

Expression (37) shows that RI has two opposing effects on the relative consumption dispersion. The direct effect is through its presence in the expression of $\Gamma\left(\theta,r^*\right)$, whereas the indirect effect is through the equilibrium interest rate (r^*). Using the expression of $\Gamma\left(\theta,r^*\right)$, it is clear that the direct effect of RI is to increase consumption volatility. The intuition is very simple: the presence of the RI-induced noise dominates the slow adjustment of consumption in determining consumption volatility at the individual level. In contrast, the indirect effect of RI will reduce consumption volatility because it reduces the general equilibrium interest rate and $\partial\Gamma\left(\theta,r^*\right)/\partial r^* > 0$.

Following the literature of precautionary savings and the estimated income process in the preceding subsection, we set $\rho = 0.04$, $\alpha = 3$, $\sigma = 0.175$, and $\phi_1 = 0.919$. Table 2 reports how the interest rate (r^*) and the relative volatility of consumption and wealth to income vary with θ in general equilibrium. It is clear from the second row of Table 2 that RI significantly affects the equilibrium interest rate. For example, when θ decreases from 1 to 0.10, r^* decreases from 3.41 percent to 2.89 percent, which is very close to 2.97 percent, the average annual equilibrium real interest rate from 1980 to 1996 estimated in Laubach and Williams (2015) (note that if $\theta = 0.11$, the equilibrium interest rate obtained in our model is exactly the same as its empirical counterpart). Here we

[26] Note that mathematically, the cross-sectional dispersion of consumption and wealth (relative to income) can be measured by the relative volatility of consumption to income and the relative volatility of wealth to income.

focus on the $1980-1996$ period because we use it to estimate the income process and compute the relative volatility of consumption to income.

The third row of Table 2 shows that μ_{cy} increases with the degree of inattention. For example, when θ decreases from 1 to 0.1, μ_{cy} increases from 0.290 to 0.375, which is the same as the empirical counterpart. It is clear from these results that the direct effect of inattention via the $\Gamma(\theta, r^*)$ term in (37) dominates its indirect general equilibrium effect via r^*. Comparing the general equilibrium (GE) and partial equilibrium (PE) results in Table 2, we can see the values of μ_{cy} are lower in the GE case if the interest rate is fixed as θ decreases. In other words, the general equilibrium effect of RI tends to reduce the volatility of individual consumption in this case.[27] Furthermore, the second panel of Table 2 shows that the equilibrium value of μ_{cy} increases with the EIS (ψ). The main reason for this result is that the equilibrium interest rate increases with the EIS. Note that the higher the value of the EIS, the higher the dissaving effect due to impatience, and the higher the equilibrium interest rate.

Another important implication of RI in general equilibrium is that RI leads to more skewed wealth dispersion measured by μ_{ay}, the relative volatility of financial wealth to labor income. The fourth row of Table 2 shows that when θ is reduced from from 1 to 0.1, μ_{ay} increases from 1.748 to 2.620, which is much closer to the empirical counterpart. (For example, μ_{ay} is 3.11 in 1993 and is 2.59 in 1998.) From (29), it is clear that the main driving force behind this result is the presence of the estimation error, $s_t - \widehat{s}_t$, because $\partial \operatorname{var}(s_t - \widehat{s}_t)/\partial\theta < 0$. Note that although $\partial r^*/\partial\theta > 0$, the estimation error channel dominates the general equilibrium channel and increases wealth dispersion. Therefore, RI also increases relative wealth dispersion, which improves the model's fit to the data.[28]

We can also see from the second panel of Table 2 that the equilibrium value of μ_{ay} increases with the EIS, which is due to that the equilibrium interest rate increases with the EIS. We can clearly inspect this mechanism by considering a special case when $\phi_1 = 1$. Specifically, in this case, (29) reduces to

$$\mu_{ay} = \sqrt{\frac{1-\theta}{1-(1-\theta)(1+r^*)^2}}, \tag{39}$$

which implies that μ_{ay} increases with ψ and r^*.

To briefly summarize the key discussions above, Table 3 compares the performances of the FI-RE model, GE-RI model, and the partial equilibrium rational inattention model (PE-RI) with

[27] We cannot examine the stochastic properties of aggregate consumption dynamics because all idiosyncratic shocks (income shocks and RI-induced noise shocks) cancel out after aggregating across consumers.

[28] The literature has found that simple models based on standard CRRA preferences and on measured uninsurable shocks to labor income cannot account for the observed U.S. wealth distribution. For example, Aiyagari (1994) finds considerably less wealth concentration in a model with only idiosyncratic labor earnings uncertainty. Given the CARA-Gaussian setting, the model here is not suitable to address the issue like why the top 1 percent or 5 percent richest families hold a large fraction of financial wealth in the U.S. economy.

the data. Overall, it shows under the estimated income process and at a single value of rational inattention parameter (θ), the GE-RI model can do a significantly better job than the FI-RE model in generating a lower interest rate, a higher consumption volatility, and a higher wealth volatility, matching the data much closer. In terms of welfare loss, as the last row in Table 3 shows and will be discussed in detail in the next subsection, the PE model significantly underestimates the welfare loss, though the welfare loss is generally small.

Table 4 reports how elastic Kalman gain, the equilibrium interest rate, and the relative volatility of consumption and wealth to income vary with different values of income uncertainty measured by σ (and σ_y). We have reached four key findings. First, as the second row of Table 4 shows, the Kalman gain increases with income volatility, meaning that agents optimally allocate more attention to the state variable when income uncertainty increases. Second, RI has significant effects on the equilibrium interest rate. Specifically, in the elastic capacity case, an increase in income volatility affects the equilibrium interest rate via two channels: (i) the direct channel (the ω_ζ^2 term in (36)) and (ii) the indirect channel (the elastic capacity θ term in (36)). The third panel of Table 4 reports the results when we shut down the indirect channel and assume that $\theta = 1$. Comparing the first and third panels of Table 4 shows that the indirect channel is more important when σ is relatively low. Third, EIS can significantly affect the dispersions of consumption and wealth via affecting the optimal attention level and the equilibrium interest rate. For example, when $\sigma = 0.2$ and ψ increases from 0.54 to 0.8, the optimal attention level is reduced from 11% to 8% and the equilibrium interest rate is reduced from 2.79 percent to 2.73 percent. There are two channels through which EIS affects the equilibrium interest rate under RI: (i) the optimal attention channel and (ii) the impatience-induced dissaving channel. In this quantitative analysis, we find that the optimal attention channel dominates the impatience-induced dissaving channel. Fourth, μ_{cy} decreases with the value of σ in general equilibrium. That is, consumption becomes smoother when income becomes more volatile. For example, in the equilibrium RI economy when $\psi = 0.54$, μ_{cy} decreases from 0.34 to 0.21 when σ increases from 0.2 to 0.4.[29]

The last finding highlighted above might provide a potential explanation for the empirical evidence documented in Blundell, Pistaferri, and Preston (2008) that income and consumption inequality diverged over the sampling period they study.[30] To explore this issue in our model, we do the following exercise. First, we divide the full sample into two sub-samples ($1980 - 1986$ and $1987 - 1996$) and apply the same estimation procedure to re-estimate σ and ϕ_1 (see the first and second rows of Table 1 for the estimation results). Household income is more volatile in late sub-periods than earlier ones. Specifically, the standard deviation of y is 0.386 in the sub-sample

[29]It is not surprising that μ_{cy} is greater in the equilibrium RI economy than in the equilibrium FI economy because the value of θ is less than 1 in the RI case. This result is the same as that we obtained in the fixed capacity case and reported in Table 2.

[30]Other mechanisms have been proposed for this decline; see Krueger and Perri (2006) and Athreya, Tam, and Young (2009) for examples.

$(1980-1986)$, while it is 0.427 in the sub-sample $(1987-1996)$. The average values of μ_{cy} are 0.46 and 0.30 in the first and second sub-samples, respectively. In the elastic capacity case, using the estimated income processes in the first sub-sample, we first use $\mu_{cy}=0.46$ to calibrate $\lambda=0.38$; the corresponding value of θ is 0.08 in the first sub-sample. Using this calibrated value of λ, we find that μ_{cy} is reduced to 0.38 in the second sub-sample, which is much closer to the empirical counterpart than the value obtained in the fixed capacity case (0.41). Note that here we assume that the marginal information-processing cost is invariant across sub-samples.

4.3. Welfare Losses due to RI in Equilibrium

We now turn to the welfare cost of RI – how much utility does a consumer lose if the actual consumption path he chooses under RI deviates from the first-best FI-RE path in which $\theta=1$? To answer this question, we follow Barro (2007) and Luo and Young (2010) by computing the marginal welfare cost due to RI. The following proposition summarizes the main result.

Proposition 5. *Given the initial value of the state, $\widehat{s}_0$, the marginal welfare cost* (mwc) *due to RI is given by*

$$\mathrm{mwc}(\theta)\equiv\frac{(\partial v(\widehat{s}_0)/\partial\theta)\,\theta}{(\partial v(\widehat{s}_0)/\partial\widehat{s}_0)\,\widehat{s}_0}=\frac{\theta\psi}{r^{*2}}\left[\frac{r^*}{\psi}+\frac{1}{(1+r^*)\,\widehat{s}_0}\right]\frac{dr^*}{d\theta}, \tag{40}$$

where $dr^*/d\theta$ is given in (32) and $\widehat{v}(\widehat{s}_0)=-\exp(-r^*\alpha\widehat{s}_0+\ln(1+r^*))/(r^*\alpha)$. The monthly dollar loss due to deviating from the FI-RE path $(\theta=1)$ can be written as

$$\$\,\mathrm{loss}\,(\theta<1)\equiv\frac{r^*}{12}\mathrm{mwc}(1)(1-\theta)\,\widehat{s}_0. \tag{41}$$

Proof. See Appendix 7.3. Since we are interested in the deviation from the FI-RE path, $\theta=1$ is considered as the starting point. If we change from 1 to θ, the percentage change is $(\theta-1)$. $\widehat{s}_0$ is initial total wealth. Finally, we need to convert the change in the $\widehat{s}_0$ term to monthly rates by multiplying by $r^*/12$. ■

Expression (40) gives the proportionate reduction in the initial level of the perceived state $(\widehat{s}_0)$ that compensates, at the margin, for a percentage decrease in θ (i.e., stronger degree of RI) — in the sense of preserving the same effect on welfare for a given $\widehat{s}_0$. To do quantitative welfare analysis we need to know the value of $\widehat{s}_0$. First, we set $\widehat{y}_0\equiv E[y_t]=1$, $\phi_1=0.919$, and the ratio of the initial level of financial wealth $(\widehat{a}_0)$ to mean income $(\widehat{y}_0)$ equal to 5.[31] Second, given that $\widehat{s}_0=\widehat{a}_0+\widehat{y}_0/(1+r^*-\phi_1)+\overline{y}/r^*$, we can calculate the values of the monthly dollar loss ($\$$ loss) for different values of θ and the corresponding values of the general equilibrium interest rate. The fifth row of Table 2 reports the welfare losses (measured by the dollar) for different degrees of

[31] This number varies largely for different individuals, from 2 to 20. 5 is the average wealth/income ratio in the Survey of Consumer Finances 2001. We find that changing the value of this ratio only has minor effects on the welfare calculation.

inattention. For example, when $\theta = 0.1$, the welfare loss due to RI in general equilibrium is about \$14.64 per month, or about 0.06 percent of mean income per month, which is relatively small.[32] This welfare loss decreases to \$8.51 when $\theta = 0.6$, or 0.04 percent of monthly income. This result is similar to the findings by Pischke (1995), Luo (2008), Luo and Young (2010), and Luo, Nie, and Young (2015), and is robust to changes in the income process and the degrees of patience and risk aversion.[33] Thus, it seems reasonable for agents to devote low channel capacity to observing and processing information because the welfare improvement from increasing capacity would be trivial.

Another implication of the welfare losses due to RI reported in Table 2 is that there is a general equilibrium effect of RI on the welfare loss. For example, when r is set to be 3.41 percent (the general equilibrium interest rate obtained under FI) in the partial equilibrium (PE) case, the monthly dollar loss due to RI is significantly less than that obtained in the GE case.[34] When $\theta = 0.2$, the welfare loss in the GE case is \$15.64, while it is close to 0.01 in the PE case. The main intuition behind this result is that the general equilibrium channel governed by $dr^*/d\theta$ is shut down in the PE model.

4.4. Policy Implications

In this section, we discuss the effects of changes in public policy on the size of precautionary savings, the interest rate, and the cross-sectional distribution of consumption, wealth, and income in general equilibrium. Some public policies are important for ordinary consumers because they cover such risks as unemployment, health, and longevity, and may affect the need for the consumers to accumulate financial wealth.

Here for simplicity we consider a public policy that can be used to provide social insurance by reducing the income variance. Specifically, we assume that the government now imposes a marginal tax rate on labor income (τ) by increasing it from 0 to 1/3. In this case, the labor income risk measured by $(1-\tau)\,\sigma$ is reduced from σ to $(2/3)\,\sigma$. Using the expression for the precautionary saving demand, (20), it is clear that the change in the tax rate leads to a reduction in precautionary savings, holding other parameter values fixed. From the individual consumption function, it is clear that the presence of rational inattention measured by $\Gamma(\theta, r) > 1$ can amplify the impact of this public policy on the precautionary saving demand. In the general equilibrium, given

$$\frac{1}{2} r^* \alpha \Gamma(\theta, r^*) (1-\tau)^2 \omega_\zeta^2 - \frac{\psi}{r^*} \ln\left(\frac{1+\rho}{1+r^*}\right) = 0, \tag{42}$$

[32]In our estimation, we normalize the household income to the 1982 unit. The average value of real disposable personal income per capita is \$24,146 from 1980 to 1996.

[33]Pischke (1995) found that in most cases the utility losses arising from households having no information about aggregate income shocks are less than \$1 per quarter in the LQ permanent income model.

[34]See Appendix 7.3 for the derivations of the welfare loss due to RI in partial equilibrium.

it is clear that the reduction in the labor income risk has two opposite effects on the equilibrium interest rate: (i) it increases the interest rate by reducing the uncertainty about labor income from ω_{ζ}^2 to $(1-\tau)^2\omega_{\zeta}^2$ (the direct effect) and (ii) it reduces the interest rate by reducing the optimal attention level, θ, and then increasing $\Gamma(\theta, r^*)$ (the indirect effect). In the quantitative analysis reported in Table 4 in which we set $\sigma = 0.3$, it is clear that the direct channel dominates the indirect channel, which drives up the interest rate in the RI model. It is worth noting that in the FI-RE case, the indirect channel disappears, and this public policy unambiguously increases the equilibrium interest rate. Comparing Columns 2 and 3 of Table 4, we can see that under RI, r^* increases by 20%, while under FI-RE, it is increased by 23%.

Furthermore, it is clear from Expression (37) that this public policy can increase μ_{cy} via two channels: (i) increasing the equilibrium interest rate and (ii) increasing $\Gamma(\theta, r^*)$ by reducing θ.[35] In the FI-RE case, the second channel disappears and only the interest rate channel matters in determining μ_{cy}. For example, Table 4 shows that μ_{cy} increases by 31% under RI, while it only increases by 17% under FI-RE, which means that under RI, the public policy has larger impact on μ_{cy} than under FI-RE.

However, this policy may have opposite effects on the relative volatility of wealth to income, μ_{ay}, in the RI and FI-RE models. In the FI-RE case, this ratio is only determined by the equilibrium interest rate. As shown in Expression (38), it is clear that the higher the value of the interest rate, the smaller the ratio is. In contrast, in the RI case, the RI-induced noises term drives up μ_{ay}. In Table 4, we can see that the RI-induced noise channel may dominate the interest rate channel, and the ratio increases with the tax rate in the RI case. In contrast, in the FI-RE case, the RI-induced noise channel disappears and only the interest rate channel remains, which makes μ_{ay} decrease with the tax rate. As shown in Table 4, when the policy is implemented, μ_{ay} increases from 2.39 to 2.53 under RI, while it is reduced from 1.88 to 1.78 under FI-RE.

5. Comparison with Alternative Models

5.1. Comparison with Habit Formation

An alternative structure that delivers slow consumption dynamics is the habit formation (HF) model of Constantinides (1990). With HF preferences, households try to smooth consumption growth (roughly speaking), rather than the level of consumption; the result is that consumption tends to respond slowly to changes in permanent income. Luo (2008) shows that RI and HF deliver identical aggregate consumption growth movements, but at the individual level RI models deliver more consumption volatility due to the noise shocks (which are canceled out by aggregation).

In this section, we compare the different implications of HF and RI in general equilibrium.

[35]Note that $\Gamma(\theta, r^*)$ is also increasing in r^* for given θ.

Following Alessie and Lusardi (1997), we introduce HF into the FI-RE model specified in Section 2.1 by assuming that the utility function takes the following form:

$$f(U_t) = f(c_t - \gamma c_{t-1}) + \frac{1}{1+\rho} f(\mathcal{CE}_t[U_{t+1}]), \tag{43}$$

where $\gamma > 0$ is the habit parameter, $f(c_t - \gamma c_{t-1}) = (-\psi)\exp(-(c_t - \gamma c_{t-1})/\psi)$, $f(U_t) = (-\psi)\exp(-U_t/\psi)$, $\mathcal{CE}_t[U_{t+1}] = g^{-1}(E_t[g(U_{t+1})])$, and $g(U_{t+1}) = -\exp(-\alpha U_{t+1})/\alpha$. Using the same solution method used in Section 2.1, we can solve for the consumption function under HF:

$$c_t = \frac{\gamma}{1+r} c_{t-1} + r\left(1 - \frac{\gamma}{1+r}\right) s_t + \frac{\psi}{r}\ln\left(\frac{1+\rho}{1+r}\right) - \frac{1}{2}\alpha r\left(1 - \frac{\gamma}{1+r}\right)^2 \omega_\zeta^2. \tag{44}$$

(See Online Appendix D for the derivation.) The corresponding saving function can thus be written as

$$d_t = (1-\phi_1)\phi(y_t - \overline{y}) + \frac{r\gamma}{r+1}\frac{\zeta_t}{1-\gamma\cdot L} - \frac{1}{1-\gamma}\left(\frac{\psi}{r}\ln\left(\frac{1+\rho}{1+r}\right) - \frac{1}{2}\alpha r\left(1 - \frac{\gamma}{1+r}\right)^2 \omega_\zeta^2\right).$$

Following the same definition of general equilibrium in our benchmark model, it is straightforward to show that there exists a unique equilibrium interest rate r^* such that

$$\frac{\psi}{r^*}\ln\left(\frac{1+\rho}{1+r^*}\right) - \frac{1}{2}r^*\alpha\widetilde{\Gamma}(\gamma, r^*)\,\omega_\zeta^2 = 0,$$

where $\widetilde{\Gamma}(\gamma, r^*) = \left(1 - \frac{\gamma}{1+r^*}\right)^2 < 1$. In general equilibrium, it is straightforward to show that

$$\frac{dr^*}{d\gamma} > 0.$$

That is, the stronger the habit persistence, the higher the equilibrium interest rate.[36] In summary, we can conclude that although both RI and HF lead to slow adjustments in consumption, they have opposite effects on the equilibrium interest rate.[37] RI reduces the equilibrium interest rate, while HF increases it. The second row of Table 5 reports the general equilibrium interest rates for different values of γ.[38] We can see from the table that r^* increases as the degree of habit formation increases. For example, if γ is raised from 0.4 to 0.9, r^* increases from 3.57 percent to 3.79 percent.

The following proposition summarizes the implications of habit formation for the relative volatility of consumption and wealth to income:

[36]In a partial equilibrium model, Alessie and Lusardi (1997) show that the stronger the habit, the smaller the effect of income uncertainty on the precautionary saving term.

[37]The mechanisms of RI and HF that generate slow adjustment are distinct. Under RI, slow adjustment is forced upon the agent due to finite information processing capacity (learning is slow). In contrast, slow adjustment is optimal under HF because consumers are assumed to prefer to smooth consumption growth.

[38]Here we also set $\gamma = 3$, $\psi = 0.54$, $\phi_1 = 0.92$, $\sigma = 0.175$, and $\rho = 0.04$.

Proposition 6. *Under habit formation, the relative volatility of individual consumption growth to income growth is*

$$\mu_{cy} \equiv \frac{sd\left(\Delta c_t^*\right)}{sd\left(\Delta y_t\right)} = \frac{r^*\left(1+r^*-\gamma\right)}{1+r^*}\sqrt{\frac{\left(1+\phi_1\right)\left\{\frac{\left[\left(1-\phi_1\right)\phi+\left(r^*\phi-1\right)\right]^2}{1-\phi_1^2}+\frac{\left(r^*\gamma\phi\right)^2}{\left(1-\gamma^2\right)\left(1+r^*\right)^2}+2\frac{\left(r^*\gamma\phi\right)\left[\left(1-\phi_1\right)\phi+\left(r^*\phi-1\right)\right]}{\left(1+r^*\right)\left(1-\gamma\phi_1\right)}\right\}}{2\left[1-\left(\frac{\gamma}{1+r^*}\right)^2\right]}}, \tag{45}$$

and the relative volatility of financial wealth to income is

$$\mu_{ay} \equiv \frac{sd\left(\Delta a_t^*\right)}{sd\left(\Delta y_t\right)} = \frac{\phi}{\sqrt{2}}\sqrt{\left(1+\phi_1\right)\left[\frac{1-\phi_1}{1+\phi_1}+\left(\frac{r^*\gamma}{1+r^*}\right)^2\frac{1}{1-\gamma^2}+\frac{2\left(1-\phi_1\right)r^*\gamma}{\left(1+r^*\right)\left(1-\gamma\phi_1\right)}\right]}. \tag{46}$$

Proof. See Online Appendix D for the derivation. ■

Expressions (45) and (46) show that habit formation affects μ_{cy} via two channels. The first channel is direct through the presence of γ, whereas the second channel is indirect and operates through the equilibrium interest rate r^*. Given the complexity of these expressions, we cannot obtain explicit results about how habit formation affects μ_{cy}. We therefore use the same parameter values as used in the preceding subsection to do a quantitative analysis. The first panel of Table 5 reports the general equilibrium and partial equilibrium interest rates for different values of γ. It is clear from the table that μ_{cy} is decreasing with the degree of habit formation. For example, when γ is increased from 0.4 to 0.9, μ_{cy} is reduced from 0.21 to 0.09 in general equilibrium; as with RI, the general equilibrium effects are small and the slow adjustment in consumption channel dominates the general equilibrium channel. Thus, the value of μ_{cy} is even lower than that predicted in the FI-RE case, and makes the model fit the data worse in this dimension.

From the fourth row of the first panel in Table 5, we can see that the relative volatility of wealth to income (μ_{ay}) increases with the degree of habit formation. For example, if γ is increased from 0.4 to 0.9, μ_{ay} increases from 1.76 to 2.30 in general equilibrium. The main reason for this result is that habit formation affects the wealth accumulation response to an income shock by slowing the adjustment of consumption to the shock, which creates a temporary gap between consumption and permanent income, which exaggerates the wealth accumulation effect and thereby increases the volatility of wealth accumulation that results from income shocks. Even for a high degree of habit ($\gamma = 0.9$), the model's prediction on μ_{ay} is still well below its empirical counterpart (3.28). In addition, values of γ that high cannot be reconciled with some of the large changes in consumption observed at the individual level; in effect, one is forced to suppose that these movements are almost entirely due to measurement errors.[39]

[39]Technically, this statement holds only if the utility function does not permit "effective consumption" $c_t - \gamma c_{t-1}$ to be negative, as would the case with CRRA preferences. CARA preferences are defined for negative values.

5.2. Comparison with Incomplete Information about Income

In this subsection, we consider an incomplete information (IC) model in which the income process has two components and consumers cannot distinguish the two components. Following Muth (1960) and Pischke (1991), we assume that observed (measured) labor income includes a unit root and the whole income process has two kinds of structural shocks to labor income: One has a permanent impact on the level of labor income and the other has only transitory impact.[40] Specifically, the income process can be written as:

$$y_{t+1} = y_{t+1}^p + y_{t+1}^i, \tag{47}$$

$$y_{t+1}^p = y_t^p + \varepsilon_{t+1}, \tag{48}$$

$$y_{t+1}^i = \overline{y} + \xi_{t+1}, \tag{49}$$

where y_{t+1}^p and y_{t+1}^i are permanent and transitory components in measured income, respectively, ε_{t+1} and ξ_{t+1} are orthogonal permanent and transitory iid shocks with mean 0 and variance ω_ε^2 and ω_ξ^2, respectively. Note that here we can interpret the iid component, ξ_{t+1}, as measurement error which would destroy the identification of the true level of labor income.

Given that the change in income is $\Delta y_{t+1} = \varepsilon_{t+1} + \epsilon_{t+1} - \epsilon_t$, the best forecast is to recognize that Δy_{t+1} is a moving-average process of order one:

$$\Delta y_{t+1} = \nu_{t+1} - \tau \nu_t, \tag{50}$$

where the innovation, $\nu_t \sim N\left(0, \omega_\nu^2\right)$, contains information on current and lagged permanent and transitory income shocks. Equating the variances and autocorrelation coefficients of (**??**) and (50), we have

$$\omega_\nu^2 = \frac{\text{var}\left(\Delta y_{t+1}\right)}{1+\tau^2} = \frac{\omega_\epsilon^2}{\tau},$$

where $\tau = -\left(1 - \sqrt{1-4\varrho^2}\right)/\left(2\varrho\right) \in [0,1]$ and $\varrho = -\omega_\xi^2/\left(\omega_\varepsilon^2 + 2\omega_\xi^2\right) \in (-0.5, 0]$. τ will be large if the variance of the transitory shock ω_ξ^2 is large relative to the variance of the permanent shock ω_ε^2 and will converge to 0 as ω_ξ^2 approaches 0. In the following analysis, we use τ to measure the relative importance of measurement error and thus the degree of incomplete information.

Following the same procedure in Section 2.1, we define a new state variable, $s_t = a_t + y_t/r - \tau\nu_t/\left(r\left(1+r\right)\right)$, for the IC problem, and rewrite the original budget constraint as follows:

$$s_{t+1} = \left(1+r\right)s_t - c_t + \zeta_{t+1}, \tag{51}$$

[40] Wang (2004) considered a similar incomplete information problem in continuous-time, and assumed that the two individual components in income follow different Ornstein-Uhlenbeck processes. Here for simplicity we just consider the permanent-transitory decomposition in income.

where $\zeta_{t+1} = \frac{1+r-\tau}{r(1+r)}\nu_{t+1}$. Maximizing the typical consumer's lifetime utility subject to (51) leads to the following consumption function:

$$d_t^* = \tau\nu_t - \frac{\psi}{r}\ln\left(\frac{1+\rho}{1+r}\right) + \frac{1}{2}\alpha r\omega_\zeta^2.$$

Since ν_t is an idiosyncratic innovation with mean zero, following the same definition of general equilibrium in our benchmark model, it is straightforward to show that there exists a unique equilibrium interest rate r^* such that

$$\frac{1}{2}\alpha r^*\omega_\zeta^2 - \frac{\psi}{r^*}\ln\left(\frac{1+\rho}{1+r^*}\right) = 0, \tag{52}$$

which implies that $dr^*/d\omega_\zeta^2 < 0$. Given that $\omega_\zeta^2 = \left[\frac{1+r-\tau}{r(1+r)}\right]^2 \frac{\mathrm{var}(\Delta y_{t+1})}{1+\tau^2}$ is decreasing with τ, it is clear that the higher the degree of incomplete information, the higher the equilibrium interest rate.

In summary, we can conclude that although both RI and IC lead to slow adjustments in consumption, they have opposite effects on the equilibrium interest rate. RI reduces the equilibrium interest rate, while IC increases it. The second panel of Table 5 reports the equilibrium interest rates for different values of τ.[41] We can see from the table that r^* increases as the degree of incomplete information increases. For example, when τ is raised from 0.4 to 0.9, r^* increases from 1.30 percent to 4.10 percent.

The following proposition summarizes the implications of incomplete information for the relative volatility of consumption and wealth to income:

Proposition 7. *Under incomplete information, the relative volatility of individual consumption growth to income growth is*

$$\mu_{cy} \equiv \frac{sd\left(\Delta c_t^*\right)}{sd\left(\Delta y_t\right)} = \left(1 - \frac{\tau}{1+r^*}\right)\sqrt{\frac{1}{1+\tau^2}}, \tag{53}$$

and the relative volatility of financial wealth to income is

$$\mu_{ay} \equiv \frac{sd\left(\Delta a_t^*\right)}{sd\left(\Delta y_t\right)} = \frac{\tau}{1+r^*}\sqrt{\frac{1}{1+\tau^2}}. \tag{54}$$

Proof. Using the expressions for the equilibrium consumption and asset accumulation functions, it is straightforward to show that

$$\Delta c_{t+1}^* = r\zeta_{t+1} = \left(1 - \frac{\tau}{1+r^*}\right)\nu_{t+1} = \left(1 - \frac{\tau}{1+r^*}\right)\frac{\Delta y_{t+1}}{1-\tau\cdot L}, \tag{55}$$

$$\Delta a_{t+1}^* = \frac{\tau}{1+r^*}\nu_t = \frac{\tau}{1+r^*}\frac{\Delta y_t}{1-\tau\cdot L}. \tag{56}$$

[41] Here we also set $\gamma = 3$, $\psi = 0.54$, $\phi_1 = 0.92$, $\sigma = 0.175$, and $\rho = 0.04$.

Taking unconditional variance on both sides of (55) and (56) yields (53) and (54). ■

From (53) and (54), we can see that incomplete information about current income affects μ_{cy} via two channels. The first channel is direct through the presence of τ, whereas the second channel is indirect and operates through the equilibrium interest rate r^*. Using the same parameter values as used in the preceding subsection, the second panel of Table 5 reports the general equilibrium results for different values of τ. It is clear from the table that μ_{cy} decreases with τ. For example, if τ is increased from 0.4 to 0.9, μ_{cy} falls from 0.56 to 0.10 in general equilibrium.

From this panel, we can also see that μ_{ay} increases with τ. For example, if τ is increased from 0.4 to 0.9, μ_{ay} increases from 0.37 to 0.64 in general equilibrium. The main reason for this result is that incomplete information about income affects the wealth accumulation response to an income shock by slowing the adjustment of consumption to the shock, which creates a temporary gap between consumption and permanent income and thereby increases the volatility of wealth that results from income shocks. Even for a high degree of incomplete information (e.g., $\tau = 0.9$), the model's prediction on μ_{ay} is still well below its empirical counterpart (3.28).

5.3. Comparison with Models with Borrowing Constraints

Our version of Huggett (1993) abstracts from borrowing constraints. As noted in many papers, borrowing constraints can deliver consumption dynamics that display excess sensitivity to predictable movements in income, although Ludvigson and Michaelides (2001) and Hryshko (2014) show that the basic model with borrowing constraints does not deliver the observed excess sensitivity in micro or macro data. We argue in this subsection that borrowing constraint models make predictions regarding the dispersion of consumption relative to income that is inconsistent with our data, and therefore that RI models are to be preferred.

Standard models with borrowing constraints (Huggett 1993, Aiyagari 1994) imply that poor households (those close to the borrowing constraint) have a relatively high marginal utility of consumption which leads to these households responding very strongly to increases in their income; the result is that for a given dispersion in income, consumption changes are more volatile and wealth changes less volatile among the poor.[42] Indeed, using a simple benchmark version of Huggett (1993) with a uniform borrowing constraint set to twice average income, we find that μ_{cy} for the poor (those with below median wealth, who are borrowers) is roughly twice μ_{cy} for the rich (those with above median wealth); similarly, μ_{ay} is about 50 percent smaller for the poor than the rich. And these relationships are even stronger if we consider percentiles further out in the tails.[43] Tighter borrowing constraints lead to even larger discrepancies, as more of the below-average-wealth households are close to the borrowing limit; if we set the borrowing limit sufficiently low

[42]While the opposite is true for income decreases, mean reversion in income leads to the first effect dominating.
[43]See Online Appendix E for details on the specification of the Huggett (1993) model we use.

it plays little role and the economy is very similar to our benchmark.

This variation across wealth is strongly rejected in our data. Table 6, based on the PSID data, shows that the relative dispersion of consumption for the whole sample is almost the same as that of the top 50 percent by income levels. That is, even if we exclude the bottom 50 percent of households by income the ratio is nearly unchanged, which suggests borrowing constraints do not play a significant role in the dynamics of consumption relative to income. The dispersion ratio varies slightly more if we select households by their wealth levels, although our sample size shrinks significantly due to limited wealth information in our data.

6. Concluding Remarks

In this paper we have studied how rational inattention affects the interest rate and the joint dynamics of consumption, income, and wealth in a Huggett-type general equilibrium model with recursive utility. We highlight our two main results here in the conclusion. First, RI helps the basic model deliver a good fit to the dispersions of consumption and wealth relative to income; more inattention leads to more dispersion, pushing the models closer to the data, and we find that there is a common value of the inattention parameter that delivers a good fit for both. The effects on the equilibrium interest rate are modest, and work to slightly offset the added dispersion. These results are robust to the exact way attention is modeled (elastic or inelastic), and the elastic attention version also captures the observed movements in consumption dispersion relative to income dispersion evident in US data. Second, we compare RI to three alternative models – habit formation, incomplete information about income, and borrowing constraints. We find that the RI model delivers better predictions regarding these dispersions than the alternatives.

7. Appendix

7.1. Deriving the Consumption, Saving, and Value Functions in the Cabellero-Huggett Model with RU and RI

Using the income process, the original budget constraint (3) can be rewritten as

$$
\begin{aligned}
a_{t+1} + \phi y_{t+1} + \frac{\phi\phi_0}{r} &= (1+r)\, a_t + y_t - c_t + \phi\left(\phi_0 + \phi_1 y_t + w_{t+1}\right) + \frac{\phi\phi_0}{r} \\
&= (1+r)\left(a_t + \phi y_t + \frac{\phi\phi_0}{r}\right) - c_t + \zeta_{t+1},
\end{aligned}
$$

where the $(t+1)$-innovation $\zeta_{t+1} = \phi w_{t+1}$ is Gaussian innovation process with mean zero and variance $\phi^2\sigma^2$. Denote $s_t = a_t + \phi y_t + \phi\phi_0/r$, the new budget constraint can be rewritten as (6) in the main text. As shown in Section 2.2, under RI, the typical consumer uses the Kalman filter, (13), to update the perceived state, $\widehat{s}_t$.

The objective of the consumer is to solve the following Bellman equation based on the recursive utility defined in Section 2.1:

$$f\left(J\left(\widehat{s}_t\right)\right)=\max_{c_t}\left\{f\left(c_t\right)+\frac{1}{1+\rho}f\left(\mathcal{CE}_t\left[J\left(\widehat{s}_{t+1}\right)\right]\right)\right\}, \tag{57}$$

subject to (13). We first conjecture that $J\left(\widehat{s}_t\right)=A\widehat{s}_t+A_0$, where A and A_0 are undetermined coefficients. Substituting the guessed function into the definition of the certainty equivalent: $\exp\left(-\alpha\mathcal{CE}_t\right)=E_t\left[\exp\left(-\alpha J\left(\widehat{s}_{t+1}\right)\right)\right]$, we have

$$\exp\left(-\alpha\mathcal{CE}_t\right)=\exp\left(-\alpha A\left[\left(1+r\right)\widehat{s}_t-c_t\right]+\frac{1}{2}\alpha^2A^2\omega_{\widehat{\zeta}}^2-\gamma A_0\right),$$

which implies that

$$\mathcal{CE}_t=A\left[\left(1+r\right)s_t-c_t-\frac{1}{2}\alpha A\omega_{\widehat{\zeta}}^2\right]+A_0.$$

Substituting these expressions into (57) yields:

$$f\left(J\left(\widehat{s}_t\right)\right)=\max_{c_t}\left\{f\left(c_t\right)+\frac{1}{1+\rho}f\left(J\left(\left(1+r\right)\widehat{s}_t-c_t-\frac{1}{2}\alpha A\omega_{\widehat{\zeta}}^2\right)\right)\right\}. \tag{58}$$

The FOC for c_t is thus

$$f'\left(c_t\right)=\frac{A}{1+\rho}f'\left(A\left[\left(1+r\right)s_t-c_t-\frac{1}{2}\alpha A\omega_{\widehat{\zeta}}^2\right]+A_0\right).$$

The Envelop theorem is

$$f'\left(J\left(\widehat{s}_t\right)\right)=\frac{1+r}{1+\rho}f'\left(A\left[\left(1+r\right)s_t-c_t-\frac{1}{2}\alpha A\omega_{\widehat{\zeta}}^2\right]+A_0\right).$$

Combining these two conditions yields

$$c_t=A\widehat{s}_t+A_0-\psi\ln\left(\frac{A}{1+r}\right). \tag{59}$$

Substituting (59) into (58) yields:

$$\begin{aligned}\exp\left(-\frac{1}{\psi}\left(A\widehat{s}_t+A_0\right)\right)=&\exp\left(-\frac{1}{\psi}\left(A\widehat{s}_t+B-\psi\ln\left(\frac{A}{1+r}\right)\right)\right)\\&+\frac{1}{1+\rho}\exp\left(-\frac{1}{\psi}\left[A\left(\left(1+r\right)\widehat{s}_t-\left(A\widehat{s}_t+B-\psi\ln\left(\frac{A}{1+r}\right)\right)-\frac{1}{2}\alpha A\omega_{\widehat{\zeta}}^2\right)+A_0\right]\right).\end{aligned}$$

Matching the $\widehat{s}_t$ terms in the exponential functions, we obtain that $A=r$. Matching the constant coefficient terms yields:

$$A_0 = \frac{\psi}{r}\ln\left(\frac{1+\rho}{1+r}\right) + \psi\ln\left(\frac{r}{1+r}\right) - \frac{1}{2}\alpha r\omega_{\widehat{\zeta}}^2$$

Substituting the expressions for A and A_0 into (59) yields (20) in the main text. The corresponding value function is just (19) in the main text.

Using (13), (20), and (59), we can derive the savings function:

$$\begin{aligned}
d_t^* &= ra_t + y_t - c_t^* \\
&= ra_t + y_t - c_t + (c_t - c_t^*) \\
&= ra_t + y_t - r\left[a_t + \phi y_t + \frac{\phi\phi_0}{r} + \frac{1}{r^2\alpha}\left(\ln\left(\frac{1+\rho}{1+r}\right) - \ln\left(E_t\left[\exp\left(-r\alpha\phi w_{t+1}\right)\right]\right)\right)\right] + \\
&\left\{\begin{array}{c} r\left[a_t + \phi y_t + \phi\frac{\phi_0}{r} + \frac{1}{r^2\alpha}\left(\ln\left(\frac{1+\rho}{1+r}\right) - \ln\left(E_t\left[\exp\left(-r\alpha\phi w_{t+1}\right)\right]\right)\right)\right] \\ -r\left[\widehat{s}_t + \frac{1}{r^2\alpha}\left(\ln\left(\frac{1+\rho}{1+r}\right) - \ln\left(E_t\left[\exp\left(-r\alpha\widehat{\zeta}_{t+1}\right)\right]\right)\right)\right] \end{array}\right\} \\
&= (1-\phi_1)\phi(y_t - \overline{y}) + r(s_t - \widehat{s}_t) + \frac{1}{r\alpha}\left[\ln\left(E_t\left[\exp\left(-r\alpha\widehat{\zeta}_{t+1}\right)\right]\right) - \ln\left(\frac{1+\rho}{1+r}\right)\right].
\end{aligned}$$

To derive the relative volatility of financial wealth (a_t) to labor income (y_t) in general equilibrium, we first rewrite the above saving equation as follows:

$$\Delta a_{t+1}^* = d_t^* = (1-\phi_1)\phi(y_t - \overline{y}) + r^*(s_t - \widehat{s}_t).$$

Taking unconditional variance on both sides yields:

$$\begin{aligned}
\operatorname{var}(d_t^*) &= \operatorname{var}\left((1-\phi_1)\phi(y_t - \overline{y}) + r^*(s_t - \widehat{s}_t)\right) \\
&= \left[\frac{1-\phi_1}{1+\phi_1} + \frac{(1-\theta)r^{*2}}{1-(1-\theta)(1+r^*)^2} + \frac{2r^*(1-\phi_1)(1-\theta)}{1-\phi_1(1-\theta)(1+r^*)}\right]\frac{\omega^2}{(1+r^*-\phi_1)^2},
\end{aligned}$$

which is just (38) in the main text, where we use the expression for $s_t - \widehat{s}_t$ specified in (17). Furthermore, using (16) and (28), it is straightforward to show that the relative volatility of consumption growth to income growth is (37) in the main text.

7.2. Proof of Uniqueness of General Equilibrium in the Benchmark Model

To prove uniqueness, consider the derivative of the aggregate saving function, $D(\theta, r) = \Pi(\theta, r) - \psi\Psi(r)/r$, with respect to r: we have

$$\frac{dD}{dr} = \frac{\psi}{r(1+r)} + \frac{\psi}{r^2}\ln\left(\frac{1+\rho}{1+r}\right)\left[\frac{2(1-\phi_1)}{1+r-\phi_1} + \frac{2r(1-\theta)(1+r)}{1-(1-\theta)(1+r)^2}\right] > 0$$

when θ is fixed, because $\phi_1 < 1$, where the last line is obtained by using the general equilibrium condition, $\frac{1}{2}r\Gamma(\theta,r)\alpha\left(\frac{\sigma}{1+r-\phi_1}\right)^2 + \frac{\psi}{r}\ln\left(\frac{1+r}{1+\rho}\right) = 0$. From this expression, we can see that fixed capacity does not change the equilibrium property of the model because $\frac{2r(1-\theta)(1+r)}{1-(1-\theta)(1+r)^2}$ is always positive.[44]

7.3. Computing the Welfare Loss due to RI

Given that the value function under RI in general equilibrium is

$$\widehat{v}(\widehat{s}_0) = -\frac{\psi(1+r^*)}{r^*}\exp\left(-\frac{r^*}{\psi}\widehat{s}_t\right),$$

we can compute the following partial derivatives:

$$\frac{\partial\widehat{v}(\widehat{s}_0)}{\partial\theta} = \frac{\exp(-r^*\widehat{s}_0/\psi)}{r^{*2}/\psi}\left[1+\frac{r^*(1+r^*)}{\psi}\widehat{s}_0\right]\frac{dr^*}{d\theta} \text{ and } \frac{\partial\widehat{v}(\widehat{s}_0)}{\partial\widehat{s}_0} = (1+r^*)\exp(-r^*\widehat{s}_0/\psi).$$

The marginal welfare cost due to RI can thus be written as:

$$\text{mwc} \equiv \frac{(\partial v(\widehat{s}_0)/\partial\theta)\,\theta}{(\partial v(\widehat{s}_0)/\partial\widehat{s}_0)\,\widehat{s}_0} = \frac{\theta\psi}{r^{*2}}\left[\frac{r^*}{\psi}+\frac{1}{(1+r^*)\,\widehat{s}_0}\right]\frac{dr^*}{d\theta},$$

where we use the facts that in general equilibrium (i.e., $\ln\left(\frac{1+\rho}{1+r}\right) = \ln\left(E_t\left[\exp\left(-r^*\widehat{\zeta}_{t+1}/\psi\right)\right]\right)$), and $dr^*/d\theta$ is given in (32).

In the partial equilibrium setting in which $r^* = r$ is fixed,

$$\widehat{v}(\widehat{s}_t) = -\frac{\psi(1+r)}{r}\exp\left(-\frac{r}{\psi}\widehat{s}_t\right)\exp\left(-\frac{1}{r}\ln\left(\frac{1+\rho}{1+r}\right)+\frac{1}{2}\frac{\theta}{1-(1-\theta)(1+r)^2}\frac{r}{\psi^2}\omega_\zeta^2\right).$$

Note that in partial equilibrium, $\ln\left(\frac{1+\rho}{1+r}\right)$ and $\ln\left(E_t\left[\exp\left(-r\widehat{\zeta}_{t+1}/\psi\right)\right]\right)$ do not cancel out. The marginal welfare cost due to RI in partial equilibrium can thus be written as:

$$\text{mwc} = \frac{\theta\omega_\zeta^2}{2\psi\widehat{s}_0}\frac{(1+r)^2-1}{\left[1-(1-\theta)(1+r)^2\right]^2}.$$

The month dollar loss can be thus written as:

$$\$\,\text{loss}(\theta<1) \equiv \frac{1}{12}r\text{mwc}(1)(1-\theta)\,\widehat{s}_0 = \frac{1}{12}r\left(\frac{\omega_\zeta^2}{2\psi}\right)\left((1+r)^2-1\right)(1-\theta).$$

[44]For the elastic RI case, the derivatives are too complicated to sign because θ itself is a function of r. However, given the plausible parameter values used in this paper, the equilibrium is unique.

References

[1] Aiyagari, S. Rao (1994), "Uninsured Idiosyncratic Risk and Aggregate Saving," *Quarterly Journal of Economics* 109 (3), 658-684.

[2] Alessie, Rob and Annamaria Lusardi (1997), "Consumption, Saving and Habit Formation," *Economics Letters* 55(1), 103-108.

[3] Andrade, Philippe and Le Bihan, Hervé (2013), "Inattentive Professional Forecasters," *Journal of Monetary Economics* 60(8), 967-982.

[4] Angeletos, George-Marios and Laurent-Emmanuel Calvet (2006), "Idiosyncratic Production Risk, Growth and the Business Cycle," *Journal of Monetary Economics* 53(6), 1095-1115.

[5] Athreya, Kartik, Xuan S. Tam, and Eric R. Young (2009), "Unsecured Credit Markets Are Not Insurance Markets," *Journal of Monetary Economics* 56(1), 83-103.

[6] Barro, Robert J. (2007), "On the Welfare Costs of Consumption Uncertainty," manuscript.

[7] Blundell Richard, Luigi Pistaferri, and Ian Preston (2008), "Consumption Inequality and Partial Insurance," *American Economic Review* 98(5), 1887-1921.

[8] Caballero, Ricardo J. (1990), "Consumption Puzzles and Precautionary Savings," *Journal of Monetary Economics* 25, 113-136.

[9] Campbell, John Y. (2003) "Consumption-Based Asset Pricing." In *Handbook of the Economics of Finance Vol. 1B*, edited by George Constantinides, Milton Harris, and Rene Stulz, pp. 803-887. North-Holland Press.

[10] Carroll, Christopher D. (2011), "Theoretical Foundations of Buffer Stock Saving," manuscript.

[11] Cheremukhin, Anton, Popova, Anna, and Tutino, Antonella (2015), "A Theory of Discrete Choice with Information Costs," *Journal of Economic Behavior and Organization* 113(C), 34-50.

[12] Coibion, Olivier and Yuriy Gorodnichenko (2015), "Information Rigidity and the Expectations Formation Process: A Simple Framework and New Facts," *American Economic Review* 105(8), 2644–2678.

[13] Constantinides, George M. (1990), "Habit Formation: A Resolution of the Equity Premium Puzzle," *Journal of Political Economy* 98(3), 519-543.

[14] Crump, Richard K., Stefano Eusepi, Andrea Tambalotti, and Giorgio Topa (2015), "Subjective Intertemporal Substitution," Federal Reserve Bank of New York Staff Reports, Number 734.

[15] Epstein, Larry G. and Stanley E. Zin (1989), "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework," *Econometrica* 57(4), 937-969.

[16] Floden, Martin and Jesper Linde (2001), "Idiosyncratic Risk in the United States and Sweden: Is There a Role for Government Insurance?" *Review of Economic Dynamics* 4, 406-437.

[17] Friedman, Milton (1957), "A Theory of the Consumption Function," *Princeton, NJ: Princeton University Press*.

[18] Guvenen, Fatih (2006), "Reconciling Conflicting Evidence on the Elasticity of Intertemporal Substitution: A Macroeconomic Perspective," *Journal of Monetary Economics* 53(7), 1451-1472.

[19] Guvenen, Fatih and Anthony A. Smith, Jr., (2014), "Inferring Labor Income Risk and Partial Insurance from Economic Choices," *Econometrica* 82(6), 2085-2129.

[20] Hall, Robert E. (1978), "Stochastic Implications of the Life Cycle-Permanent Income Hypothesis: Theory and Evidence," *Journal of Political Economy* 91(6), 249-265.

[21] Hall, Robert E. (1988), "Intertemporal Substitution in Consumption," *Journal of Political Economy* 96(2), 339-57.

[22] Hong, Harrison, Walter Torous, and Rossen Valkanov (2007), "Do Industries Lead Stock Markets?" *Journal of Financial Economics* 83(2), 367-396.

[23] Hryshko, Dmytro (2014), "Correlated Income Shocks and Excess Smoothness of Consumption," *Journal of Economic Dynamics and Control* 48, 41-62.

[24] Huggett, Mark (1993), "The Risk-Free Rate in Heterogeneous-Agent Incomplete-Insurance Economies," *Journal of Economic Dynamics and Control* 17(5-6), 953-969.

[25] Katz, Lawrence F. and David H. Autor (1999), "Changes in the Wage Structure and Earnings Inequality," in O. Ashenfelter and D. Card (eds.), *Handbook of Labor Economics*, vol. 3A, North-Holland, Amsterdam.

[26] Krueger Dirk and Fabrizio Perri (2006), "Does Income Inequality Lead to Consumption Inequality? Evidence and Theory," *Review of Economic Studies* 73(1), 163-193.

[27] Laubach, Thomas and John C. Williams (2015), "Measuring the Natural Rate of Interest Redux," Working Paper 2015-16, Federal Reserve Bank of San Francisco.

[28] Ludvigson, Sydney C. and Alexander Michaelides (2001), "Does Buffer-Stock Saving Explain the Smoothness and Excess Sensitivity of Consumption?" *American Economic Review* 91(3), 631-647.

[29] Luo, Yulei (2008), "Consumption Dynamics under Information Processing Constraints," *Review of Economic Dynamics* 11, 366-385.

[30] Luo, Yulei (2010), "Rational Inattention, Long-Run Consumption Risk, and Portfolio Choice," *Review of Economic Dynamics* 13(4), 843-860.

[31] Luo, Yulei and Eric R. Young (2010), "Risk-sensitive Consumption and Savings under Rational Inattention," *American Economic Journal: Macroeconomics* 2(4), 281-325.

[32] Luo, Yulei and Eric R. Young (2016), "Long-run Consumption Risk and Asset Allocation under Recursive Utility and Rational Inattention," *Journal of Money, Credit, and Banking* 48(2-3), 325-362.

[33] Luo, Yulei, Jun Nie, and Eric R. Young. (2015), "Slow Information Diffusion and the Inertial Behavior of Durable Consumption," *Journal of the European Economic Association* 13(5), 805-840.

[34] Maćkowiak, Bartosz and Mirko Wiederholt. (2009), "Optimal Sticky Prices under Rational Inattention." *American Economic Review*, 99, 769-803.

[35] Maćkowiak, Bartosz and Mirko Wiederholt (2015), "Business Cycle Dynamics under Rational Inattention," *Review of Economic Studies* 82(4): 1502-1532.

[36] Matejka, Filip and Alisdair McKay (2015), "Rational Inattention to Discrete Choices: A New Foundation for the Multinomial Logit Model," *American Economic Review* 105(1), 272-98.

[37] Mondria, Jordi (2010), "Portfolio Choice, Attention Allocation, and Price Comovement," *Journal of Economic Theory* 145(5), 1837–1864.

[38] Pischke, Jorn-Steffen (1995), "Individual Income, Incomplete Information, and Aggregate Consumption," *Econometrica* 63, 805-840.

[39] Shafieepoorfard, Ehsan and Maxim Raginsky (2013), "Rational Inattention in Scalar LQG Control," *Proceedings of the IEEE Conference on Decision and Control* 52, 5733-5739.

[40] Sims, Christopher A. (2003), "Implications of Rational Inattention," *Journal of Monetary Economics* 50 (3), 665-690.

[41] Sims, Christopher A. (2010), "Rational Inattention and Monetary Economics," *Handbook of Monetary Economics*.

[42] Sun, Yeneng (2006), "The Exact Law of Large Numbers via Fubini Extension and Characterization of Insurable Risks," *Journal of Economic Theory* 126, 31-69.

[43] Toda, Alexis Akira (2017), "Huggett Economies with Multiple Stationary Equilibria," manuscript.

[44] Van Nieuwerburgh, Stijn and Laura Veldkamp (2009), "Information Immobility and the Home Bias Puzzle," *Journal of Finance* 64(3), 1187-1215.

[45] Van Nieuwerburgh, Stijn and Laura Veldkamp (2010), "Information Acquisition and Under-Diversification," *Review of Economic Studies* 77(2), 779-805.

[46] Veldkamp, Laura (2011), *Information Choice in Macroeconomics and Finance*, Princeton University Press.

[47] Vissing-Jørgensen, Annette and Orazio P. Attanasio (2003), "Stock-Market Participation, Intertemporal Substitution, and Risk-Aversion," *American Economic Review* 93(2), 383-391.

[48] Wang, Neng (2003), "Caballero Meets Bewley: The Permanent-Income Hypothesis in General Equilibrium," *American Economic Review* 93(3), 927-936.

[49] Wang, Neng (2004), "Precautionary Saving and Partially Observed Income," *Journal of Monetary Economics* 51, 1645–1681.

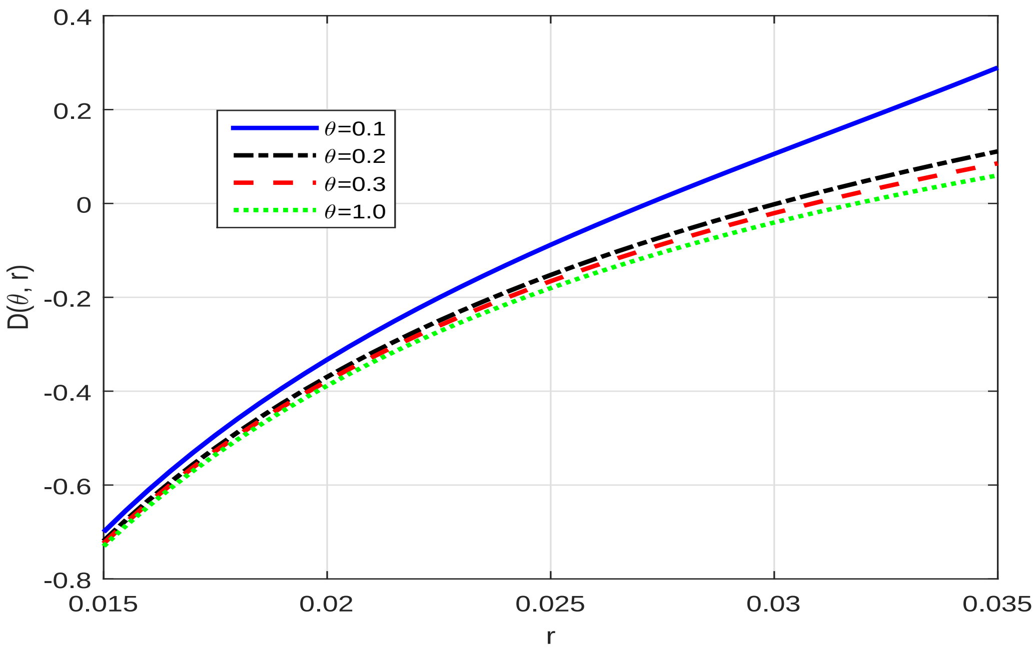

Figure 1. *Effects of RI on Aggregate Saving*

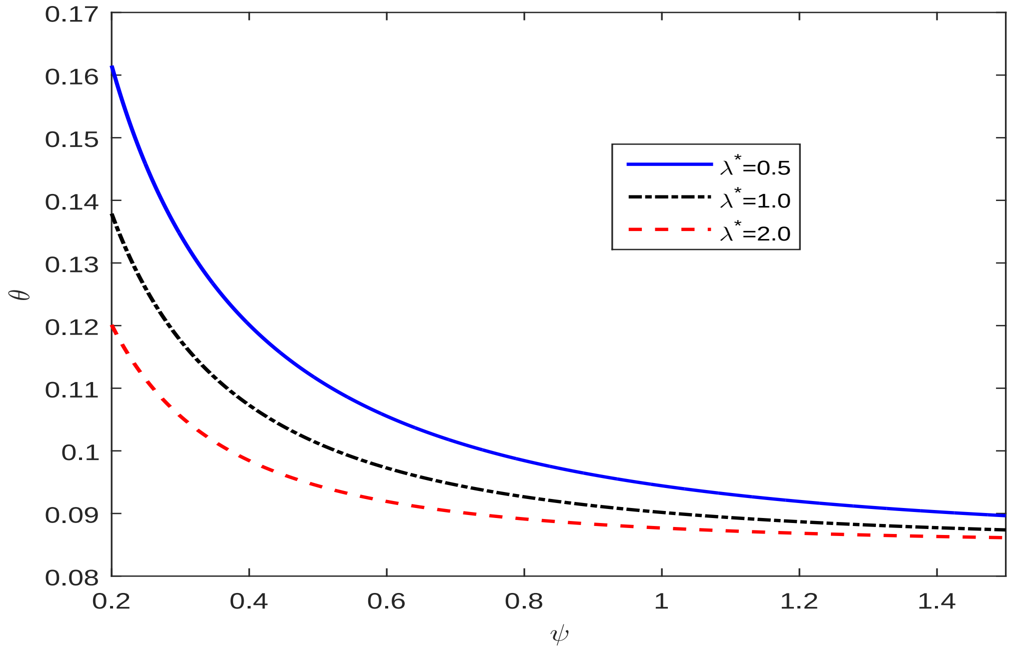

Figure 2. *Effects of ψ on Elastic Attention*

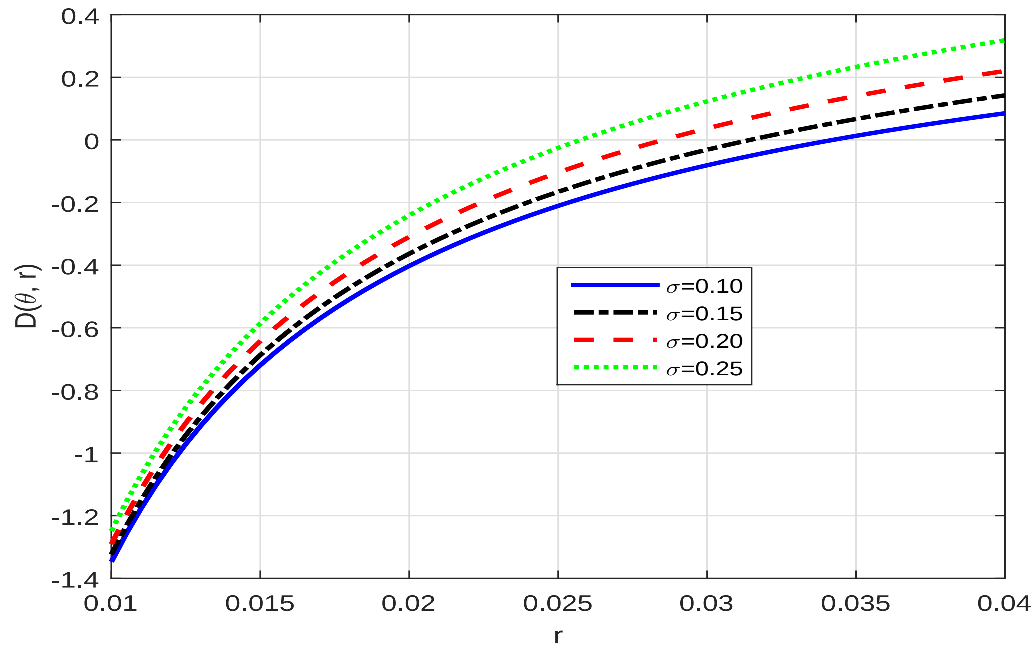

Figure 3. *Effects of Income Volatility on the Interest Rate in GE (Elastic κ)*

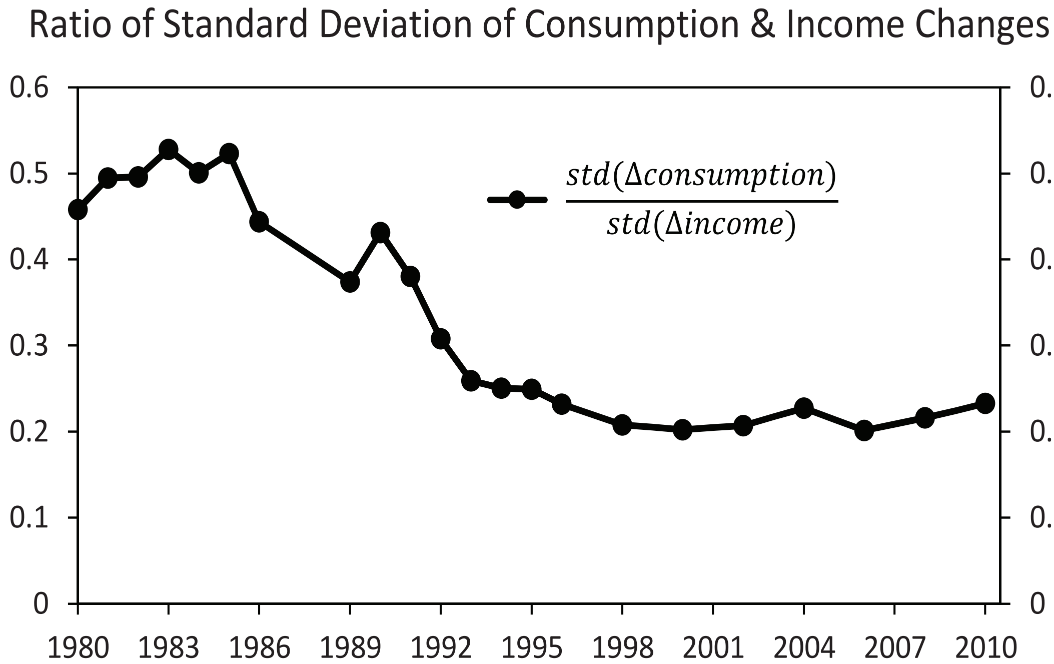

Figure 4. *Relative Consumption Dispersion*

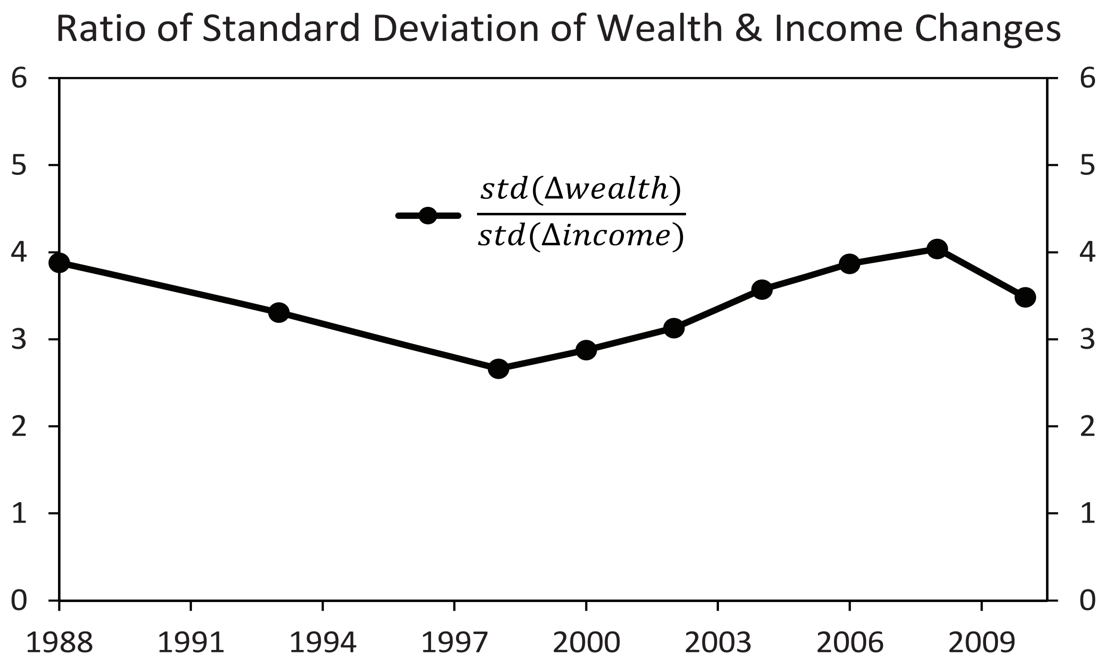

Figure 5. *Relative Dispersion of Wealth to Income*

Table 1. *Estimation of the Income Process*

	std (ϵ_{it})	ϕ_{it}	std (y_{it})
Period 1 (1980 − 1986)	0.154	0.917	0.386
Period 2 (1987 − 1996)	0.175	0.912	0.427
Full Period (1980 − 1996)	0.175	0.919	0.444

Table 2. *Implications of RI for Interest rates, the Relative Volatility of Consumption and Wealth to Income, and Welfare*

	θ	10%	20%	40%	60%	100%
GE ($\psi = 0.54$)	$r^*(\%)$	2.89	3.22	3.35	3.38	3.41
	μ_{cy}	0.375	0.324	0.302	0.296	0.290
	μ_{ay}	2.620	2.208	1.945	1.840	1.748
	$loss	14.64	15.64	12.53	8.51	0
GE ($\psi = 0.8$)	$r^*(\%)$	3.12	3.46	3.57	3.60	3.63
	μ_{cy}	0.416	0.346	0.318	0.309	0.303
	μ_{ay}	2.671	2.203	1.922	1.812	1.716
	$loss	15.78	16.58	13.11	8.87	0
PE ($\psi = 0.54$)	μ_{cy}	0.474	0.342	0.307	0.297	0.290
($r = 3.41\%$)	μ_{ay}	2.751	2.204	1.938	1.837	1.748
	$loss	0.006	0.005	0.004	0.003	0

Table 3. *Model Comparison*

	Data	FI-RE ($\psi = 0.54$)	FI-RE ($\psi = 0.8$)	RI-GE ($\theta = 0.1$, $\psi = 0.54$)	RI-GE ($\theta = 0.1$, $\psi = 0.8$)	RI-PE ($\theta = 0.1$, $\psi = 0.54$)
$r^*(\%)$	2.97	3.41	3.63	2.89	3.12	n.a.
μ_{cy}	0.38	0.29	0.30	0.38	0.42	0.47
μ_{ay}	3.28	1.75	1.72	2.62	2.67	2.75
$loss	n.a.	0	0	14.64	15.78	0.006

Table 4. *Implications of RI (Elastic κ and θ)*

	$\sigma\left(\sigma_y\right)$	0.2 (0.51)	0.3 (0.77)	0.4 (1.02)
GE-RI	θ	11%	14%	15%
($\lambda = 0.38$, $\psi = 0.54$)	$r^*(\%)$	2.79	2.32	1.94
	μ_{cy}	0.34	0.26	0.21
	μ_{ay}	2.53	2.39	2.35
GE-RI	θ	8%	10%	12%
($\lambda = 0.38$, $\psi = 0.8$)	$r^*(\%)$	2.73	2.46	2.14
	μ_{cy}	0.43	0.31	0.25
	μ_{ay}	2.87	2.56	2.46
GE-FI	r^*	3.23	2.62	2.14
($\theta = 1$, $\psi = 0.54$)	μ_{cy}	0.28	0.24	0.20
	μ_{ay}	1.78	1.88	1.97

Table 5. *Implications of Habit Formation and Incomplete Information*

Habit Formation	γ	0.4	0.6	0.8	0.9
GE ($\psi = 0.54$)	$r^*(\%)$	3.85	4.02	4.14	4.18
	μ_{cy}	0.21	0.17	0.12	0.09
	μ_{ay}	1.76	1.83	2.03	2.30

Incomplete Info	τ	0.4	0.6	0.8	0.9
GE ($\psi = 0.54$)	$r^*(\%)$	1.30	3.01	3.89	4.10
	μ_{cy}	0.56	0.36	0.18	0.10
	μ_{ay}	0.37	0.50	0.60	0.64

Table 6. *Relative Consumption Dispersion*

μ_{cy}	By Income Level	(Obs.)	By Wealth Level (with wealth data)	(Obs.)
Entire sample	0.321	(22,370)	0.247	(9,401)
Top 75%	0.328	(16,785)	0.235	(7,067)
Top 50%	0.315	(11,195)	0.212	(4,708)
Top 25%	0.293	(5,600)	0.179	(2,354)