Growth and Welfare Gains from Financial Integration under Model Uncertainty*

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Abstract

We build a robustness (RB) version of the Obstfeld (1994) model to study the effects of financial integration on growth and welfare. Our model can account for, both theoretically and empirically, the observed relationships between growth and volatility in developing and advanced countries. Our model also reconciles the equity premium puzzle with a reasonable risk aversion parameter. Our calibrated model shows that financial integration leads to significantly larger gains in growth and welfare for advanced countries than developing countries. Our analytical solutions help uncover the key mechanisms by which this happens.

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1 Introduction

Obstfeld (1994) proposes a general equilibrium version of the Merton-type model with stochastic production technology that links the financial diversification of an economy (openness) to long-run growth, and then uses his model to measure the welfare gains associated with financial integration. In his model, an economy with an interior portfolio of risky and riskless capital (a diversified equilibrium) will unambiguously generate a positive relationship between the average growth rate and the volatility of real GDP per capita, while an undiversified equilibrium (with only risky capital) can generate a negative relationship provided the intertemporal elasticity of substitution exceeds one.

Recent empirical studies challenge the above predictions. Ramey and Ramey (1995) document a robust negative relationship between the average growth rate of an economy and the volatility of output; this relationship holds after controlling for a number of country-specific factors.[1] More recently, Kose et al. (2006) further show that the growth-volatility relationship is negative in developing countries while it is positive in more advanced countries. Using more recent data over the period $1962 - 2011$ from the Penn World Tables 9.0, we find these relationships continue to hold (see Figures 2 - 4). Furthermore, cross-country holdings of US government debt (essentially a risk-free asset) are large and widespread, indicating that an undiversified equilibrium does not look empirically plausible.[2]

We reconsider Obstfeld's model by introducing a fear of model misspecification, and study how the household's preference for robustness interacts with stochastic production technology and affects optimal consumption-portfolio rules and the equilibrium growth rate. This is largely motivated by recent findings in the literature that introducing model uncertainty helps solve the excess volatility puzzle (Djeutem and Kasa 2013) and the international consumption correlation puzzle (Luo et al. 2014), and the welfare costs due to model uncertainty can be significant. (See, for example, Barillas, Hansen, and Sargent (2009) and Ellison and Sargent (2012)). Our goal in this paper is to characterize whether model uncertainty due to the preference for robustness can help the model explain the different growth-volatility relationships in developing and advanced countries, and to quantify the growth and welfare effects of financial integration under model uncertainty.

Hansen and Sargent (1995) first introduced the preference for robustness (RB) into linear-quadratic-Gaussian (LQG) economic models. In robust control problems, agents are concerned

[1] See Aghion and Banerjee (2005) for a recent survey on volatility and growth.

[2] As we will discuss below, the theoretical prediction on the negative volatility-growth relationship depends on the magnitude of the elasticity of intertemporal substitution (EIS). Specifically, to generate a negative relationship, the value of the EIS in our model need to be greater than one in the undiversified equilibrium. It is worth noting that the evidence on the IES is mixed. (The estimates range so widely that almost any value between 0 and 2 looks empirically reasonable.)

about the possibility that their true model is misspecified in a manner that is difficult to detect statistically; consequently, they choose their decisions as if the subjective distribution over shocks was chosen by an evil agent to minimize their utility. More specifically, under RB, agents have a best estimated reference model (called the approximating model) in their mind and consider a range of models (the distorted models) surrounding the approximating model. As discussed in Hansen, Sargent, and Tallarini (1999) and Luo and Young (2010), RB models can produce precautionary savings even within the class of linear-quadratic-Gaussian (LQG) models, which leads to analytical simplicity. Many recent papers have shown the usefulness of viewing agents as having (potentially) misspecified models of the economy and being aware of this fact; Hansen and Sargent (2007) provide a book-length introduction and discussion of the literature.[3]

A desire for robust decision rules complicates the link between average growth and volatility. If we interpret the model misspecification fears as "entirely in the head" of the agents, then robustness only affects the magnitude of the correlation between volatility and growth; this model therefore cannot replicate the negative relationship between mean growth and volatility, as it becomes observationally equivalent to a standard full-information rational expectations (FI-RE) model with a higher coefficient of risk aversion. If instead we interpret the model-misspecification fears as justified, which means the distorting model governs the true dynamics of the economy, then the cutoff value for the IES to generate the negative relationship between volatility and growth is smaller than one and is decreasing with the degree of concerns about model uncertainty. Under this interpretation we can calibrate the model to capture the observed negative correlation in developing countries. There is no definitive justification for either perspective – Hansen and Sargent (2007) usually adopt the "entirely in the head" perspective, but estimation via an indirect reference approach, which takes into account key features in the data, suggests that the distorted model is an empirically-plausible model of the data. Put another way, once we entertain the idea that agents do not trust their models, there is no "true" model anymore. Our robustness model fits the data on volatility and growth only if the distorted model generates the data.

Our robustness version of Obstfeld's model implies that the growth rate and volatility of real GDP are negatively correlated in the diversified equilibrium if

$$\vartheta > \frac{1+\psi}{1-\psi}\gamma,$$

where ϑ, γ, and ψ are the parameter governing the degree of robustness, the coefficient of relative risk aversion, and the elasticity of intertemporal substitution, respectively. To better quantify the

[3] There are three main ways to model ambiguity and robustness in the literature: the multiple priors model (Gilboa and Schmeidler 1989), the "smooth ambiguity" model (Klibanoff, Marinacci, and Mukerji 2005), and the multiplier utility and robust control/filtering model (Hansen and Sargent 2001). See Epstein and Schneider (2010) for a recent review. In this paper, for tractability, we follow Hansen and Sargent (2007) and use the robust control method to model concerns about model misspecification. Chen and Epstein (2002) and Ju and Miao (2012) examine how ambiguity affects portfolio choices and asset prices.

values of these key parameters, we jointly calibrate these three parameters to match three key moments in the data - output growth, volatility of growth, and the growth-volatility correlation - for developing and advanced countries separately. Our analytical solutions greatly facilitate this calibration process. Our calibration results suggest developing countries face significantly larger model uncertainty. Using the detection error probability (DEP) proposed by Hansen and Sargent (2007), the corresponding DEP for developing and developed countries are 0.3 percent and 17 percent, respectively. Intuitively speaking, this result means the probability that a likelihood-ratio test cannot distinguish competing models in developing countries is 0.3 percent, much lower than the probability in advanced countries. In other words, the agents in developing countries face greater model uncertainty and thus take into account a *larger* set of models than the agents in advanced countries when making consumption-investment decisions.

We then quantitatively evaluate the growth and welfare gains associated with financial integration. In the Obstfeld model, financial integration increases the span of assets available in a given country, leading to a portfolio shift away from the risk-free capital to the risky assets that can hedge local risks. The model not only predicts an increase in growth, since risky assets have higher returns, but also predicts higher volatility as the portfolio share of the risky capital increases. Consistent with this prediction, Schularick and Steger (2010) show that integration can boost growth through higher investment and Eozenou (2008) presents evidence for a positive connection between integration and macroeconomic volatility.

Since the basic model predicts a positive correlation between volatility and growth, it will likely overstate the welfare gains associated with financial integration (see also Epaulard and Pommeret 2005). Other papers also find large gains from international financial integration, even when growth rates are not affected in the long run; see Hoxha, Kalemli-Ozcan, and Vollrath (2013) for an example. Gourinchas and Jeanne (2006) show that the gains are generally smaller in models with exogenous growth, which is a manifestation of the old Lucas observation that the welfare costs of aggregate consumption risk are magnitudes smaller than the welfare costs of low growth.

Using our preferred specification – the calibrated distorted model – we find that growth and welfare gains are also positive, but much smaller than the standard Obstfeld model. In addition, we find the growth and welfare gains are much larger in advanced countries than in developing countries. Our close-form solution for equilibrium growth and welfare make it easy to explain these results. Our closed-form formula shows that financial integration influences growth through two channels. The first channel is the interest rate channel. Our calibrated model shows the equilibrium interest rate increases after the financial integration. This is because the higher demand for risky assets reduces the demand for the risk-free asset and thus pushes up the equilibrium risk-free rate. A higher return to the risk-free asset increases the wealth accumulation pace which, in the balanced path, also increases the output growth. The second channel is the volatility channel.

The volatility is influenced by the share and volatility of the risky assets. As financial integration reduces overall volatility for the international risk-asset combination (i.e., the world-wide mutual fund), it also leads to an increase in the share of the risky assets in both developing and advanced countries. On balance, the higher risky share and lower volatility cause the growth volatility to increase in both developing and advanced countries. Finally, the difference in the growth-volatility relationship plays a key role in explaining the effect of the second channel on growth–the higher growth-volatility relationship increases growth in advanced countries but reduces growth in developing countries. This is why the net effect on growth through the two channels is larger in advanced countries than in developing countries.

Regarding the welfare implications, our formula shows that the welfare improvement due to financial integration is an increasing function of growth and the equilibrium risk-free rate. As the equilibrium growth rate increases more in advanced countries, it is not surprising that the welfare improvement from financial integration is also larger in advanced countries. In our calibration model economy, since we have set the initial risk-free rate to be the same in the two groups of countries, the effect from a higher interest rate plays a similar role in welfare improvement.

This paper is organized as follows. Section 2 presents a robustness version of the Obstfeld-type model with recursive utility in a closed economy and discusses how the presence of robustness can have the potential to generate the observed negative relationship between growth and volatility of the macroeconomy. Section 3 presents the theoretical results on growth and welfare allowing financial integration. Section 4 shows our quantitative analysis. Section 5 concludes.

2 A Risk-Sharing Model with Recursive Utility and Model Uncertainty

2.1 The Model Setting

Following Obstfeld (1994), in this paper we consider a continuous-time risk-sharing model with multiple assets. Specifically, we assume that individuals save by accumulating capital and by making risk-free loans that pay a real return i_t. There are two types of capital: one is risk free with a constant return and one is risky with a stochastic return; households are prevented from shorting either type. The value of the risk free asset (b_t) follows the process

$$\frac{db_t}{dt} = rb_t \tag{1}$$

for some constant $r > 0$. There is a simple stochastic production technology that is linear in risky capital ($k_{e,t}$):

$$dy_t = ak_{e,t}dt + \sigma k_{e,t}dB_t, \tag{2}$$

where dy_t is the instantaneous output flow, $k_{e,t}$ denotes the stock of capital, $a > r$ is the expected technology level, σ is the standard deviation of the production technology, and B_t is a standard Brownian motion defined over the complete probability space. It is worth noting that the AK specification, (2), can be regarded as a reduced form of the following stochastic Cobb-Douglas production function specification when labor is supplied inelastically:

$$dy_t = Ak_{e,t}^{1-\alpha}\left(\overline{k}_{e,t}l\right)^{\alpha}\left(dt + \sigma_y dB_{y,t}\right), \alpha \in (0,1), \tag{3}$$

where $Ak_{e,t}^{1-\alpha}\left(\overline{k}_{e,t}l\right)^{\alpha}$ is the *deterministic* flow of production, $k_{e,t}$ denotes individual firm's stock of capital, $\overline{k}_{e,t}$ is the average economy-wide stock of capital, and $\overline{k}_{e,t}l$ measures the (inelastic) supply of efficiency labor units, σ_y is the standard deviation of the technology innovation, and $B_{y,t}$ is a standard Brownian motion. This production function exhibits constant returns to scale at the individual level. Furthermore, in equilibrium, $k_{e,t} = \overline{k}_{e,t}$ and the stochastic production is linear in capital:

$$dy_t = Al^{\alpha}k_{e,t}\left(dt + \sigma_y dB_{y,t}\right), \alpha \in (0,1), \tag{4}$$

which is just the specification of (2) if we set $a = Al^{\alpha}$ and $\sigma = a\sigma_y$. We assume that the wage rate, w, over $(t, t+dt)$ is determined at the beginning of t and is set to be equal to the expected marginal product of labor,

$$w = E\left[\frac{\partial\left(Ak_{e,t}^{1-\alpha}\left(l\overline{k}_{e,t}\right)^{\alpha}\right)}{\partial l}\right]_{k_{e,t}=\overline{k}_{e,t}} = A\alpha l^{\alpha-1}k_{e,t} = \alpha\mu\frac{k_{e,t}}{l},$$

and the total rate of return to labor during this period is determined by wdt.

In the absence of adjustment costs, the rate of return to the risky capital can be written as

$$r_{e,t} \equiv \frac{dy_t - \varrho k_{e,t} - lwdt}{k_{e,t}} = \mu dt + \sigma dB_t, \tag{5}$$

where ϱ is the depreciation rate and $\mu = (1-\alpha)a - \varrho$ is the expected return of the risky capital. If $i_t < r$, there is no equilibrium because this condition implies an arbitrage profit from issuing loans and investing the proceeds in the risk free asset. If $i_t > r$ there exists an equilibrium with no risk-free assets if and only if there exists a short sale constraint on capital, which we implicitly impose. Finally, when $i_t = r$, the division between the risk free asset and the loan is indeterminate. Consequently, the individuals only need to choose from two assets: the risky capital and a composite safe asset offering a return i_t. Later we will show that the real interest rate (i_t) is constant in equilibrium.[4]

[4]In many ways our model will be similar to Cagetti and DeNardi (2008), in which households can choose to save in the form of riskless corporate capital, riskless bonds, or risky entrepreneurial capital. Our model is also related to Mendoza, Quadrini, and Ríos-Rull (2009), who study the connection between financial openness and gross capital positions in a model with risky assets in multiple countries. Both papers have models that are not amenable to analytical solutions.

The budget constraint for the representative consumer can thus be written as

$$dk_t = \left[\left(i + \alpha_t (\mu - i)\right) k_t - c_t\right] dt + \alpha_t k_t \sigma dB_t, \tag{6}$$

where $k_t = k_{e,t} + b_{f,t}$ is total wealth, $b_{f,t}$ is holdings of the composite safe asset, and α_t is the fraction of wealth invested in risky capital.

Following Campbell and Viceira (2002), and Campbell, Chacko, Rodriguez, and Viceira (henceforth, CCRV 2004), we adopt the Duffie-Epstein (1992) parameterization of recursive utility in continuous-time:

$$J_t = E_t \left[\int_t^\infty f(c_s, J_s)\, ds\right], \tag{7}$$

where J is continuation utility and $f(c, J)$ is a normalized aggregator of current consumption and continuation utility given by

$$f(c, J) = \frac{\delta (1-\gamma) J}{1 - 1/\psi} \left[\left(\frac{c}{\left[(1-\gamma) J\right]^{1/(1-\gamma)}}\right)^{1-1/\psi} - 1\right], \tag{8}$$

where $\delta > 0$ is the discount rate, $\gamma > 0$ is the coefficient of relative risk aversion, and $\psi > 0$ is the elasticity of intertemporal substitution. When $\psi = 1/\gamma$, the above recursive utility reduces to the standard time-separable power utility. As $\psi \to 1$, J converges to

$$f(c, J) = \delta (1-\gamma) J \left[\ln(c) - \frac{1}{1-\gamma} \ln\left((1-\gamma) J\right)\right]. \tag{9}$$

The only difference from the standard additive power utility case is that here we use the aggregator $f(c, J)$ to replace the instantaneous utility function $u(c)$ from the standard expected utility case.

In the full-information case, the Bellman equation is

$$\sup_{c_t, \alpha_t} \left\{ f(c_t, J_t) + \mathcal{D}J(k_t) \right\}, \tag{10}$$

subject to (6), where $f(c_t, J_t)$ is specified in (8) and

$$\mathcal{D}J(k_t) = J_k \left[\left(i + \alpha_t (\mu - i)\right) k_t - c_t\right] + \frac{1}{2} J_{kk} \sigma^2 \alpha_t^2 k_t^2. \tag{11}$$

We can now solve for the consumption and portfolio rules and the expected growth rate. The following proposition summarizes the main results, which are identical to those in Obstfeld (1994).

1 *In the full-information case, the portfolio rule is*

$$\alpha^* = \frac{\mu - i}{\gamma \sigma^2}, \tag{12}$$

the consumption function is

$$c_t^* = \psi \left\{ \delta - \left(1 - \frac{1}{\psi}\right) \left[i + \frac{(\mu - i)^2}{2\gamma\sigma^2}\right] \right\} k_t, \tag{13}$$

the evolution of risky capital is

$$\frac{dk_t}{k_t} = \left[\psi\left(i-\delta\right) + \frac{1}{2}\left(1+\psi\right)\left(\mu-i\right)\alpha^*\right]dt + \alpha^*\sigma dB_t, \tag{14}$$

and the mean and standard deviation of the growth rate are defined as:

$$g \equiv E\left[\frac{dk_t}{k_t}\right]/dt = \psi\left(i-\delta\right) + \frac{1}{2}\left(1+\psi\right)\left(\mu-i\right)\alpha^*, \tag{15}$$

and

$$\Sigma = \alpha^*\sigma, \tag{16}$$

respectively.

Proof. See Appendix 6.1. ∎

From (13), if we define the marginal propensity to consume as

$$m \equiv \psi\left\{\delta - \left(1-\frac{1}{\psi}\right)\left[i + \frac{\left(\mu-i\right)^2}{2\gamma\sigma^2}\right]\right\}, \tag{17}$$

the expected growth rate can be written as

$$g = r_p - m, \tag{18}$$

and the value function is

$$J\left(k\right) = \left(\delta^{-\psi}m\right)^{\frac{1-\gamma}{1-\psi}}\frac{k^{1-\gamma}}{1-\gamma}, \tag{19}$$

where $r_p \equiv i + \alpha\left(\mu-i\right)$ is the return to the market portfolio.

Following Obstfeld (1994), we first consider a closed-economy equilibrium in which the two capital goods can be interchanged in one-to-one ratio and the amount of asset supply can always be adjusted to accommodate the equilibrium asset demand, (12). There are two types of equilibrium: (i) one in which both types of capital are held (*diversified*) and (ii) one in which only risky capital is held (*undiversified*). In the diversified equilibrium, the interest rate i is equal to r and $\alpha^* = \frac{\mu-r}{\gamma\sigma^2} \leq 1$. In the undiversified equilibrium, $\frac{\mu-r}{\gamma\sigma^2} > 1$, which means that the interest rate i will rise above r until the excess supply of the risk free asset is eliminated (that is, until $\frac{\mu-i}{\gamma\sigma^2} = 1$). Therefore, the interest rate will be constant and equal to

$$i = \mu - \gamma\sigma^2 > r. \tag{20}$$

The following proposition summarizes the results about the expected growth rate in the two types of equilibria.[5]

[5] Note that the two types of equilibria do not co-exist; the economy only has one equilibrium for a given set of parameters.

2 *In the diversified equilibrium, the expected growth rate is*

$$g = \psi (r - \delta) + (1 + \psi) \frac{(\mu - r)^2}{2\gamma\sigma^2}. \tag{21}$$

In the undiversified equilibrium the expected growth rate is

$$g = \psi (\mu - \delta) + \frac{1}{2} (1 - \psi) \gamma\sigma^2. \tag{22}$$

Comparing (21) with (22), it is clear that the effects of the volatility of the fundamental shocks (σ^2) on the growth rate (g) are different in the two equilibria. In the diversified equilibrium, it is immediate that

$$\frac{\partial (g)}{\partial (\gamma\sigma^2)} < 0,$$

where $\gamma\sigma^2$ measures total amount of uncertainty (uncertainty about the return on the risky capital) facing the agent. In contrast, in the undiversified equilibrium, the effect of volatility on the growth rate depends on the value of the elasticity of intertemporal substitution. Specifically, a fall in σ^2 increases growth if $\psi > 1$ but lowers growth when $\psi < 1$. The magnitude of the EIS (ψ) is a key issue in macroeconomics and finance. We can find examples in the literature that find values for ψ that range well below 1 (Hall 1988, Campbell 1989) to well above 1 (Vissing-Jørgenson and Attanasio 2003, Gourinchas and Parker 2002). There does not seem to be much consensus here, despite the clear importance of this parameter in growth models (Lucas 1990).[6]

Now we connect GDP volatility to growth. Since the standard deviation of the growth rate of real GDP, Σ, can be written as $\Sigma = \alpha^* \sigma$ and the Sharpe ratio, η, is defined as $\eta \equiv \frac{\mu - r}{\sigma}$, we can link η to Σ:

$$\Sigma = \frac{\eta}{\gamma}. \tag{23}$$

Using these relationships, in the diversified equilibrium, the expected growth rate can be written as

$$g = \psi (r - \delta) + \frac{(1 + \psi) \gamma}{2} \Sigma^2. \tag{24}$$

Note that the relationship between average growth and GDP volatility is unambiguously positive, in direct contrast to the data we examine below. In contrast, using the undiversified equilibrium we get a negative relationship only if $\psi > 1$ (since $\alpha^* = 1$ GDP volatility equals shock volatility).

If we look at the data, it seems to us that an undiversified equilibrium is not reasonable. For example, in the US the stock of government debt has historically hovered around 50 percent of GDP and is currently at close to 100 percent; US debt is generally considered as close to risk-free as any asset and is widely held across the world (see Figure 5, which is taken from US Treasury

[6]Guvenen (2006) finds that stockholders have a higher EIS (around 1.0) than non-stockholders (around 0.1). Crump, Eusepi, Tambalotti, and Topa (2015) find that the EIS is precisely and robustly estimated to be around 0.8 in the general population using the newly released FRBNY Survey of Consumer Expectations (SCE).

data on foreign holdings of US government debt; we eliminate countries that lack data for June 2016). Once we rule out undiversified equilibria, the basic Obstfeld model does not replicate the negative correlation between growth and volatility.

2.2 Introducing RB

To introduce robustness into the above model, we follow Anderson, Hansen, and Sargent (2003) (henceforth, AHS) and Maenhout (2004) and introduce a preference for robustness (RB) by adding an endogenous distortion $\upsilon(k_t)$ to the law of motion of the state variable k_t:

$$dk_t = [(i + \alpha_t(\mu - i))k_t - c_t]\,dt + \alpha_t\sigma k_t(\alpha_t\sigma k_t\upsilon(k_t)\,dt + dB_t). \tag{25}$$

As shown in AHS (2003), the objective $\mathcal{D}J$ defined in (11) plays a crucial role in introducing robustness. $\mathcal{D}J$ can be thought of as $E[dJ]/dt$ and is easily obtained using Itô's lemma. A key insight of AHS (2003) is that this differential expectations operator reflects a particular underlying model for the state variable. The consumer accepts the approximating model, (6), as the best approximating model, but is still concerned that it is misspecified. He therefore wants to consider a range of models (i.e., the distorted model, (25)) surrounding the approximating model when computing the continuation payoff. A preference for robustness is then achieved by having the agent guard against the distorting model that is reasonably close to the approximating model. The drift adjustment $\upsilon(k_t)$ is chosen to minimize the sum of the expected continuation payoff, but adjusted to reflect the additional drift component in (25), and of an entropy penalty:

$$\inf_{\upsilon}\left[\mathcal{D}J(k_t) + \upsilon(k_t)(\alpha_t k_t\sigma)^2 J_k + \frac{1}{2\vartheta(k_t)}(\alpha_t k_t\sigma)^2\upsilon(k_t)^2\right], \tag{26}$$

where the first two terms are the expected continuation payoff when the state variable follows (6), i.e., the alternative model based on drift distortion $\upsilon(k_t)$.[7] $\vartheta(k_t)$ is fixed and state independent in AHS (2003), whereas it is state-dependent in Maenhout (2004). The key reason of replacing fixed ϑ with a state-dependent counterpart $\vartheta(k_t)$ in Maenhout (2004) is to assure the homotheticity (scale invariance) of the decision problem, a property which is required for the model to display balanced growth. As emphasized in AHS (2003) and Maenhout (2004), the last term in the HJB above is due to the agent's preference for robustness and reflects a concern about the quadratic variation in the partial derivative of the value function weighted by the robustness parameter, $\vartheta(k_t)$. The following proposition summarizes the solution.

3 *Under RB, the portfolio rule is*

$$\alpha^* = \frac{\mu - i}{\widetilde{\gamma}\sigma^2}, \tag{27}$$

[7]Note that $\vartheta(k_t) = 0$ here corresponds to the expected utility case.

the consumption function is

$$c_t^* = \psi \left\{ \delta - \left(1 - \frac{1}{\psi}\right) \left[i + \frac{(\mu - i)^2}{2\widetilde{\gamma}\sigma^2} \right] \right\} k_t, \tag{28}$$

the evolution of risky capital for the approximating model and the distorted model are

$$\left(\frac{dk_t}{k_t}\right)^a = \left[\psi (i - \delta) + (1 + \psi) \frac{(\mu - i)^2}{2\widetilde{\gamma}\sigma^2} \right] dt + \alpha\sigma dB_t, \tag{29}$$

and

$$\left(\frac{dk_t}{k_t}\right)^d = \left[\psi (i - \delta) + \left(1 + \psi - \frac{2\vartheta}{\gamma + \vartheta}\right) \frac{(\mu - i)^2}{2\widetilde{\gamma}\sigma^2} \right] dt + \alpha\sigma dB_t, \tag{30}$$

respectively, where the effective coefficient of absolute risk aversion $\widetilde{\gamma}$ *is defined as:* $\widetilde{\gamma} \equiv \gamma + \vartheta$.

Proof. See Appendix 6.3. ■

In the RB economy, we again consider diversified and undiversified equilibria. In the diversified equilibrium, the interest rate i is equal to r and $\alpha^* = \frac{\mu - r}{\widetilde{\gamma}\sigma^2} \leq 1$. In the undiversified equilibrium, $\frac{\mu - r}{\widetilde{\gamma}\sigma^2} > 1$, so the interest rate is given by

$$i = \mu - \widetilde{\gamma}\sigma^2 > r. \tag{31}$$

Since $\widetilde{\gamma} = \gamma + \vartheta > \gamma$, the equilibrium interest rate under RB is lower than that under FI-RE. The intuition is simple: the additional amount of precautionary savings due to robustness drives down the equilibrium interest rate. The proposition summarizes the results about the expected growth rate under RB in the two equilibria.

4 *Under RB, in the diversified equilibrium, the expected growth rate is*

$$g^a = \psi (r - \delta) + \frac{(1 + \psi)\widetilde{\gamma}}{2}\Sigma^2 \tag{32}$$

under the approximating model and

$$g^d = \psi (r - \delta) + \left(1 + \psi - \frac{2\vartheta}{\gamma + \vartheta}\right) \frac{\widetilde{\gamma}}{2}\Sigma^2 \tag{33}$$

under the distorted model. The value function is

$$J(k) = \Omega \frac{k^{1-\gamma}}{1 - \gamma}, \tag{34}$$

where $\Omega = \left(\frac{2\psi\delta + (1 - \psi)(g + r)}{1 + \psi}\right)^{\frac{1-\gamma}{1-\psi}}$ *and* $g = g^a$ *(*g^d*) under the approximating model (the distorted model).*

In the undiversified equilibrium, the expected growth rate is

$$g^a = \psi(\mu - \delta) + \frac{1}{2}(1-\psi)\widetilde{\gamma}\Sigma^2 \tag{35}$$

under the approximating model and

$$g^d = \psi(\mu - \delta) + \frac{1}{2}\left(1 - \psi - \frac{2\vartheta}{\gamma + \vartheta}\right)\widetilde{\gamma}\Sigma^2 \tag{36}$$

under the distorted model. The corresponding value function is

$$J(k) = \Omega\frac{k^{1-\gamma}}{1-\gamma}, \tag{37}$$

where $\Omega = (\mu - g)^{\frac{1-\gamma}{1-\psi}}$ *and* $g = g^a$ (g^d) *under the approximating model (the distorted model).*

Proof. See Appendix 6.3. ■

In the diversified equilibrium, as shown in (32), under the approximating model the growth rate is decreasing with σ^2 and γ for any value of ψ because $\frac{\partial g^a}{\partial(\gamma\sigma^2)} < 0$. Furthermore, we also have

$$\frac{\partial g^a}{\partial\vartheta} < 0.$$

We can see from these results that the stronger the degree of model uncertainty, the more negative the correlation between the volatility of the fundamental shock and economic growth. It is clear from (33) that, under the distorted model, the growth rate is decreasing with σ^2 when $\psi > \frac{\vartheta-\gamma}{\gamma+\vartheta}$, and is increasing with σ^2 when $\psi < \frac{\vartheta-\gamma}{\gamma+\vartheta}$. Furthermore, under the distorted model, we can also conclude that

$$\frac{\partial g^d}{\partial\vartheta} < 0$$

because $\frac{\partial(-\vartheta/(\gamma+\vartheta))}{\partial\vartheta} < 0$ and $\frac{\partial\widetilde{\gamma}}{\partial\vartheta} > 0$. From (34), we can see that the total uncertainty, σ^2, influences the lifetime utility only through its effect on the growth rate, g. Specifically, in the diversified equilibrium, it is straightforward to show that given the initial level of k,

$$\text{sign}\left(\frac{\partial J}{\partial\sigma^2}\right) = \text{sign}\left(\frac{\partial g}{\partial\sigma^2}\right),$$

where $g = g^a$ or g^d, for any value of γ, ψ, and ϑ because $\frac{\partial J}{\partial g} > 0$. Figures 6 and 7 show that both the growth rate and lifetime utility measured by Ω are decreasing with the degree of RB, ϑ under both the approximating and distorted models for given γ and k_0. It is clear from these two figures that the economy would experience much lower economic growth and lower lifetime welfare if it is governed by the distorted model rather than the approximating model. For example, when $\vartheta = 1$, the growth rate is 2.1 percent in the approximating model, while it is only 0.4 percent in the distorted model.

In the undiversified equilibrium, under the approximating model, a fall in σ^2 increases growth when $\psi > 1$ but lowers it when $\psi < 1$. In contrast, under the distorting model, a fall in σ^2 increases growth when $\psi > 1 - \frac{2\vartheta}{\gamma+\vartheta}$ but lowers it when $\psi < 1 - \frac{2\vartheta}{\gamma+\vartheta}$. In other words, the presence of RB weakens the condition on ψ such that economic growth is inversely related to fundamental uncertainty. When the preference for RB is strong enough, a small value of ψ can still guarantee the inverse relationship between growth and volatility. Figure 8 illustrates the inverse relationship between EIS and RB for different values of risk aversion when $1 - \psi - \frac{2\vartheta}{\gamma+\vartheta} = 0$. It clearly shows that the critical value of ψ for generating the negative relationship between volatility and growth decreases with the value of ϑ.

In the undiversified equilibrium, we can see from (37) that the total uncertainty, σ^2, also influences the lifetime utility only through its effect on the growth rate, g. It is straightforward to show that given the initial level of k, when $\gamma > 1$,

$$\operatorname{sign}\left(\frac{\partial J}{\partial \sigma^2}\right) = -\operatorname{sign}\left(\frac{\partial g}{\partial \sigma^2}\right) \text{ when } \psi < 1;$$
$$\operatorname{sign}\left(\frac{\partial J}{\partial \sigma^2}\right) = \operatorname{sign}\left(\frac{\partial g}{\partial \sigma^2}\right) \text{ when } \psi > 1.$$

where $g = g^a$ or g^d, because $\frac{\partial J}{\partial g} > 0$ for any value of ϑ. Figures 9 and 10 plot how the growth rate and lifetime utility vary with the degree of RB in the undiversified equilibrium when $\psi = 0.2$. They clearly show that the growth rate is increasing with ϑ and lifetime utility measured by Ω is decreasing with ϑ under the approximating model, whereas the growth rate is decreasing and lifetime utility is increasing under the distorted model, given γ and k_0. This result is not surprising because RB affects the growth rate via two channels: (1) increasing the effective coefficient of relative risk aversion $\widetilde{\gamma}$ and (2) reducing the $1 - \psi - \frac{2\vartheta}{\gamma+\vartheta}$ term.

The differing effects of ψ under the approximating and distorted models lead us naturally to consider how to view the fears expressed by agents in the model. One possible interpretation is that these fears are unjustified (they are "entirely in the head" of the agents); in this case, which is the usual one applied by Hansen and Sargent (2007), the connection between volatility and growth is unambiguous for diversified economies and is unambiguous for undiversified economies once we know the value of ψ. However, the distorted model cannot be easily dismissed as a description of the world; we show later that calibration of ϑ via detection error probabilities means that agents cannot reject the hypothesis that the distorted model describes the data, so their fears need not be ignored as imaginary.

The following proposition summarizes the relationship between γ and ϑ in the RB model:

5 *In the RB version of the Obstfeld model, the parameters governing risk aversion and uncertainty aversion, γ and ϑ, are observationally equivalent in the sense that they lead to the same growth rate and lifetime utility if the true economy is governed by the approximating model. In* *contrast, the observational equivalence does not hold if the true economy is governed by the distorted model.*

Proof. The proof is straightforward by inspecting Equations (32), (33), (35), and (36). ■

From (27) and (28), it is clear that the RB model with the coefficient of relative risk aversion γ and the degree of RB ϑ and the FI model with $\widetilde{\gamma} = \gamma + \vartheta > \gamma$ are observationally equivalent in the sense that they lead to the same consumption and portfolio rules.[8] However, the two model economies lead to different state transition dynamics when the true economy is governed by the distorted model.

3 International Integration under RB

In this section we extend the above benchmark closed-economy model to a multi-country economy. Following Obstfeld (1994), we now assume that there are N countries, indexed by $j = 1, 2, \cdots, N$, and the representative agent in country j has a coefficient of relative risk aversion γ_j, an elasticity of intertemporal substitution ψ_j, a discount rate δ_j, and a parameter governing the preference for robustness ϑ_j. We will generally suppose that preferences are homogeneous; however, for reasons we outline in the next section, that will imply that ϑ will generally not be the same across countries.[9] To completely understand the mechanisms of the model, we will consider heterogeneity in ϑ directly without necessarily assuming heterogeneity in σ. Here we use ϑ to denote the degree of robustness, and use p to denote the resulting amount of model uncertainty.

In an integrated global equilibrium, there is a single risk-free interest rate i^*. Country j's expected growth rate can be written as

$$g^a = \psi_j \left(i^* - \delta_j\right) + \left(1 + \psi_j\right) \frac{\left(\mu^* - i^*\right)^2}{2\widetilde{\gamma}_j \left(\sigma^*\right)^2}, \tag{38}$$

where $\widetilde{\gamma}_j = \gamma_j + \vartheta_j$, if the economy is governed by the approximating model, and

$$g^d = \psi_j \left(i^* - \delta_j\right) + \left(1 + \psi_j - \frac{2\vartheta_j}{\gamma_j + \vartheta_j}\right) \frac{\left(\mu^* - i^*\right)^2}{2\widetilde{\gamma}_j \left(\sigma^*\right)^2}, \tag{39}$$

if the economy is governed by the distorted model. If there exists risk free capital, we have $i^* = r$; if not, $i^* = \mu^* - \gamma^* \left(\sigma^*\right)^2 > r$.

Following Obstfeld (1994), the welfare gain from financial integration can be calculated as an equivalent variation: by what percentage must financial wealth be increased under financial

[8] Maenhout (2004) obtained the same conclusion in an otherwise standard Merton model.

[9] We have studied the effect of heterogeneous ϑ in our other work, in particular Luo, Nie, and Young (2012, 2014).

autarky to leave the representative household indifferent to financial integration? Using (34), the equivalent variation for country j, Λ_j, can be written as

$$\Lambda_j = \left(\frac{m_j^*}{m_j}\right)^{1/(1-\psi_j)} - 1 = \left(\frac{2\psi_j\delta_j + (1-\psi_j)\left(g_j^* + i^*\right)}{2\psi_j\delta_j + (1-\psi_j)(g_j + i_j)}\right)^{1/(1-\psi_j)} - 1, \tag{40}$$

where m_j and m_j^* are the marginal propensity to consume before and after financial integration.

From the expression of the welfare gain, Λ_j, we can see that both an increase in growth and an increase in risk-free interest rate contribute to the welfare gain. In addition, the growth improvement can be different in an economy governed by the approximating model and an economy governed by the distorted model. Depending on the original growth rate before the financial integration and how much growth is improved after the financial integration, an economy could experience larger welfare gains under the distorted model.

In general, we can prove that if the economy remains in a diversified equilibrium after integration, the welfare gain is larger if the economy is governed by the approximating model than if it is governed by the distorted model.

6 *Let $\psi < 1$. The welfare gain from financial integration, measured by Λ_j, declines with the degree of robustness in the diversified equilibrium. In addition, for the same country, the welfare gain from financial integration under the distorted model is lower than that under the approximating model.*

Proof. We first prove that the welfare improvement after financial integration for country j, Λ_j, is a decreasing function of the degree of robustness, ϑ, under the diversified equilibrium:

$$\begin{aligned}\Lambda_j &= \left(\frac{2\psi_j\delta_j + (1-\psi_j)\left(g_j^* + i^*\right)}{2\psi_j\delta_j + (1-\psi_j)(g_j + i_j)}\right)^{1/(1-\psi_j)} - 1 \\ &\equiv \left(\frac{\chi + g_j^* + r}{\chi + g_j + r}\right)^{1/(1-\psi_j)} - 1,\end{aligned} \tag{41}$$

where $\chi \equiv \frac{2\psi_j\delta_j}{1-\psi_j}$, and we have used the fact that $i = r$ in the diversified equilibrium.

For convenience, we drop all subscripts in the equations for economic growth. In the diversified equilibrium, growth rates under the approximating model and the distorted model are given by

$$g^a = \psi(r-\delta) + \frac{(1+\psi)\widetilde{\gamma}}{2}\Sigma^2 \tag{42}$$

and

$$g^d = \psi(r-\delta) + \left(1 + \psi - \frac{2\vartheta}{\gamma+\vartheta}\right)\frac{\widetilde{\gamma}}{2}\Sigma^2. \tag{43}$$

Using the expression for Σ, we can rewrite these equations as

$$g = p + q^i(\vartheta)\frac{(\mu - r)^2}{\sigma^2}, (i = a, d) \tag{44}$$

where $p = \psi(r - \delta)$, $q^a(\vartheta) = \frac{(1+\psi)}{2(\gamma+\vartheta)}$, and $q^d(\vartheta) = \left(1 + \psi - \frac{2\vartheta}{\gamma+\vartheta}\right)\frac{1}{2(\gamma+\vartheta)}$.

Notice that the effect of the financial integration is to change country-specific σ and μ to the equilibrium σ^* and μ^*, which is independent of the degree of robustness ϑ^j in country j. Define $n = \frac{(\mu-r)^2}{\sigma^2}$ and $n^* = \frac{(\mu^*-r)^2}{\sigma^{*2}} \equiv h \cdot n$. Without loss of generality, we assume $h > 1$ which means growth rises after financial integration. Then we can rewrite growth before financial integration as

$$g = p + q^i(\vartheta)n$$

and growth after financial integration as

$$g = p + q^i(\vartheta)hn.$$

Substituting the above expressions into (41), we get

$$\begin{aligned}\Lambda_j &= \left(\frac{\chi + p + q(\vartheta)nh + r}{\chi + p + q(\vartheta)n + r}\right)^{1/(1-\psi_j)} - 1 \\ &\equiv \left(h - \frac{(h-1)s}{s + q(\vartheta)n}\right)^{\nu} - 1\end{aligned} \tag{45}$$

where $s \equiv \chi + p + r$, $\nu = 1/(1 - \psi_j)$, and therefore we have

$$\frac{\partial \Lambda_j}{\partial \vartheta} = (\nu - 1)\left(h - \frac{(h-1)s}{s + q(\vartheta)n}\right)^{\nu-1}\frac{(h-1)sn}{(s + q(\vartheta)n)^2}q'(\vartheta). \tag{46}$$

It is easy to see $q'(\vartheta) < 0$. In addition, if $\psi_j < 1$, $\nu > 1$. Thus, $\frac{\partial \Lambda_j}{\partial \vartheta} < 0$.

Similarly, we have

$$\frac{\partial \Lambda_j}{\partial q} = (\nu - 1)\left(h - \frac{(h-1)s}{s + qn}\right)^{\nu-1}\frac{(h-1)sn}{(s + qn)^2} > 0. \tag{47}$$

As $q^a > q^d$, we have $\Lambda_j^a > \Lambda_j^d$. ■

However, if the economy switches from a diversified equilibrium to an undiversified equilibrium after financial integration, the relative size of the welfare gain depends on the actual change in growth rates.

4 Quantitative Analysis

In this section we calibrate the model taking into account the key facts we want to explain. In particular, we first document the stylized facts on the correlation between economic growth and growth volatility across countries. Using more recent data we confirm previous studies' findings that this relationship is negative in developing countries but positive in developed countries.[10] We then utilize the analytical solution in our model to jointly calibrate the key parameters based on the explicit relationships derived from the model.

Regarding the key parameter on the degree of robustness, as a double check, we also follow the literature to calculate the corresponding detection error of probability (DEP) at the parameter values we calibrate for developing and advanced countries, respectively. As DEP provides an intuitive explanation on the amount of model uncertainly, our approach is not only consistent with the cross-country evidence (similar to the indirect reference approach) but also provides an alternative explanation based on a statistical tool.

Section 4.1 presents empirical evidence on the correlation between growth and volatility in developing and advanced countries. Section 4.2 discusses how to jointly calibrate the key parameters. Section 4.3 quantitatively computes and compares growth and welfare gains due to financial integration between developing and developed countries.

4.1 Empirical Evidence

In this section we present evidence on the relationships between growth and volatility in developing countries and advanced countries. We use both simple scatterplots and a regression analysis controlling the across-country heterogeneity to show these relationships.[11]

The data we use come from the Penn World Tables (version 8.0), which contains national accounts data on a wide set of countries that has been chained and converted to $USD. This conversion allows us to compare GDP levels between countries as well as across time. We construct a sample that is as close as possible to Ramey and Ramey (1995) but extend the time horizon to cover more recent years. In particular, our sample consists of 80 countries and covers the $1962 - 2011$ period.[12] The list of countries is reported in Figure 1. Figures 2 and 3 provide a graphical view on the relationships between GDP growth and its volatility by plotting the mean real per-capita GDP growth rate in the $1962 - 2011$ period for each country against its standard deviation. Figure 2 shows a negative relationship between GDP growth and its volatility for developing countries, while Figure 3 shows a positive relationship for advanced countries. This finding, using more recent data, confirms the finding by Kose et al. (2006). If we pool all countries'

[10] The whole sample shows a negative correlation, similar to Ramey and Ramey (1995) which use early data.

[11] We are not the first to study this relationship: other papers include Hnatkovska and Loayza (2003), Imbs (2006), and Miranda-Pinto (2016).

[12] There are 167 countries in the Penn World Tables dataset, 134 of which contain all of the variables used in the analysis (most countries excluded were missing the human capital variable). Of these 134 countries, 98 contain all variables of interest for the full sample period of 1962-2011. An additional 18 countries were removed since they were not contained in the Ramey and Ramey (1995) analysis. This procedure leaves us with a sample of 80 countries with observations from 1962 to 2011, yielding a total of 4000 observations.

data together and plot the data in one figure, as shown in Figure 4, it does suggests a negative correlation between growth and volatility, which is consistent with the finding in Ramey and Ramey (1995). To sum up, using data covering a longer horizon, our simple scatter plot charts confirm the findings in the previous literature.

However, this simple correlation may be biased due to the heterogeneity across countries. To provide a more accurate measure of the volatility and to isolate the connection to growth, we follow Ramey and Ramey (1995) by controlling for country-specific effects. Our control variables include mean investment share of GDP for each country over the sample period, real per-capita GDP in 1962 (logged), and a human capital index, all data which is included in the Penn World Tables. We conduct a two-stage regression. In the first stage, we regress real per-capita GDP growth on the set of control variables and compute the residuals, which represent the GDP growth uncorrelated with our explanatory series. We compute the standard deviation of these residuals for each country to provide a measure of the volatility of unexplained growth. In the second stage, we put the measured volatility back into the original regression with the control variables for the second stage of the regression. Motivated by the above evidence that developing countries and advanced countries show different relationships between growth and volatility, we include a dummy variable to capture the difference. As seen in Table 1, the volatility of GDP growth has a significant negative effect on per-capita real GDP growth for developing countries.[13] In particular, a 1 percentage point increase in the standard deviation of growth leads to a 0.085 percentage point *decline* in growth for developing countries. In contrast, for advanced countries, a 1 percentage point increase in the standard deviation of growth leads to a 0.077 (0.152 minus 0.085) percentage point *increase* in growth.

In addition, our control variables are largely irrelevant – only investment's share of GDP is correlated with growth in any important way (initial GDP is significant but clearly not quantitatively important). Consistent with some AK-style endogenous growth models (such as Rebelo 1991), we do find that high investment countries grow faster. See McGrattan (1998) for a discussion of how this observation provides support for AK-style endogenous growth models; see also Farmer and Lahiri (2006).

As we noted above, from Equation (24) it is clear that the standard full-information Obstfeld model cannot generate the negative relationship between the volatility and mean growth rate of real GDP per capita we observed in the data unless the economy is in an undiversified equilibrium with an EIS larger than 1. As discussed before, there is some uncertainty regarding the value of the EIS, so it is not clear that the model is a good instrument for measuring the welfare gains from diversification as they depend critically on the relationship between growth and volatility. In the next subsections, we will explore how and to what extent introducing RB can help make

[13] See also Imbs (2006), Dabušinskas, Kulikov, and Randveer (2012), Hnatkovska and Loayza (2004), and Miranda-Pinto (2016) for alternative approaches that also find a negative correlation between growth and volatility.

the model fit the data better in this aspect.

4.2 Calibrating the Key Parameters

To quantitatively study the changes in growth and welfare associated with financial integration under model uncertainty, we need to use reasonable values of the model parameters. We divide parameters into two groups to estimate and calibrate separately. In the first group, we calibrate each of them to a particular moment or take a value from the existing literature. In the second group, we jointly calibrate them so that the model-predicted moments can match the empirical counterparts.

Our first group of parameters consists of 4 parameters: the return to risk-free capital, r, the return to risky capital, μ, the utility discount rate, δ, and the standard deviation of the risky-capital return, σ. We use the same value of r as in Wang et al (2016). We choose μ to match an equity premium of 6%. We set the value of δ based on the survey paper Zhuang et al (2007). We take values of σ from Table 3 in Obstfeld (1994). In particular, we take the average of North America and North Europe as the value for advanced countries, and the average of other regions as the value for developing countries. These results are reported in Table 3.

The second group of parameters consists of 3 parameters: the robustness parameter (ϑ), the EIS parameter (ψ), and the risk aversion parameter (γ). These three parameters are more difficult to pin down separately because they do not have direct targets to match individually. Therefore, we conduct a joint estimation by choosing their values so the model-predicted output growth, standard deviation of growth, and the elasticity of growth to the variance of growth –the three core moments– match their empirical counterparts. Notice that the elasticity of growth to variance of growth is the regression coefficients reported in Table 2. The idea of matching the estimated elasticity is in line with the indirect reference approach. It also takes the advantage of our analytical solutions which deliver an explicit expression of growth as a function of growth volatility. The calibration results are reported in Table 4 which show model fits the key moments in the data perfectly. Comparing the parameters in the two groups of countries, the EIS is similar in two groups of countries; the degree of risk aversion is stronger in advanced countries; and the largest difference lies in the value of the robustness parameter.

The values of the RB parameter, ϑ, do not have an intuitive explanation without further guidance. For example, even though we can see ϑ is larger in developing countries than in advanced countries, what does that mean? How can we link the difference in parameter values in the two group of countries to the difference in the amount of model uncertainty in the two groups? To answer these questions and to better interpret the key parameter, we adopt the procedure outlined in HSW (2002) and AHS (2003) to calculate the the detection error probability (DEP) associated with the value of the RB parameter (ϑ). The technical details are listed in Appendix 6.3. The intuition is as follows. The DEP is a statistic concept to measure the difference between the two models (in our case, the approximating model and the distorted model). It tells us the probability that a likelihood ratio test can not distinguish one model from the other model. In other words, a larger DEP means it is more difficult to distinguish two models. This also means the distance between the two models is smaller, or the agent is taking into account a smaller range of model when making decisions. Similarly, a smaller DEP means the agent is taking into account a larger range of models in making decisions, or the agent has stronger preference for robustness.

Following this procedure, the corresponding DEP for developing countries is 0.3%, much lower than the DEP for advanced countries which is 17%. Given the negative relationship between ϑ and DEP, this suggests the amount of model uncertainty in developing countries is significantly greater than in advanced countries. In other words, to explain the data (growth, volatility, and the correlation), we need greater model uncertainty in developing countries.

4.3 Quantifying Growth and Welfare Gains

Using the estimated parameters for developing and advanced countries, we can quantitatively explore the effects of financial integration on growth and welfare under model uncertainty. The results are reported in Table 5. As the top panel of the table shows, both developing and advanced countries experience an increase in economic growth after financial integration, with the improvement in advanced countries much larger than in developing countries. In particular, advanced countries' growth increases from 2.36 percent to 3.37 percent, representing a 42.8 percent increase, compared to the 5.5 percent increase in developing countries.

With our analytical solutions, we can explicitly show why the growth improvement in advanced countries is larger than in developing countries. Recall our derived growth formula (39) which shows growth is determined by two terms. The first term on the right hand side captures the effect of changes in the equilibrium risk-free rate, i^*, on growth, while the second term highlights the effects of changes in the volatility of the risky-capital return, σ^*, on growth. The bottom panel of Table 5 show that the equilibrium risk-free rate increases slightly after financial integration, from 3.5 percent to 3.7 percent. In addition, as the fourth panel of the table shows, the volatility of the risky-capital return declines in both groups of countries, but the decline in volatility for advanced countries (from 0.148 to 0.065) is much larger than developing countries (which only declines from 0.0723 to 0.065). According to (39), the increase in the risk-free rate pushes up growth for both developing and advanced countries. However, the decline in volatility has different effects on growth for the two groups of countries. The decline in volatility raises growth for advanced countries because the coefficient on the volatility term is calibrated to be positive to match the data. On the other hand, the decline in volatility reduces growth for developing countries as the coefficient is negative which is also guided by the data. This is why the growth improvement is much smaller in developing countries. In addition, the larger decline in volatility also helps amplify the increase in growth for advanced countries.

Next, a comparison in welfare improvement in the two groups shows a similar pattern, as shown in the second panel in Table 5. The improvement in welfare is 60 percent in advanced countries, much higher than the improvement in developing countries which is 15.6 percent. It is easy to understand this difference by using our formula (40) which shows both an increase in growth (g^*) and the risk-free rate (i^*) can enhance the welfare improvement. As advanced countries experience a larger growth improvement, it is not surprising their welfare improvement is also larger than developing countries.

Financial integration also leads to different changes in portfolio choices in developing and advanced countries. As the third panel in Table 5 shows, advanced countries' share of risky assets increases from 0.2 to 1, while developing countries' share increases from 0.69 to 0.83. The larger increase in advanced countries' share in risky assets is due to their larger drop in volatility in the risky-capital return (the fourth panel). After financial integration, the international risky asset is a combination of the two types of risky capital in developing and advanced countries.[14] The optimal weight in this combination depends on the volatility of each risky capital. Because the risky capital return is more volatile in advanced countries than in developing countries, the optimal weight in the combined risk assets after financial integration shows a smaller share in advanced countries' risky capital. This explains why the volatility of risky capital declines after financial integration.

Finally, it is worth noting that in order to focus on the key channels through which robustness influences growth we set the return to the risk-free capital (r), the return to the risky capital (μ), and the discount rate (δ) to be the same for developing and developed countries. This greatly simplifies the analysis so we can better explain the key channel through which RB affects growth and welfare. If we allow these parameters to be different in two groups of countries, then as the formula (39) and (40) show, $i^{ast} - \delta$ and $\mu^* - i^*$ could become different in two groups of countries. These changes may slightly change our quantitative results but will not alter the key mechanisms we highlighted above.

5 Conclusion

The relationship between average growth and average volatility is negative in developing countries but positive in advanced countries. We show introducing model uncertainty due to a preference for robustness into an otherwise standard Obstfeld (1994) model helps explain the negative relationship between the growth rate and the volatility of real GDP; in contrast, the basic model cannot replicate this relationship. Our calibrated model shows advanced countries benefit more than developing countries in financial integration, and the difference in the growth-volatility re-

[14] The weights of risky assets in the international risky asset combination is 0.81 and 0.19 for developing and advanced countries, respectively.

lationship is the key to explaining this result. Our model could be extended to include other sources of risk, such as fiscal policy (Eaton 1981), and then to assess which countries gain from integration and how that is related to policy choices.

6 Appendix

6.1 Solving the Two-Asset Case in the Full-information Case

In the full-information case, the FOCs for consumption and portfolio choice are

$$c_t = \delta^{\psi} J_k^{-\psi} \left[(1-\gamma) J\right]^{\frac{1-\gamma\psi}{1-\gamma}}, \tag{48}$$

$$\alpha_t = -\frac{J_k (\mu - i)}{J_{kk} k_t \sigma_k^2}, \tag{49}$$

respectively. Substituting these FOCs into the Bellman equation,

$$0 = \frac{\delta (1-\gamma) J}{1-1/\psi} \left[\left(\frac{c}{\left[(1-\gamma) J\right]^{1/(1-\gamma)}} \right)^{1-1/\psi} - 1 \right] + J_k r k_t - J_k c_t + J_k \alpha_t (\mu - i) k_t + \frac{1}{2} J_{kk} \alpha_t^2 k_t^2 \sigma^2,$$

yields the following ODE:

$$0 = \frac{\delta (1-\gamma) J}{1-1/\psi} \left[J_k^{1-\psi} \left[(1-\gamma) J\right]^{\frac{\gamma(1-\psi)}{1-\gamma}} \delta^{\psi-1} - 1 \right] + J_k r k_t - J_k J_k^{-\psi} \left[(1-\gamma) J\right]^{\frac{1-\gamma\psi}{1-\gamma}} \delta^{\psi} +$$
$$J_k \left(-\frac{J_k (\mu - i)}{J_{kk} k_t \sigma^2} \right) (\mu - i) k_t + \frac{1}{2} J_{kk} \left(-\frac{J_k (\mu - i)}{J_{kk} k_t \sigma^2} \right)^2 k_t^2 \sigma^2.$$

Conjecture that the value function is $J(k_t) = \frac{A k_t^{1-\gamma}}{1-\gamma}$ for some constant A. Dividing by J_k and k_t on both sides of the above ODE yields

$$0 = \frac{\delta}{1-1/\psi} \left(A^{\frac{1-\psi}{1-\gamma}} \delta^{\psi-1} - 1 \right) + \left(r - A^{\frac{1-\psi}{1-\gamma}} \delta^{\psi} \right) + \frac{(\mu - i)^2}{\gamma \sigma^2} - \frac{1}{2} \gamma \left(\frac{\mu - i}{\gamma \sigma^2} \right)^2 \sigma^2,$$

which implies that

$$A = \left\{ \delta^{-\psi} \ (1-\psi) \left[r - \frac{\delta}{1-1/\psi} + \frac{(\mu - r)^2}{2\gamma\sigma^2} \right] \right\}^{\frac{1-\gamma}{1-\psi}}.$$

Substituting it back to into $c_t = \delta^{\psi} J_k^{-\psi} \left[(1-\gamma) J\right]^{\frac{1-\gamma\psi}{1-\gamma}}$ yields the consumption function, (13), in the main text. Substituting the consumption function into the resource constraint gives (14) in the main text.

6.2 Solving the Two-Asset Case in the RB Case

Solving first for the infimization part of (26) yields:

$$\upsilon^*(k_t) = -\vartheta(k_t) J_k.$$

Substituting for $\upsilon^*(k_t)$ in the robust HJB equation gives:

$$\sup_{c_t,\alpha_t}\left\{\begin{array}{c}\frac{\delta(1-\gamma)J}{1-1/\psi}\left[\left(\frac{c}{[(1-\gamma)J]^{1/(1-\gamma)}}\right)^{1-1/\psi}-1\right]+\\ J_k\left[(r+\alpha_t(\mu-i))k_t-c_t\right]+\frac{1}{2}J_{kk}\alpha_t^2k_t^2\sigma^2-\frac{1}{2}\vartheta(k_t)(\alpha_t k_t\sigma)^2J_k^2\end{array}\right\} \tag{50}$$

From (50), the FOCs for consumption and portfolio choice are:

$$c_t = \delta^{\psi} J_k^{-\psi}\left[(1-\gamma)J\right]^{\frac{1-\gamma\psi}{1-\gamma}}, \tag{51}$$

$$\alpha_t = -\frac{J_k(\mu-i)}{J_{kk}k_t\sigma^2-\vartheta(k_t)k_t\sigma^2J_k^2}, \tag{52}$$

respectively.

Substituting these FOCs into the Bellman equation yields the following ODE:

$$0=\frac{\delta(1-\gamma)J}{1-1/\psi}\left[\left(\frac{\delta^{\psi}J_k^{-\psi}\left[(1-\gamma)J\right]^{\frac{1-\gamma\psi}{1-\gamma}}}{\left[(1-\gamma)J\right]^{1/(1-\gamma)}}\right)^{1-1/\psi}-1\right]+J_krk_t-J_kc_t-J_k\left(\frac{J_k(\mu-i)}{J_{kk}k_t\sigma^2-\vartheta(k_t)k_t\sigma^2J_k^2}\right)(\mu-r)k_t$$
$$\frac{1}{2}J_{kk}\left(-\frac{J_k(\mu-i)}{J_{kk}k_t\sigma^2-\vartheta(k_t)k_t\sigma^2J_k^2}\right)^2k_t^2\sigma_k^2-\frac{1}{2}\vartheta\left[\frac{\mu-i}{(\gamma+\vartheta)\sigma^2}\right]^2k_t\sigma^2J_k,$$

where we assume that $\vartheta(k_t)=\frac{\vartheta}{(1-\gamma)J(k_t)}>0$. Conjecture that the value function is $J(k_t)=\frac{Ak_t^{1-\gamma}}{1-\gamma}$. Divided by J_k and k_t on both sides of the above ODE yields:

$$0=\frac{\delta}{1-1/\psi}\left(A^{\frac{1-\psi}{1-\gamma}}\delta^{\psi-1}-1\right)+\left(i-A^{\frac{1-\psi}{1-\gamma}}\delta^{\psi}\right)+\frac{(\mu-i)^2}{(\gamma+\vartheta)\sigma^2}-\frac{1}{2}(\gamma+\vartheta)\left[\frac{\mu-i}{(\gamma+\vartheta)\sigma^2}\right]^2\sigma^2,$$

which implies that

$$A^{\frac{1-\psi}{1-\gamma}}=\left\{\delta^{-\psi}\ (1-\psi)\left[i-\frac{\delta}{1-1/\psi}+\frac{1}{2}\frac{(\mu-i)^2}{\widetilde{\gamma}\sigma^2}\right]\right\}^{\frac{1-\gamma}{1-\psi}}.$$

Substituting it back to into $c_t=\delta^{\psi}J_k^{-\psi}\left[(1-\gamma)J\right]^{\frac{1-\gamma\psi}{1-\gamma}}$ yields the consumption function, (28), in the main text.

Under the approximating model, substituting the consumption function, (28), into the resource constraint gives the following expression for the expected growth rate:

$$g=\psi(i-\delta)+(1+\psi)\frac{(\mu-i)^2}{2\widetilde{\gamma}\sigma^2}.$$

In contrast, under the distorted model, substituting (28) into the following evolution equation,

$$dk_t = \left[(\alpha (\mu - i) + r) k_t - c_t + (\alpha_t k_t \sigma)^2 \upsilon (k_t) \right] dt + \sigma \alpha k_t dB_t,$$

we have

$$\begin{aligned} \frac{dk_t}{k_t} &= \left\{ \alpha (\mu - i) + r - (1 - \psi) \left[i - \frac{\delta}{1 - 1/\psi} + \frac{1}{2} \frac{(\mu - i)^2}{\widetilde{\gamma} \sigma^2} \right] - \alpha_t^2 k_t^2 \sigma^2 \frac{\vartheta}{(1 - \gamma) J} J_k \right\} dt + \alpha \sigma dB_t \\ &= \left\{ \frac{(\mu - i)^2}{\widetilde{\gamma} \sigma^2} + r - (1 - \psi) \left[i - \frac{\delta}{1 - 1/\psi} + \frac{1}{2} \frac{(\mu - i)^2}{\widetilde{\gamma} \sigma^2} \right] - \alpha_t^2 \sigma^2 \vartheta \right\} dt + \alpha \sigma dB_t \\ &= \left[\psi (i - \delta) + \left(1 + \psi - \frac{2\vartheta}{\widetilde{\gamma}} \right) \frac{(\mu - i)^2}{2 \widetilde{\gamma} \sigma^2} \right] dt + \alpha \sigma dB_t \end{aligned}$$

6.3 Calculating the DEP

The model detection error probability denoted by p is a measure of how far the distorted model can deviate from the approximating model without being discarded; low values for this probability mean that agents are unwilling to discard many models, implying that the cloud of models surrounding the approximating model is large. In this case, it is easier for the consumer to distinguish the two models. The value of p is determined by the following procedure. Let model P denote the approximating model, (29):

$$\left(\frac{dk_t}{k_t} \right)^a = \left[\psi (i - \delta) + (1 + \psi) \frac{(\mu - i)^2}{2 \widetilde{\gamma} \sigma^2} \right] dt + \alpha \sigma dB_t,$$

and model Q be the distorted model, (25):

$$\left(\frac{dk_t}{k_t} \right)^d = \left[\psi (i - \delta) + \left(1 + \psi - \frac{2\vartheta}{\gamma + \vartheta} \right) \frac{(\mu - i)^2}{2 \widetilde{\gamma} \sigma_k^2} \right] dt + \alpha \sigma dB_t,$$

Define p_P as

$$p_P = \text{Prob} \left(\ln \left(\frac{L_Q}{L_P} \right) > 0 \middle| P \right), \tag{53}$$

where $\ln \left(\frac{L_Q}{L_P} \right)$ is the log-likelihood ratio. When model P generates the data, p_P measures the probability that a likelihood ratio test selects model Q. In this case, we call p_P the probability of the model detection error. Similarly, when model Q generates the data, we can define p_Q as

$$p_Q = \text{Prob} \left(\ln \left(\frac{L_P}{L_Q} \right) > 0 \middle| Q \right). \tag{54}$$

Given initial priors of 0.5 on each model and that the length of the sample is N, the detection error probability, p, can be written as:

$$p(\vartheta; N) = \frac{1}{2} (p_P + p_Q), \tag{55}$$

where ϑ is the robustness parameter used to generate model Q. Given this definition, we can see that $1-p$ measures the probability that econometricians can distinguish the approximating model from the distorted model.

The general idea of the calibration procedure is to find a value of ϑ such that $p(\vartheta; N)$ equals a given value (for example, 10%) after simulating model P, (29), and model Q, (25).[15] In the continuous-time model with the iid Gaussian specification, $p(\vartheta; N)$ can be easily computed. Because both models P and Q are arithmetic Brownian motions with constant drift and diffusion coefficients, the log-likelihood ratios are Brownian motions and are normally distributed random variables. Specifically, the logarithm of the Radon-Nikodym derivative of the distorted model (Q) with respect to the approximating model (P) can be written as

$$\ln\left(\frac{L_Q}{L_P}\right) = -\int_0^N \overline{\upsilon} dB_s - \frac{1}{2}\int_0^N \overline{\upsilon}^2 ds, \tag{56}$$

where

$$\overline{\upsilon} \equiv \upsilon^* \alpha \sigma_k = \left(-\frac{\vartheta}{\gamma+\vartheta}\right)\left(\frac{\mu - i}{\sigma}\right). \tag{57}$$

Similarly, the logarithm of the Radon-Nikodym derivative of the approximating model (P) with respect to the distorted model (Q) is

$$\ln\left(\frac{L_P}{L_Q}\right) = \int_0^N \overline{\upsilon} dB_s + \frac{1}{2}\int_0^N \overline{\upsilon}^2 ds. \tag{58}$$

Using (53)-(58), it is straightforward to derive $p(\vartheta; N)$:

$$p(\vartheta; N) = \Pr\left(x < \frac{\overline{\upsilon}}{2}\sqrt{N}\right), \tag{59}$$

where x follows a standard normal distribution. From the expressions of $\overline{\upsilon}$, (57), and $p(\vartheta; N)$, (59), we can show that the value of p is decreasing with the value of ϑ because $\partial\overline{\upsilon}/\partial\vartheta < 0$. From (57) and (59), it is clear that the calibration of the value of ϑ is independent of both the elasticity of intertemporal substitution (ψ) and the discount rate (δ).

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Growth and Volatility by Country, 1962-2011

Country	Mean Growth	S.D. Growth	Country	Mean Growth	S.D. Growth
Argentina	1.46	5.32	Malawi	1.60	6.91
Australia	2.04	1.71	Malaysia	3.96	3.92
Austria	2.51	1.97	Malta	4.49	4.33
Bangladesh	1.55	4.04	Mauritius	3.01	5.04
Barbados	1.87	4.22	Mexico	1.83	3.49
Belgium	2.36	1.97	Mozambique	2.01	5.03
Bolivia	0.69	3.82	Nepal	1.44	2.72
Botswana	5.99	5.88	Netherlands	2.25	2.03
Brazil	2.46	3.65	New Zealand	1.23	2.72
Canada	2.05	2.09	Niger	-1.09	6.06
Chile	2.43	5.10	Norway	2.55	1.83
Colombia	1.98	2.16	Pakistan	2.47	2.06
Congo, Dem. Rep.	-2.11	5.90	Panama	3.31	4.38
Costa Rica	2.23	3.21	Paraguay	1.66	3.97
Cyprus	3.87	5.85	Peru	1.30	5.16
Denmark	1.95	2.39	Philippines	1.39	3.05
Dominican Republic	3.12	4.80	Portugal	2.93	3.38
Ecuador	2.04	4.16	Senegal	-0.11	4.18
El Salvador	1.34	3.50	Sierra Leone	0.31	7.57
Fiji	1.70	4.46	Singapore	5.15	4.25
Finland	2.57	3.21	South Africa	1.02	2.50
France	2.21	1.97	Spain	2.68	2.73
Germany	2.21	2.09	Sri Lanka	3.44	2.31
Ghana	0.81	4.56	Sweden	2.07	2.27
Greece	2.55	4.22	Switzerland	1.35	2.17
Guatemala	1.37	2.43	Syria	2.10	8.86
Honduras	1.04	3.12	Taiwan	5.56	2.94
Hong Kong	4.58	4.24	Tanzania	1.75	3.19
Iceland	2.47	4.00	Thailand	4.74	4.94
India	3.15	3.36	Togo	0.15	5.65
Iran	1.05	10.25	Trinidad & Tobago	2.34	4.97
Ireland	3.24	3.32	Tunisia	3.28	4.39
Israel	3.17	6.27	Turkey	2.65	3.98
Italy	2.32	2.74	Uganda	1.19	4.48
Jamaica	0.47	3.71	United Kingdom	1.96	2.20
Japan	3.73	5.11	United States	2.04	2.13
Jordan	0.95	6.38	Uruguay	1.68	4.23
Kenya	0.71	2.82	Venezuela	0.62	5.40
Korea, Republic of	5.80	3.61	Zambia	-0.05	4.86
Lesotho	2.79	6.39	Zimbabwe	0.17	7.13

Source: Penn World Tables and author's calculations.

Figure 1: List of Countries

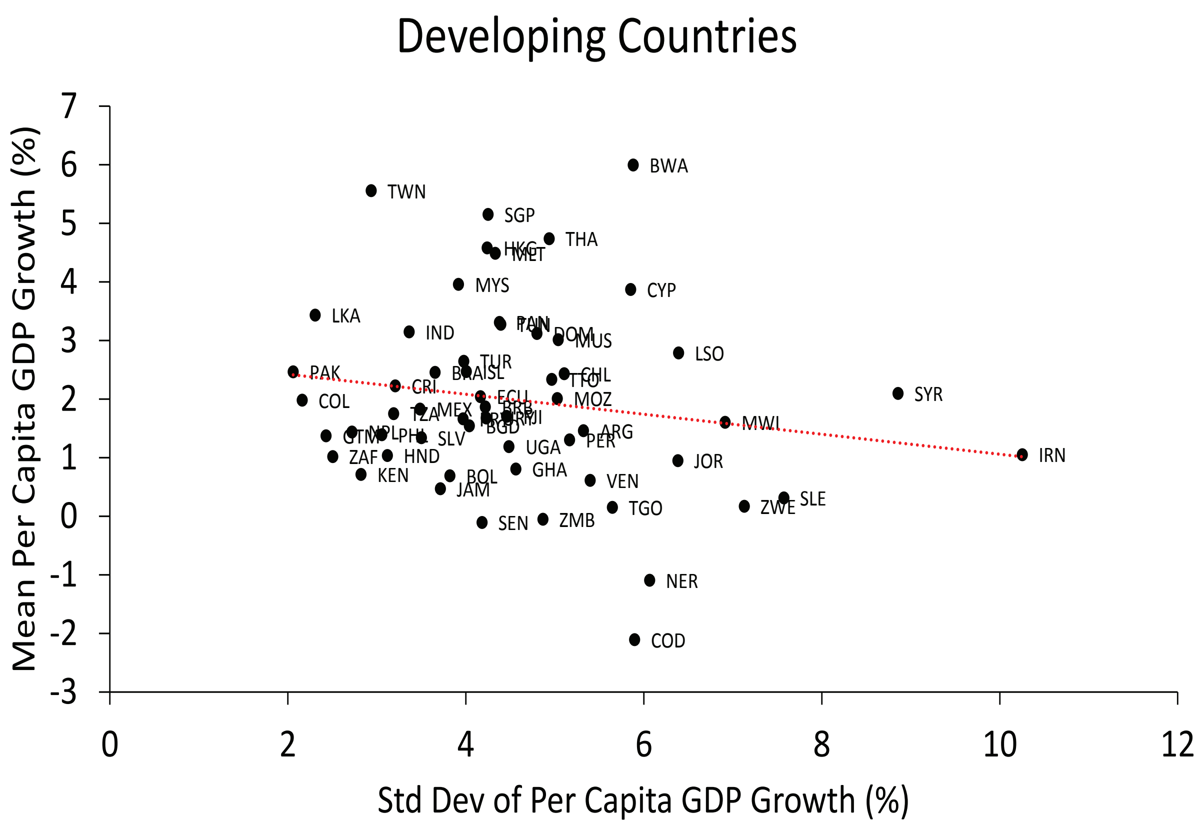

Figure 2: Growth and Volatility of Per-Capita GDP (1962-2011): Developing Countries

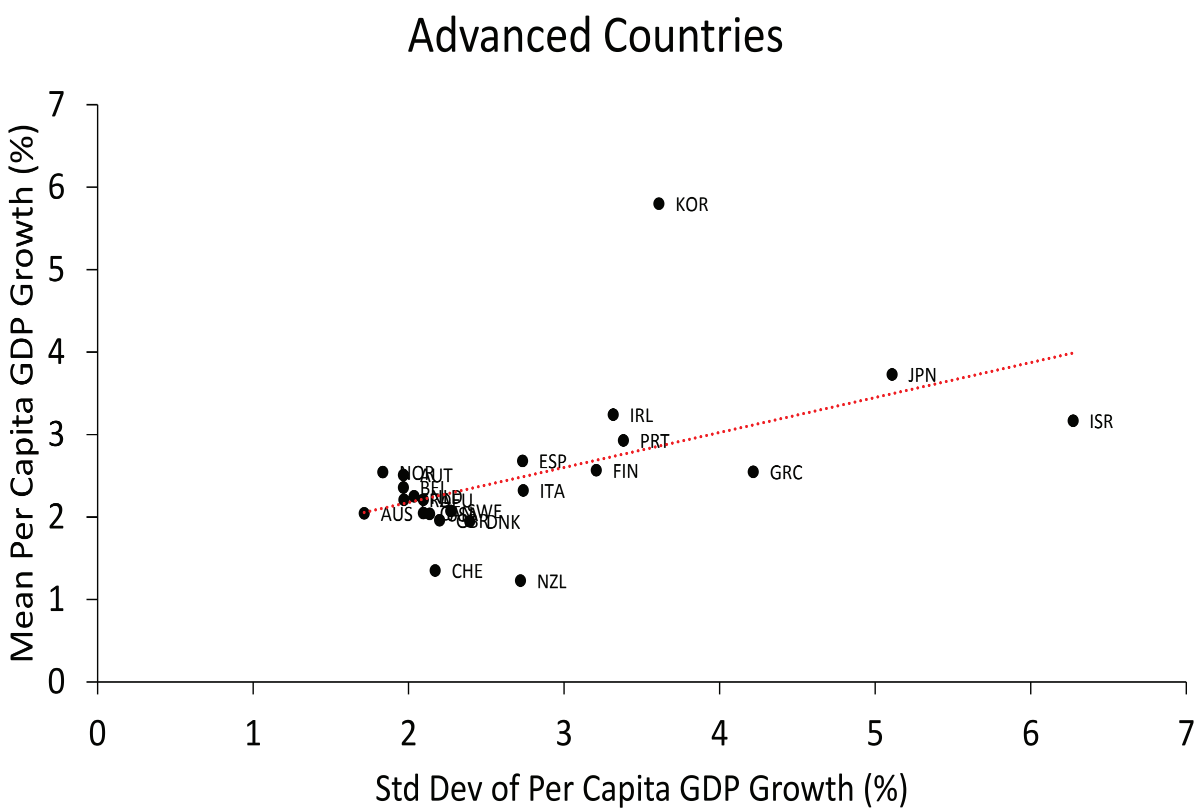

Figure 3: Growth and Volatility of Per-Capita GDP (1962-2011): Advanced Countries

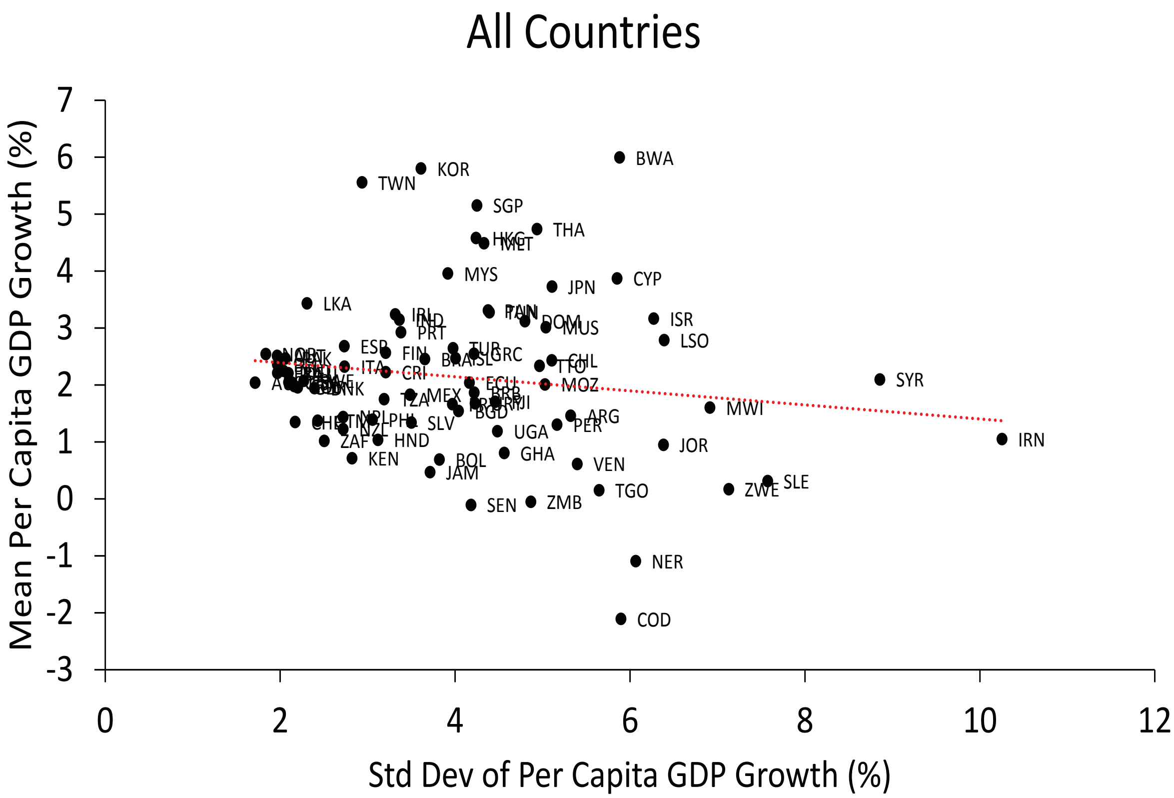

Figure 4: Growth and Volatility of Per-Capita GDP (1962-2011): All Countries

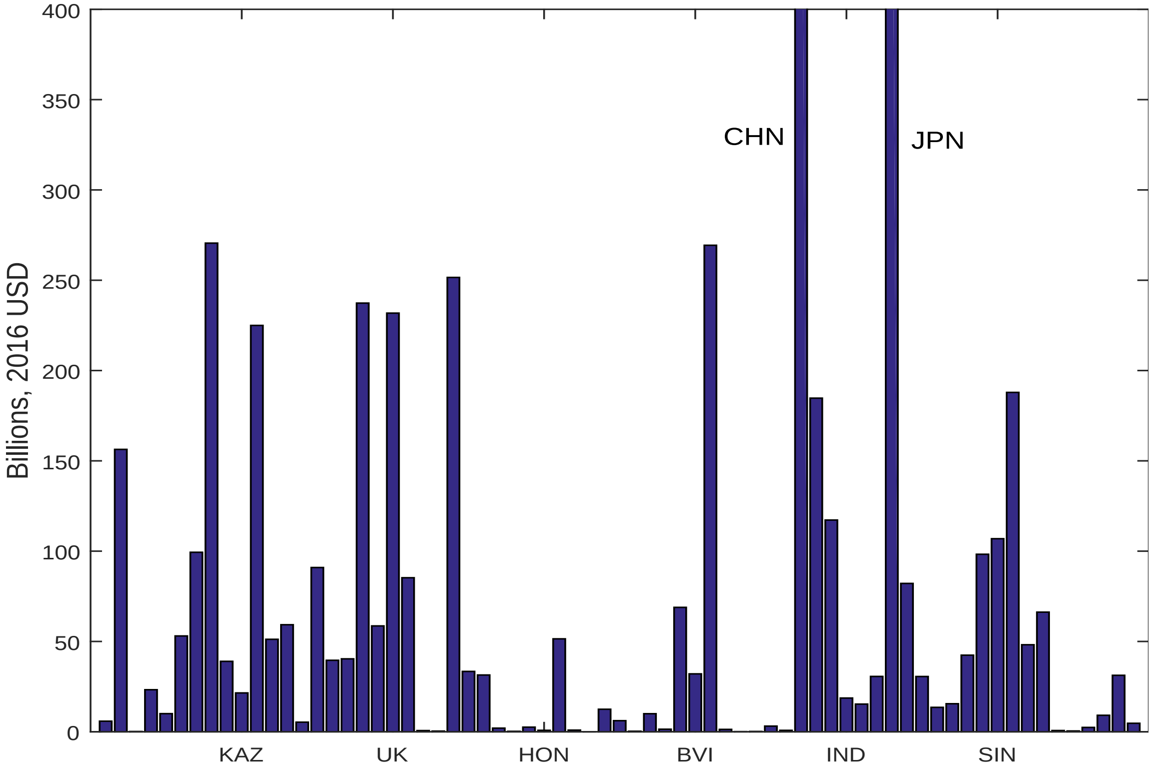

Figure 5: Distribution of US Treasury Debt

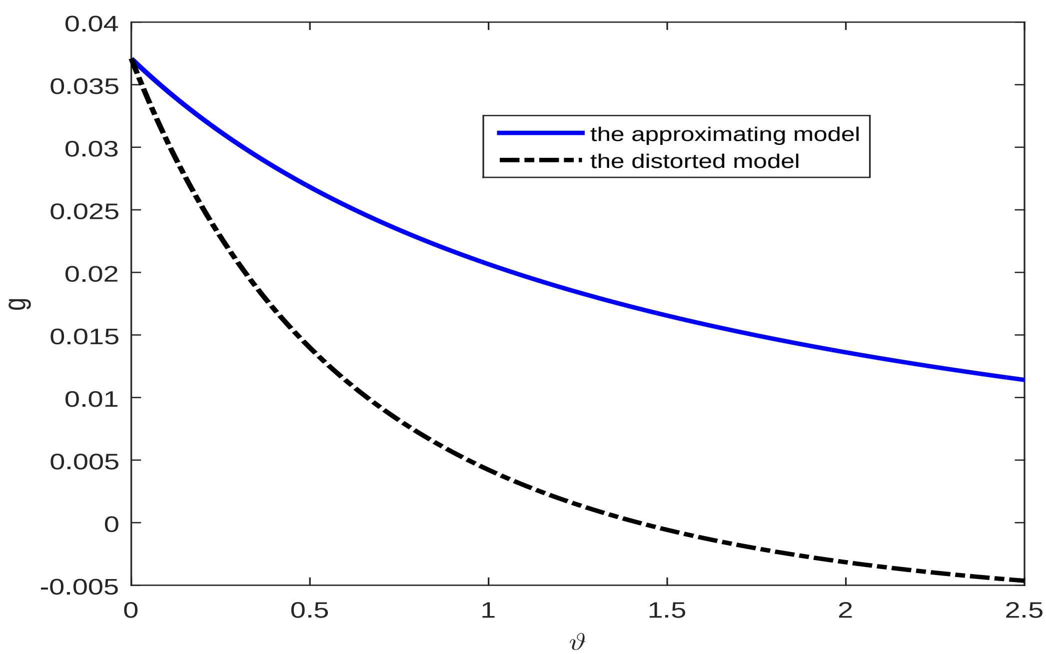

Figure 6: Effect of RB on Growth in the Diversified Equilibrium

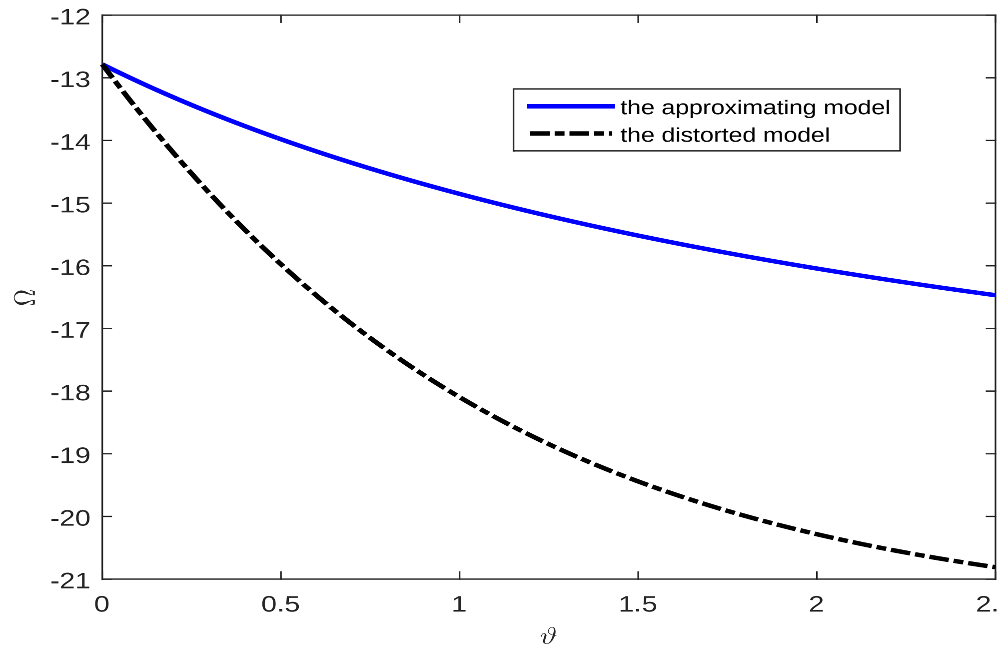

Figure 7: Effect of RB on Welfare in the Diversified Equilibrium

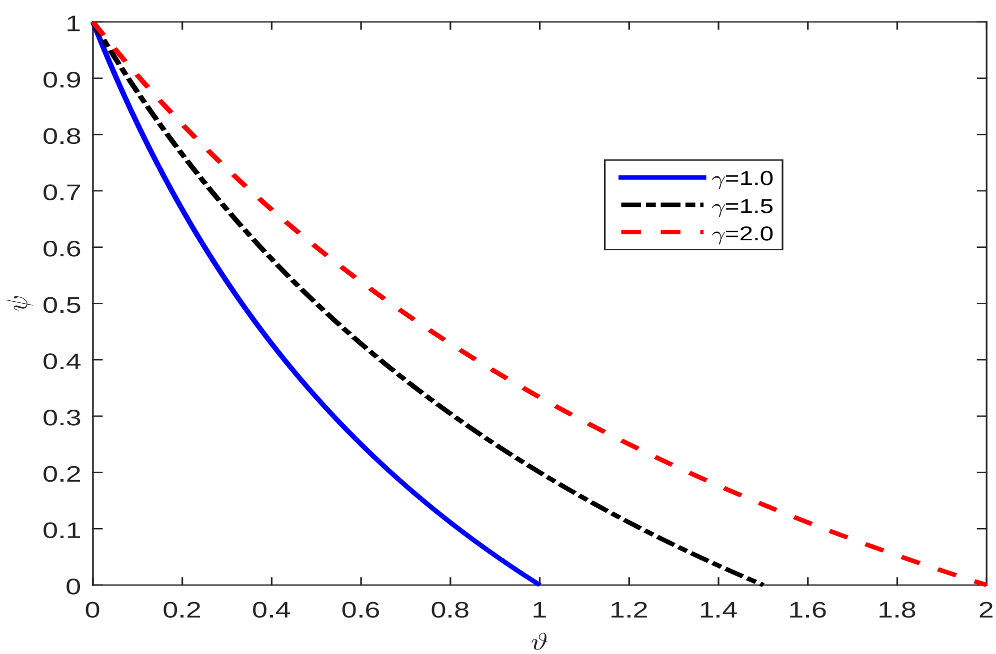

Figure 8: Relation between EIS and RB

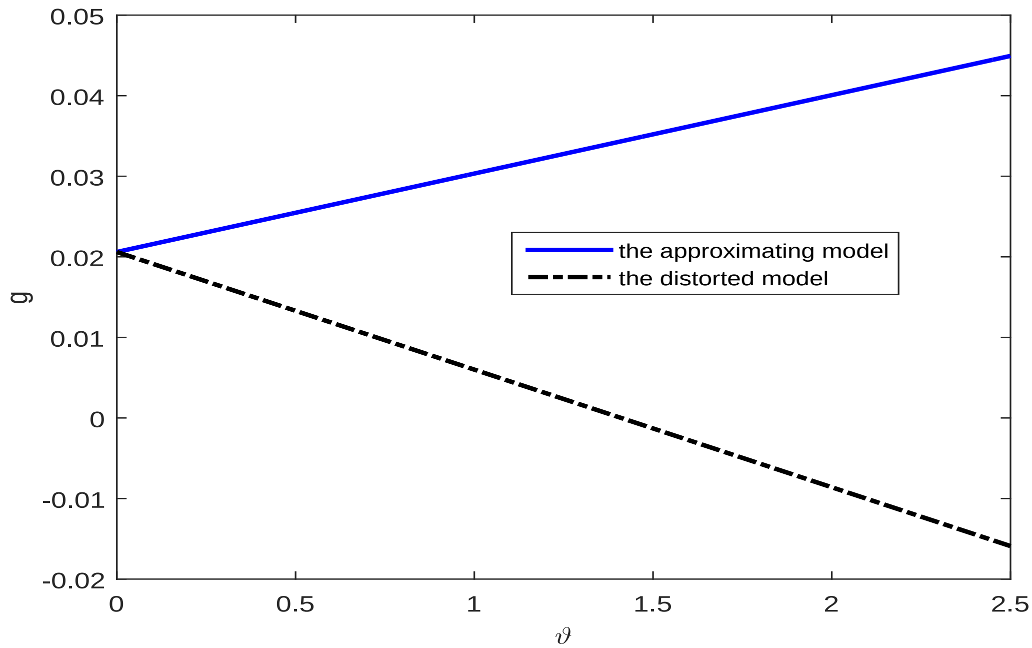

Figure 9: Effect of RB on Growth in the Undiversified Equilibrium

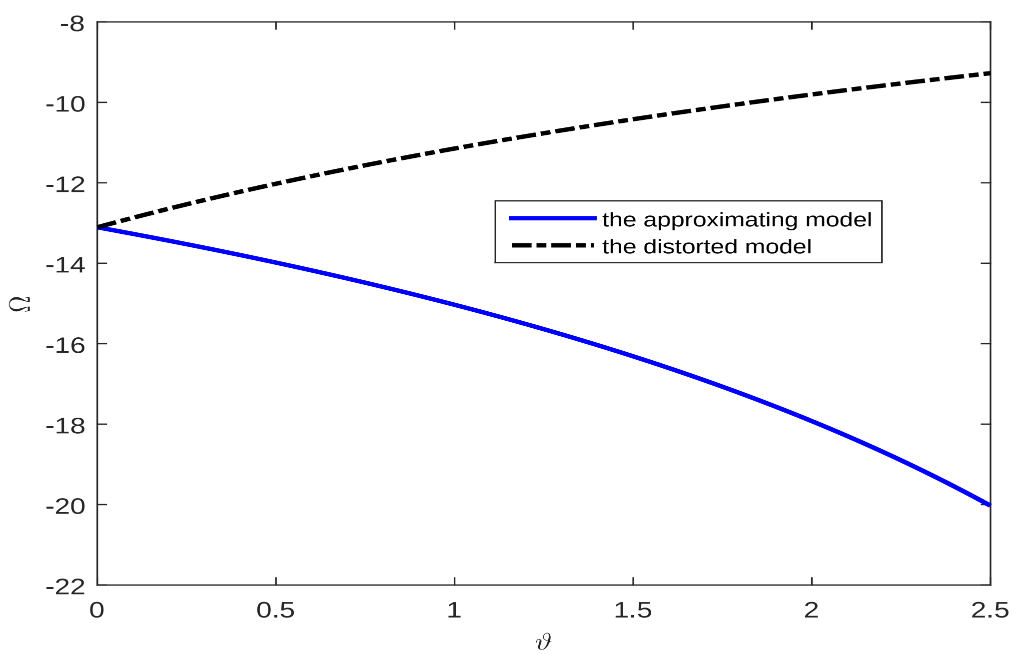

Figure 10: Effect of RB on Welfare in the Undiversified Equilibrium

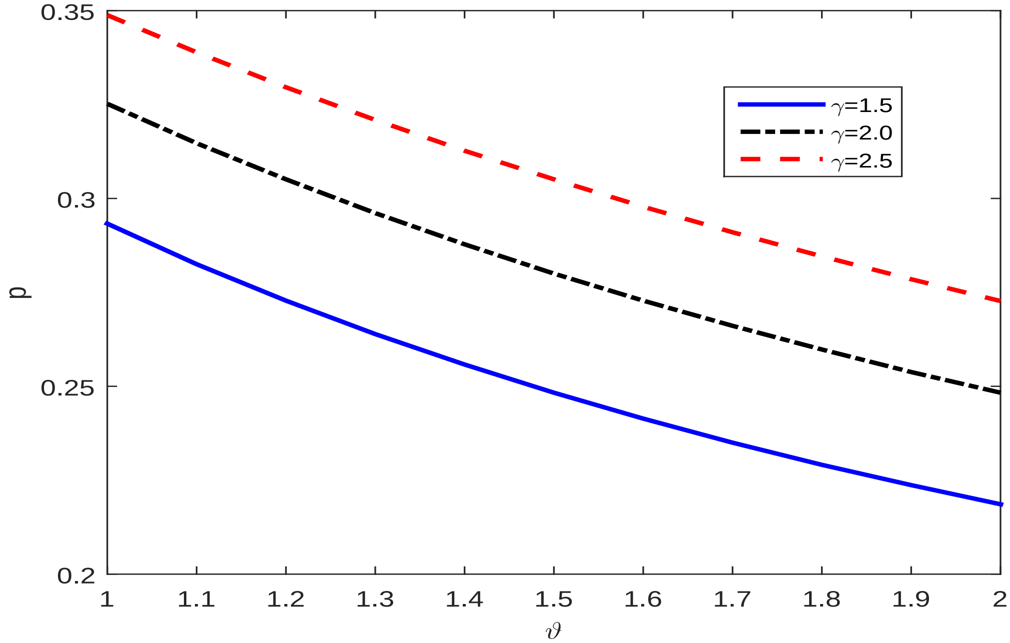

Figure 11: Relationship between ϑ and p

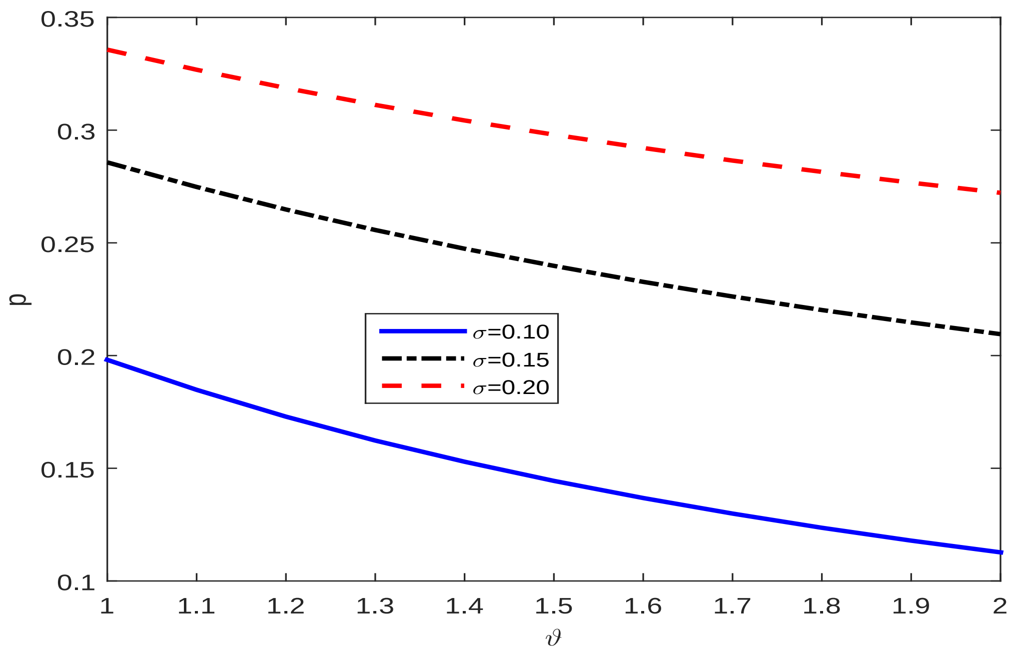

Figure 12: Relationship between ϑ and p

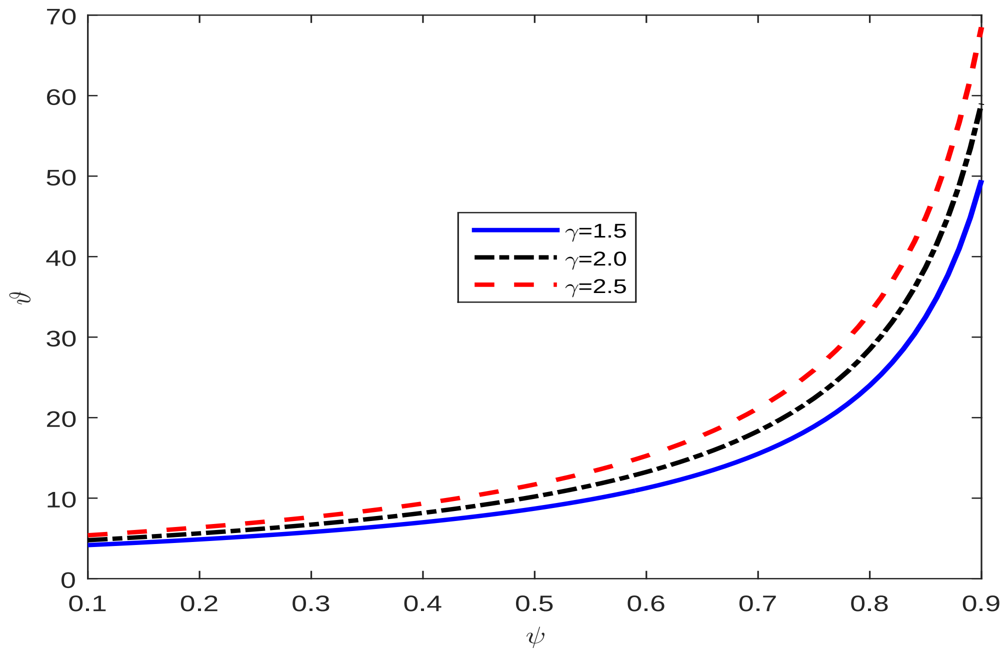

Figure 13: Inferred Values of ϑ for Given Values of ψ and γ

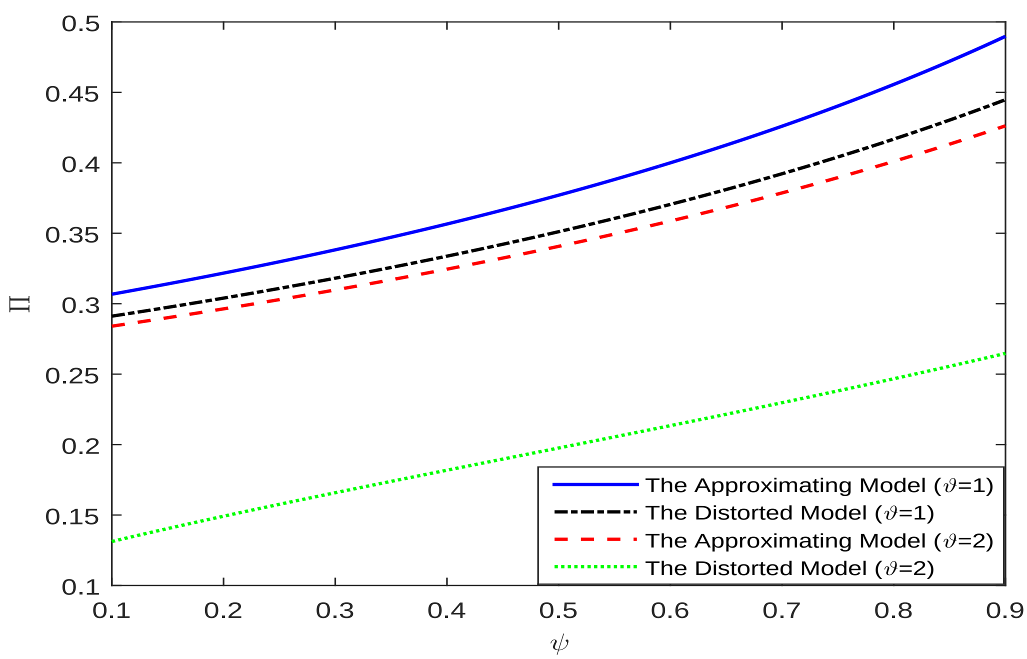

Figure 14: The Welfare Gain from Intergration under RB

Table 1: Regression Results on the Effects of Volatility on Per-Capita GDP Growth (1962-2011)

Independent Variables	Regression Coefficient	p-value
Volatility, $std(g)$	-0.085	0.064
Country Dummy, $I_{adv} \cdot std(g)$	0.153	0.015
Investment Share	0.066	0.000
Human Capital	0.002	0.181
Initial Per-Capital GDP Level	-0.003	0.001
Constant	0.029	0.000

Table 2: Regression Results on the Effects of Σ^2 on g^d (1962-2011)

Independent Variables	Regression Coefficient	p-value
Variance, Σ^2	-1.136	0.010
Country Dummy, $I_{adv} \cdot \Sigma^2$	3.702	0.005
Investment Share	0.067	0.000
Human Capital	0.019	0.268
Initial Per-Capital GDP Level	-0.003	0.003
Constant	0.025	0.000

Table 3: Values of Group 1 Parameters

Parameters	Values	Target
Developing		
r	0.035	Wang et al. (2016)
μ	0.095	6% equity premium
δ	0.01	Zhuang et al. (2007)
σ	0.07	Obstfeld (1994)
Advanced		
r	0.035	Wang et al. (2016)
μ	0.095	6% equity premium
δ	0.01	Zhuang et al. (2007)
σ	0.15	Obstfeld (1994)

Table 4: Estimation of Group 2 Parameters

Parameters	Values	Moments	Data	Model
Developing				
ϑ	15.55	Reg. Coef.	-1.14	-1.14
ψ	0.76	Growth (g)	1.99%	1.99%
γ	0.84	Std (g)	5.02%	5.02%
Advanced				
ϑ	9.02	Reg. Coef.	2.57	2.57
ψ	0.71	Growth (g)	2.36%	2.36%
γ	4.54	Std (g)	2.99%	2.99%

Table 5: Growth and Welfare Gains from Financial Integration under RB

	Before Integration	After Integration	Percent Change
Growth Rate ($g, \%$)			
Developing	1.99	2.10	5.5%
Advanced	2.36	3.37	42.8%
Welfare (Before = 100)			
Developing	100	115.64	15.6%
Advanced	100	160.04	60.0%
Share of risky invest. (α)			
Developing	0.69	0.83	
Advanced	0.20	1	
Volatility of risky-capital return (σ)			
Developing	0.0723	0.065	
Advanced	0.148	0.065	
Volatility of growth (Σ)			
Developing	0.050	0.054	
Advanced	0.030	0.065	
Risk-free Rate (i)	3.50%	3.70%	