The Negative Growth-Volatility Relationship and the Gains from Financial Integration*

Yulei Luo†
The University of Hong Kong

Jun Nie‡
Federal Reserve Bank of Kansas City

Eric R. Young§
University of Virginia

October 29, 2016

Abstract

We document a negative relationship between average growth and volatility in a cross-section of countries. We then provide an interpretation of this result using a robustness (RB) version of the Obstfeld (1994) model of financial diversification and openness. Incorporating a preference for robustness can generate the observed negative relationship between growth and volatility of real GDP. Furthermore, we show that RB reduces the expected growth rate in a diversified equilibrium (positive holdings of risk-free capital), while it could increase growth in an undiversified equilibrium. Using this framework, we calculate the growth and welfare gains associated with financial market openness, and show that international integration has a smaller effect on welfare gain under RB if the economy is in a diversified equilibrium.

JEL Classification Numbers: *C61, D81, E21*.

Keywords: Robustness, Model Uncertainty, Risk Sharing, Economic Growth.

---

*We are grateful to Evan Anderson, Martin Ellison, Simon Gilchrist, Alisdair McKay, Jianjun Miao, Tom Sargent, Stephen Terry, and seminar participants at Boston University, FRB of Kansas City, and UND for helpful discussions and comments. Luo thanks the General Research Fund (GRF, No. HKU791913) in Hong Kong for financial support. The views expressed here are the opinions of the authors only and do not necessarily represent those of the Federal Reserve Bank of Kansas City or the Federal Reserve System. All remaining errors are our responsibility.

†Faculty of Business and Economics, University of Hong Kong, Hong Kong. Email address: yulei.luo@gmail.com.

‡Research Department, The Federal Reserve Bank of Kansas City. E-mail: jun.nie@kc.frb.org.

§Department of Economics, University of Virginia, Charlottesville, VA 22904. E-mail: ey2d@virginia.edu.

1. Introduction

Obstfeld (1994) proposes a general equilibrium version of the Merton-type model with stochastic production technology that links the financial diversification of an economy (openness) to long-run growth, and then uses his model to measure the welfare gains associated with financial integration. In his model, an economy with an interior portfolio of risky and riskless capital (a diversified equilibrium) will unambiguously generate a positive relationship between the average growth rate and the volatility of real GDP per capita, while an undiversified equilibrium (with only risky capital) can generate a negative relationship provided the intertemporal elasticity of substitution exceeds one. Ramey and Ramey (1995) document a robust negative relationship between the average growth rate of an economy and the volatility of output; this relationship holds after controlling for a number of country-specific factors.[1] We revisit this relationship and find that it continues to hold – average per-capita GDP growth is significantly lower in countries that have high average GDP volatility over the period $1962 - 2011$, using data from the Penn World Tables 9.0 (see Figures 1 and 2 for the relationship between volatility and growth of per capita real GDP). Furthermore, cross-country holdings of US government debt (essentially a risk-free asset) are large and widespread, indicating that an undiversified equilibrium does not look empirically plausible, and the evidence on the IES is mixed (the estimates range so widely that almost any value between 0 and 2 looks empirically reasonable).

We reconsider Obstfeld's model by introducing a fear of model misspecification, and study how the household's preference for robustness interacts with stochastic production technology and affects optimal consumption-portfolio rules and the equilibrium growth rate. The goal is to characterize whether model uncertainty due to the preference for robustness can help the model match the observed negative relationship between the volatility and growth of real GDP per capita. Hansen and Sargent (1995) first introduced the preference for robustness (RB) into linear-quadratic-Gaussian (LQG) economic models. In robust control problems, agents are concerned about the possibility that their true model is misspecified in a manner that is difficult to detect statistically; consequently, they choose their decisions as if the subjective distribution over shocks was chosen by an evil agent to minimize their utility. More specifically, under RB, agents have a best estimated reference model (called the approximating model) in their mind and consider a range of models (the distorted models) surrounding the approximating model. As discussed in Hansen, Sargent, and Tallarini (1999) and Luo and Young (2010), RB models can produce precautionary savings even within the class of linear-quadratic-Gaussian (LQG) models, which leads to analytical simplicity. Many recent papers have shown the usefulness of viewing agents as having (potentially) misspecified models of the economy and being aware of this fact; Hansen and Sargent (2007) provide a book-length introduction and discussion of the literature.[2]

[1] See Aghion and Banerjee (2005) for a recent survey on volatility and growth.

[2] There are three main ways to model ambiguity and robustness in the literature: the multiple priors model (Gilboa

A desire for robust decision rules complicates the link between average growth and volatility. If we interpret the model misspecification fears as "entirely in the head" of the consumers, then robustness only affects the magnitude of the correlation between volatility and growth; this model cannot therefore replicate the negative relationship between mean growth and volatility, as it becomes observationally equivalent to a standard model with a higher coefficient of risk aversion. If instead we interpret the model misspecification fears as justified (i.e., when the worst-case scenario actually happens), so that the distorting model governs the true dynamics of the economy, then the cutoff value for the IES to generate the negative relationship between volatility and growth is smaller than one and is decreasing with the degree of concerns about model uncertainty; under this interpretation we can calibrate the model to capture the observed negative correlation. There is no definitive justification for either perspective – Hansen and Sargent (2007) usually adopt the "entirely in the head" perspective, but calibration via detection error probabilities constructs the distorted model in such a way that it is an empirically-plausible model of the data. Put another way, once we entertain the idea that consumers do not trust their models, there is no "true" model anymore. Our model fits the data on volatility and growth only if the distorted model is correct.

Our robust version of Obstfeld's model implies that the growth rate and volatility of real GDP are negatively correlated if

$$\vartheta > \frac{1+\psi}{1-\psi}\gamma,$$

where ϑ, γ, and ψ are the parameters governing the degree of robustness, the coefficient of relative risk aversion, and the elasticity of intertemporal substitution, respectively. Given values for ψ and γ, we can use detection error probabilities p to assess whether a reasonable value for ϑ satisfies this bound; a detection error occurs when an agent selects the distorted model when the approximating model actually generated the observed data (or vice versa). For example, when $\gamma = 1.5$ and $\psi = 0.2$, we can ensure $\vartheta > 2.25$ by setting the detection error probability $p < 21.73$ percent; Hansen and Sargent (2007) argue that $p > 10$ percent is reasonable (their argument based only on introspection, but we think the introspection of Hansen and Sargent is at worst a reasonable guideline).[3]

We then evaluate the welfare gains associated with financial integration. In the Obstfeld model, financial integration increases the span of assets available in a given country, leading to a portfolio shift away from risk-free capital to assets that can hedge local risks. The result is not only an increase in growth, since risky assets have higher returns, but also higher volatility as the portfolio

---

and Schmeidler 1989), the "smooth ambiguity" model (Klibanoff, Marinacci, and Mukerji 2005), and the multiplier utility and robust control/filtering model (Hansen and Sargent 2001). See Epstein and Schneider (2010) for a recent review. We follow Hansen and Sargent. Ju and Miao (2012) and Chen, Ju, and Miao (2014) examine how ambiguity affects portfolio choices and asset prices.

[3]Djeutem and Kasa (2013) showed that to match the observed volatility of six U.S. dollar exchange rates (the Australian dollar, the Canadian dollar, the Danish dollar, the Japanese yen, the Swiss franc, and the British pound), the detection error probability should be set between 7.5% to 13.1%.

share of risky capital increases; consistent with this prediction, Schularick and Steger (2010) show that integration can boost growth through higher investment and Eozenou (2008) presents evidence for a positive connection between integration and macroeconomic volatility. If the countries are symmetric, the risky capital shares will be equal across different countries, while if volatilities are asymmetric then countries with low volatility will command a higher share. The welfare gains in the Obstfeld model could be very large.

Since the basic model predicts a positive correlation between volatility and growth, it will likely overstate the welfare gains associated with integration (see also Epaulard and Pommeret 2005). Other papers also find large gains from international financial integration, even when growth rates are not affected in the long run; see Hoxha, Kalemli-Ozcan, and Vollrath (2013) for an example. Gourinchas and Jeanne (2006) show that the gains are generally smaller in models with exogenous growth, which is a manifestation of the old Lucas observation that the welfare costs of aggregate consumption risk are magnitudes smaller than the welfare costs of low growth.

We find that under our preferred specification – low IES, diversified equilibrium, distorted model – the welfare gains of integration are significantly smaller than under Obstfeld finds, even if the effective risk aversion of the agents is the same. The smaller welfare gains are resulted from agents' concerns on possible model misspecifications that discourage them to invest in high-return assets both before and after the financial integration. More generally, we show that gains are biased toward countries with low levels of ambiguity (low effective risk aversion) and welfare gains under distorted model are lower than under the approximating model. The key change is that the growth effect in our model is smaller than Obstfeld's model, reflecting the negative effect of volatility, and the growth effect is particularly small for the country with high uncertainty. In addition, where there is heterogeneity in the degree of model uncertainty across countries, we show that a country facing higher model uncertainty could experience larger welfare gains after the financial integration if we take into account the possibility that countries may switch from diversified equilibriums to undiversified equilibriums.

This paper is organized as follows. Section 2 presents a robustness version of the Obstfeld-type model with recursive utility in a closed economy and discusses how the presence of robustness can have the potential to generate the observed negative relationship between growth and volatility of the macroeconomy. Section 3 estimates and calibrates the model parameters and conducts the empirical analysis. Section 4 discusses the growth and welfare effects of international economic integration under robustness. Section 5 concludes.

2. A Risk-Sharing Model with Recursive Utility and Model Uncertainty

2.1. The Model Setting

Following Obstfeld (1994), in this paper we consider a continuous-time risk-sharing model with multiple assets. Specifically, we assume that individuals save by accumulating capital and by making risk-free loans that pay a real return i_t. There are two types of capital: one is risk free with a constant return and one is risky with a stochastic return; households are prevented from shorting either type. The value of the risk free asset (b_t) follows the process

$$\frac{db_t}{dt} = rb_t \tag{1}$$

for some constant $r > 0$. There is a simple stochastic production technology that is linear in risky capital $(k_{e,t})$:

$$dy_t = ak_{e,t}dt + \sigma k_{e,t}dB_t, \tag{2}$$

where dy_t is the instantaneous output flow, $k_{e,t}$ denotes the stock of capital, $a > r$ is the expected technology level, σ is the standard deviation of the production technology, and B_t is a standard Brownian motion defined over the complete probability space. It is worth noting that the AK specification, (2), can be regarded as a reduced form of the following stochastic Cobb-Douglas production function specification when labor is supplied inelastically:

$$dy_t = Ak_{e,t}^{1-\alpha}\left(\overline{k}_{e,t}l\right)^{\alpha}\left(dt + \sigma_y dB_{y,t}\right), \alpha \in (0,1), \tag{3}$$

where $Ak_{e,t}^{1-\alpha}\left(\overline{k}_{e,t}l\right)^{\alpha}$ is the *deterministic* flow of production, $k_{e,t}$ denotes individual firm's stock of capital, $\overline{k}_{e,t}$ is the average economy-wide stock of capital, and $\overline{k}_{e,t}l$ measures the (inelastic) supply of efficiency labor units, σ_y is the standard deviation of the technology innovation, and $B_{y,t}$ is a standard Brownian motion. This production function exhibits constant returns to scale at the individual level. Furthermore, in equilibrium, $k_{e,t} = \overline{k}_{e,t}$ and the stochastic production is linear in capital:

$$dy_t = Al^{\alpha}k_{e,t}\left(dt + \sigma_y dB_{y,t}\right), \alpha \in (0,1), \tag{4}$$

which is just the specification of (2) if we set $a = Al^{\alpha}$ and $\sigma = a\sigma_y$. We assume that the wage rate, w, over $(t, t+dt)$ is determined at the beginning of t and is set to be equal to the expected marginal product of labor,

$$w = E\left[\frac{\partial\left(Ak_{e,t}^{1-\alpha}\left(l\overline{k}_{e,t}\right)^{\alpha}\right)}{\partial l}\right]_{k_{e,t}=\overline{k}_{e,t}} = A\alpha l^{\alpha-1}k_{e,t} = \alpha\mu\frac{k_{e,t}}{l},$$

and the total rate of return to labor during this period is determined by wdt.

In the absence of adjustment costs, the rate of return to the risky capital can be written as

$$r_{e,t} \equiv \frac{dy_t - \delta k_{e,t} - lwdt}{k_{e,t}} = \mu dt + \sigma dB_t, \tag{5}$$

where $\mu = (1-\alpha) a - \delta$. If $i_t < r$, there is no equilibrium because this condition implies an arbitrage profit from issuing loans and investing the proceeds in the risk free asset. If $i_t > r$ there exists an equilibrium with no risk-free assets if and only if there exists a short sale constraint on capital, which we implicitly impose. Finally, when $i_t = r$, the division between the risk free asset and the loan is indeterminate. Consequently, the individuals only need to choose from two assets: the risky capital and a composite safe asset offering a return i_t. Later we will show that the real interest rate (i_t) is constant in equilibrium.[4]

The budget constraint for the representative consumer can thus be written as

$$dk_t = [(i + \alpha_t (\mu - i)) k_t - c_t] dt + \alpha_t k_t \sigma dB_t, \tag{6}$$

where $k_t = k_{e,t} + b_{f,t}$ is total wealth, $b_{f,t}$ is holdings of the composite safe asset, and α_t is the fraction of wealth invested in risky capital.

Following Campbell and Viceira (2002), and Campbell, Chacko, Rodriguez, and Viceira (henceforth, CCRV 2004), we adopt the Duffie-Epstein (1992) parameterization of recursive utility in continuous-time:

$$J_t = E_t \left[\int_t^\infty f(c_s, J_s)\, ds \right], \tag{7}$$

where J is continuation utility and $f(c, J)$ is a normalized aggregator of current consumption and continuation utility given by

$$f(c, J) = \frac{\delta (1-\gamma) J}{1 - 1/\psi} \left[\left(\frac{c}{[(1-\gamma) J]^{1/(1-\gamma)}} \right)^{1-1/\psi} - 1 \right], \tag{8}$$

where $\delta > 0$ is the discount rate, $\gamma > 0$ is the coefficient of relative risk aversion, and $\psi > 0$ is the elasticity of intertemporal substitution. When $\psi = 1/\gamma$, the above recursive utility reduces to the standard time-separable power utility. As $\psi \to 1$ J converges to

$$f(c, J) = \delta (1-\gamma) J \left[\ln(c) - \frac{1}{1-\gamma} \ln((1-\gamma) J) \right]. \tag{9}$$

[4]In many ways our model will be similar to Cagetti and DeNardi (2008), in which households can choose to save in the form of riskless corporate capital, riskless bonds, or risky entrepreneurial capital. Our model is also related to Mendoza, Quadrini, and Ríos-Rull (2009), who study the connection between financial openness and gross capital positions in a model with risky assets in multiple countries. Both papers have models that are not amenable to analytical solutions.

The only difference from the standard additive power utility case is that here we use the aggregator $f(c, J)$ to replace the instantaneous utility function $u(c)$ from the standard expected utility case.

In the full-information case, the Bellman equation is

$$\sup_{c_t,\alpha_t} \{f(c_t, J_t) + \mathcal{D}J(k_t)\}, \tag{10}$$

subject to (6), where $f(c_t, J_t)$ is specified in (8) and

$$\mathcal{D}J(k_t) = J_k[(i + \alpha_t(\mu - i))k_t - c_t] + \frac{1}{2}J_{kk}\sigma^2\alpha_t^2 k_t^2. \tag{11}$$

We can now solve for the consumption and portfolio rules and the expected growth rate. The following proposition summarizes the main results, which are identical to those in Obstfeld (1994).

Proposition 1. *In the full-information case, the portfolio rule is*

$$\alpha^* = \frac{\mu - i}{\gamma\sigma^2}, \tag{12}$$

the consumption function is

$$c_t^* = \psi\left\{\delta - \left(1 - \frac{1}{\psi}\right)\left[i + \frac{(\mu - i)^2}{2\gamma\sigma^2}\right]\right\}k_t, \tag{13}$$

the evolution of risky capital is

$$\frac{dk_t}{k_t} = \left[\psi(i - \delta) + \frac{1}{2}(1 + \psi)(\mu - i)\alpha^*\right]dt + \alpha^*\sigma dB_t, \tag{14}$$

and the mean and standard deviation of the growth rate are defined as:

$$g \equiv E\left[\frac{dk_t}{k_t}\right]/dt = \psi(i - \delta) + \frac{1}{2}(1 + \psi)(\mu - i)\alpha^*, \tag{15}$$

and

$$\Sigma = \alpha^*\sigma, \tag{16}$$

respectively.

Proof. See Appendix 6.1. ∎

From (13), if we define the marginal propensity to consume as

$$m \equiv \psi\left\{\delta - \left(1 - \frac{1}{\psi}\right)\left[i + \frac{(\mu - i)^2}{2\gamma\sigma^2}\right]\right\}, \tag{17}$$

the expected growth rate can be written as

$$g = r_p - m, \tag{18}$$

and the value function is

$$J(k) = \left(\delta^{-\psi} m\right)^{\frac{1-\gamma}{1-\psi}} \frac{k^{1-\gamma}}{1-\gamma}, \tag{19}$$

where $r_p \equiv i + \alpha(\mu - i)$ is the return to the market portfolio.

Following Obstfeld (1994), we first consider a closed-economy equilibrium in which the two capital goods can be interchanged in one-to-one ratio and the amount of asset supply can always be adjusted to accommodate the equilibrium asset demand, (12). There are two types of equilibrium: (i) one in which both types of capital are held (*diversified*) and (ii) one in which only risky capital is held (*undiversified*). In the diversified equilibrium, the interest rate i is equal to r and $\alpha^* = (\mu - r) / \left(\gamma\sigma^2\right) < 1$. In the undiversified equilibrium, $(\mu - r) / \left(\gamma\sigma^2\right) \geq 1$, which means that the interest rate i will rise above r until the excess supply of the risk free asset is eliminated (that is, until $(\mu - i) / \left(\gamma\sigma^2\right) = 1$). Therefore, the interest rate will be constant and equal to

$$i = \mu - \gamma\sigma^2 > r. \tag{20}$$

The following proposition summarizes the results about the expected growth rate in the two types of equilibria.[5]

Proposition 2. *In the diversified equilibrium, the expected growth rate is*

$$g = \psi(r - \delta) + (1 + \psi)\frac{(\mu - r)^2}{2\gamma\sigma^2}. \tag{21}$$

In the undiversified equilibrium the expected growth rate is

$$g = \psi(\mu - \delta) + \frac{1}{2}(1 - \psi)\gamma\sigma^2. \tag{22}$$

Comparing (21) with (22), it is clear that the effects of the volatility of the fundamental shocks (σ^2) on the growth rate (g) are different in the two equilibria. In the diversified equilibrium, it is immediate that

$$\frac{\partial(g)}{\partial(\gamma\sigma^2)} < 0,$$

where $\gamma\sigma^2$ measures total amount of uncertainty (uncertainty about the return on the risky capital and uncertainty about government spending) facing the consumer. In contrast, in the undiversified equilibrium, the effect of volatility on the growth rate depends on the value of the elasticity of

[5]Note that the two types of equilibria do not co-exist; the economy only has one equilibrium for a given set of parameters.

intertemporal substitution. Specifically, a fall in σ^2 increases growth if $\psi > 1$ but lowers growth when $\psi < 1$.[6]

Now we connect GDP volatility to growth. Since the standard deviation of the growth rate of real GDP, Σ, can be written as $\Sigma = \alpha^* \sigma$ and the Sharpe ratio, η, is defined as $\eta \equiv (\mu - r) / \sigma$, we can link η to Σ:

$$\Sigma = \frac{\eta}{\gamma}. \tag{23}$$

Using these relationships, in the diversified equilibrium, the expected growth rate can be written as

$$g = \psi (r - \delta) + \frac{(1+\psi)\gamma}{2}\Sigma^2. \tag{24}$$

Note that the relationship between average growth and GDP volatility is unambiguously positive, in direct contrast to the data we examine below. In contrast, using the undiversified equilibrium we get a negative relationship only if $\psi > 1$ (since $\alpha^* = 1$, GDP volatility equals shock volatility).

If we look at the data, it seems to us that an undiversified equilibrium is not reasonable. For example, in the US the stock of government debt has historically hovered around 50 percent of GDP and is currently at close to 100 percent; US debt is generally considered as close to risk-free as any asset and is widely held across the world (see Figure 3, which is taken from US Treasury data on foreign holdings of US government debt; we eliminate countries that lack data for June 2016). Once we rule out undiversified equilibria, the basic Obstfeld model does not replicate the negative correlation between growth and volatility.

2.2. Introducing RB

To introduce robustness into the above model, we follow Anderson, Hansen, and Sargent (2003) (henceforth, AHS) and Maenhout (2004) and introduce a preference for robustness (RB) by adding an endogenous distortion $v(k_t)$ to the law of motion of the state variable k_t:

$$dk_t = [(i + \alpha_t (\mu - i)) k_t - c_t] dt + \alpha_t \sigma k_t (\alpha_t \sigma k_t v(k_t) dt + dB_t). \tag{25}$$

As shown in AHS (2003), the objective $\mathcal{D}J$ defined in (11) plays a crucial role in introducing robustness. $\mathcal{D}J$ can be thought of as $E[dJ]/dt$ and is easily obtained using Itô's lemma. A key insight of AHS (2003) is that this differential expectations operator reflects a particular underlying model for the state variable. The consumer accepts the approximating model, (6), as the best approximating model, but is still concerned that it is misspecified. He therefore wants to consider a range of models (i.e., the distorted model, (25)) surrounding the approximating model when computing

[6] We can find examples in the literature that find values for ψ that range well below 1 (Hall 1988, Campbell 1989) to well above 1 (Vissing-Jørgenson and Attanasio 2003, Gourinchas and Parker 2002), and various values in between (Guvenen 2006). There does not seem to be much consensus here, despite the clear importance of this parameter in growth models (Lucas 1990).

the continuation payoff. A preference for robustness is then achieved by having the agent guard against the distorting model that is reasonably close to the approximating model. The drift adjustment $v(k_t)$ is chosen to minimize the sum of the expected continuation payoff, but adjusted to reflect the additional drift component in (25), and of an entropy penalty:

$$\inf_{v}\left[\mathcal{D}J(k_t)+v(k_t)(\alpha_t k_t\sigma)^2 J_k+\frac{1}{2\vartheta(k_t)}(\alpha_t k_t\sigma)^2 v(k_t)^2\right], \tag{26}$$

where the first two terms are the expected continuation payoff when the state variable follows (6), i.e., the alternative model based on drift distortion $v(k_t)$.[7] $\vartheta(k_t)$ is fixed and state independent in AHS (2003), whereas it is state-dependent in Maenhout (2004). The key reason of replacing fixed ϑ with a state-dependent counterpart $\vartheta(k_t)$ in Maenhout (2004) is to assure the homotheticity (scale invariance) of the decision problem, a property which is required for the model to display balanced growth. As emphasized in AHS (2003) and Maenhout (2004), the last term in the HJB above is due to the agent's preference for robustness and reflects a concern about the quadratic variation in the partial derivative of the value function weighted by the robustness parameter, $\vartheta(k_t)$. The following proposition summarizes the solution.

Proposition 3. *Under RB, the portfolio rule is*

$$\alpha^* = \frac{\mu - i}{\tilde{\gamma}\sigma^2}, \tag{27}$$

the consumption function is

$$c_t^* = \psi\left\{\delta - \left(1-\frac{1}{\psi}\right)\left[i+\frac{(\mu-i)^2}{2\tilde{\gamma}\sigma^2}\right]\right\}k_t, \tag{28}$$

the evolution of risky capital for the approximating model and the distorted model are

$$\left(\frac{dk_t}{k_t}\right)^a = \left[\psi(i-\delta)+(1+\psi)\frac{(\mu-i)^2}{2\tilde{\gamma}\sigma^2}\right]dt+\alpha\sigma dB_t, \tag{29}$$

and

$$\left(\frac{dk_t}{k_t}\right)^d = \left[\psi(i-\delta)+\left(1+\psi-\frac{2\vartheta}{\gamma+\vartheta}\right)\frac{(\mu-i)^2}{2\tilde{\gamma}\sigma^2}\right]dt+\alpha\sigma dB_t, \tag{30}$$

respectively, where the effective coefficient of absolute risk aversion $\tilde{\gamma}$ is defined as: $\tilde{\gamma} \equiv \gamma + \vartheta$.

Proof. See Appendix 6.2. ■

In the RB economy, we again consider diversified and undiversified equilibria. In the diversified equilibrium, the interest rate i is equal to r and $\alpha^* = (\mu - r)/(\tilde{\gamma}\sigma^2) < 1$. In the undiversified

[7]Note that $\vartheta(k_t) = 0$ here corresponds to the expected utility case.

equilibrium, $(\mu - r) / \left(\widetilde{\gamma}\sigma^2\right) \geq 1$, so the interest rate is given by

$$i = \mu - \widetilde{\gamma}\sigma^2 > r. \tag{31}$$

Since $\widetilde{\gamma} = \gamma + \vartheta > \gamma$, the equilibrium interest rate under RB is lower than that under FI-RE. The intuition is simple: the additional amount of precautionary savings due to robustness drives down the equilibrium interest rate. The proposition summarizes the results about the expected growth rate under RB in the two equilibria.

Proposition 4. *Under RB, in the diversified equilibrium, the expected growth rate is*

$$g^a = \psi (r - \delta) + \frac{(1 + \psi)\, \widetilde{\gamma}}{2} \Sigma^2 \tag{32}$$

under the approximating model and

$$g^d = \psi (r - \delta) + \left(1 + \psi - \frac{2\vartheta}{\gamma + \vartheta}\right) \frac{\widetilde{\gamma}}{2} \Sigma^2 \tag{33}$$

under the distorted model. The value function is

$$J(k) = \Omega \frac{k^{1-\gamma}}{1-\gamma}, \tag{34}$$

where $\Omega = \left(\frac{2\psi\delta + (1-\psi)(g+r)}{1+\psi}\right)^{\frac{1-\gamma}{1-\psi}}$ *and* $g = g^a$ (g^d) *under the approximating model (the distorted model).*

In the undiversified equilibrium, the expected growth rate is

$$g^a = \psi (\mu - \delta) + \frac{1}{2} (1 - \psi)\, \widetilde{\gamma} \Sigma^2 \tag{35}$$

under the approximating model and

$$g^d = \psi (\mu - \delta) + \frac{1}{2} \left(1 - \psi - \frac{2\vartheta}{\gamma + \vartheta}\right) \widetilde{\gamma} \Sigma^2 \tag{36}$$

under the distorted model. The corresponding value function is

$$J(k) = \Omega \frac{k^{1-\gamma}}{1-\gamma}, \tag{37}$$

where $\Omega = (\mu - g)^{\frac{1-\gamma}{1-\psi}}$ *and* $g = g^a$ (g^d) *under the approximating model (the distorted model).*

Proof. See Appendix 6.2. ∎

In the diversified equilibrium, as shown in (32), under the approximating model the growth rate is decreasing with σ^2 and γ for any value of ψ because $\frac{\partial g^a}{\partial(\gamma\sigma^2)} < 0$. Furthermore, we also have

$$\frac{\partial g^a}{\partial \vartheta} < 0.$$

We can see from these results that the stronger the degree of model uncertainty, the more negative the correlation between the volatility of the fundamental shock and economic growth. It is clear from (33) that, under the distorted model, the growth rate is decreasing with σ^2 when $\psi > \frac{\vartheta-\gamma}{\gamma+\vartheta}$, and is increasing with σ^2 when $\psi < \frac{\vartheta-\gamma}{\gamma+\vartheta}$. Furthermore, under the distorted model, we can also conclude that

$$\frac{\partial g^d}{\partial \vartheta} < 0$$

because $\frac{\partial(-\vartheta/(\gamma+\vartheta))}{\partial \vartheta} < 0$ and $\frac{\partial \widetilde{\gamma}}{\partial \vartheta} > 0$. From (34), we can see that the total uncertainty, σ^2, influences the lifetime utility only through its effect on the growth rate, g. Specifically, in the diversified equilibrium, it is straightforward to show that given the initial level of k,

$$\operatorname{sign}\left(\frac{\partial J}{\partial \sigma^2}\right) = \operatorname{sign}\left(\frac{\partial g}{\partial \sigma^2}\right),$$

where $g = g^a$ or g^d, for any value of γ, ψ, and ϑ because $\frac{\partial J}{\partial g} > 0$. Figures 4 and 5 show that both the growth rate and lifetime utility measured by Ω are decreasing with the degree of RB, ϑ under both the approximating and distorted models for given γ and k_0. It is clear from these two figures that the economy would experience much lower economic growth and lower lifetime welfare if it is governed by the distorted model rather than the approximating model. For example, when $\vartheta = 1$, the growth rate is 2.1 percent in the approximating model, while it is only 0.4 percent in the distorted model.

In the undiversified equilibrium, under the approximating model, a fall in σ^2 increases growth when $\psi > 1$ but lowers it when $\psi < 1$. In contrast, under the distorting model, a fall in σ^2 increases growth when $\psi > 1 - \frac{2\vartheta}{\gamma+\vartheta}$ but lowers it when $\psi < 1 - \frac{2\vartheta}{\gamma+\vartheta}$. In other words, the presence of RB weakens the condition on ψ such that economic growth is inversely related to fundamental uncertainty. When the preference for RB is strong enough, a small value of ψ can still guarantee the inverse relationship between growth and volatility. Figure 6 illustrates the inverse relationship between EIS and RB for different values of risk aversion when $1 - \psi - \frac{2\vartheta}{\gamma+\vartheta} = 0$. It clearly shows that the critical value of ψ for generating the negative relationship between volatility and growth decreases with the value of ϑ.

In the undiversified equilibrium, we can see from (37) that the total uncertainty, σ^2, also influences the lifetime utility only through its effect on the growth rate, g. It is straightforward to show that given the initial level of k, when $\gamma > 1$,

$$\text{sign}\left(\frac{\partial J}{\partial \sigma^2}\right) = -\,\text{sign}\left(\frac{\partial g}{\partial \sigma^2}\right) \text{ when } \psi < 1;$$
$$\text{sign}\left(\frac{\partial J}{\partial \sigma^2}\right) = \text{sign}\left(\frac{\partial g}{\partial \sigma^2}\right) \text{ when } \psi > 1.$$

where $g = g^a$ or g^d, because $\frac{\partial J}{\partial g} > 0$ for any value of ϑ. Figures 7 and 8 plot how the growth rate and lifetime utility vary with the degree of RB in the undiversified equilibrium when $\psi = 0.2$. They clearly show that the growth rate is increasing with ϑ and lifetime utility measured by Ω is decreasing with ϑ under the approximating model, whereas the growth rate is decreasing and lifetime utility is increasing under the distorted model, given γ and k_0. This result is not surprising because RB affects the growth rate via two channels: (1) increasing the effective coefficient of relative risk aversion $\widetilde{\gamma}$ and (2) reducing the $1 - \psi - \frac{2\vartheta}{\gamma+\vartheta}$ term.

The differing effects of ψ under the approximating and distorted models lead us naturally to consider how to view the fears expressed by agents in the model. One possible interpretation is that these fears are unjustified (they are "entirely in the head" of the agents); in this case, which is the usual one applied by Hansen and Sargent (2007), the connection between volatility and growth is unambiguous for diversified economies and is unambiguous for undiversified economies once we know the value of ψ. However, the distorted model cannot be easily dismissed as a description of the world; we show later that calibration of ϑ via detection error probabilities means that agents cannot reject the hypothesis that the distorted model describes the data, so their fears need not be ignored as imaginary.

The following proposition summarizes the relationship between γ and ϑ in the RB model:

Proposition 5. *In the RB version of the Obstfeld model, the parameters governing risk aversion and uncertainty aversion, γ and ϑ, are observationally equivalent in the sense that they lead to the same growth rate and lifetime utility if the true economy is governed by the approximating model. In contrast, the observational equivalence does not hold if the true economy is governed by the distorted model.*

Proof. The proof is straightforward by inspecting Equations (32), (33), (35), and (36). ■

From (27) and (28), it is clear that the RB model with the coefficient of relative risk aversion γ and the degree of RB ϑ and the FI model with $\widetilde{\gamma} = \gamma + \vartheta > \gamma$ are observationally equivalent in the sense that they lead to the same consumption and portfolio rules.[8] However, the two model economies lead to different state transition dynamics when the true economy is governed by the distorted model.

[8]Maenhout (2004) obtained the same conclusion in an otherwise standard Merton model.

3. Estimation and Calibration

Section 3.1 presents evidence on the negative correlation between growth and volatility. Section 3.2 explains how the RB parameter can be calibrated using the detection error probability and reports quantitative results. Section 3.3 provides an alternative way to estimate the RB parameter based on the key model solutions, which quantifies the degree of model uncertainty that is consistent with the data.

3.1. Empirical Evidence

In this section we present evidence on the negative relationship between growth and volatility. This negative relationship will be seen in both a simple scatter plot and a regression analysis controlling the across-country heterogeneity.[9]

The data we use come from the Penn World Tables (version 8.0), which contains national accounts data on a wide set of countries that has been chained and converted to $USD. This conversion allows us to compare GDP levels between countries as well as across time. We construct a sample that is as close as possible as in Ramey and Ramey (1995) but extend the time horizon to cover more recent years. In particular, our sample consists of 80 countries and covers the $1962-2011$ period.[10] The list of countries is reported in the Appendix. Figure 2 provides a graphical view on the relationship between GDP growth and its volatility by plotting the mean real per-capita GDP growth rate in the $1962-2011$ period for each country against its standard deviation. This figure shows there is a negative relationship between GDP growth and its volatility, but the effect is clearly not strong.

However, this simple correlation may be biased due to the heterogeneity across countries. To provide a more accurate measure of the volatility and isolate the connection to growth, we follow Ramey and Ramey (1995) by controlling for country-specific effects. Our control variables include mean investment share of GDP for each country over the sample period, real per-capita GDP in 1962 (logged), and a human capital index, data for all of which is included in the Penn World Tables. We first regress real per-capita GDP growth for each country in each year on the set of control variables and compute the residuals, which will represent the GDP growth uncorrelated with our explanatory series. We next compute the standard deviation of these residuals for each country to provide a measure of the volatility of unexplained growth. This variable serves as our proxy for volatility, and is put back into the original regression with the control variables for the

[9] We are not the first to study this relationship: other papers include Hnatkovska and Loayza (2003), Imbs (2006), and Miranda-Pinto (2016).

[10] There are 167 countries in the Penn World Tables dataset, 134 of which contain all of the variables used in the analysis (most countries excluded were missing the human capital variable). Of these 134 countries, 98 contain all variables of interest for the full sample period of 1962-2011. An additional 18 countries were removed since they were not contained in the Ramey and Ramey (1995) analysis. This procedure leaves us with a sample of 80 countries with observations from 1962 to 2011, yielding a total of 4000 observations.

second stage of the regression. As seen Table 1, volatility of growth has a significant negative effect on per-capita real GDP growth.[11]

In addition, our control variables are largely irrelevant – only investment's share of GDP is correlated with growth in any important way (initial GDP is significant but clearly not quantitatively important). Consistent with some AK-style endogenous growth models (such as Rebelo 1991), we do find that high investment countries grow faster. See McGrattan (1998) for a discussion of how this observation provides support for AK-style endogenous growth models; see also Farmer and Lahiri (2006).

As we noted above, from Equation (24) it is clear that the standard full-information Obstfeld model cannot generate the negative relationship between the volatility and mean growth rate of real GDP per capita we observed in the data unless the economy is in an undiversified equilibrium with an EIS larger than 1. As noted also, there is some uncertainty regarding the value of the EIS, so it is not clear that the model is a good instrument for measuring the welfare gains from diversification as they depend critically on the relationship between growth and volatility. In the next subsections, we will explore how and to what extent introducing RB can help make the model fit the data better in this aspect.

3.2. Calibrating the Robustness Parameter

To fully explore how RB affects the relationship between volatility and growth, we adopt the calibration procedure outlined in HSW (2002) and AHS (2003) to calibrate the value of the RB parameter (ϑ) that governs the degree of robustness. Specifically, we calibrate ϑ by using the detection error probability (DEP) based on a statistical theory of model selection. We can then infer what values of ϑ imply reasonable fears of model misspecification for empirically-plausible approximating models. The model detection error probability denoted by p is a measure of how far the distorted model can deviate from the approximating model without being discarded; low values for this probability mean that agents are unwilling to discard many models, implying that the cloud of models surrounding the approximating model is large. In this case, it is easier for the consumer to distinguish the two models. The value of p is determined by the following procedure. Let model P denote the approximating model, (29):

$$\left(\frac{dk_t}{k_t}\right)^a = \left[\psi\left(i-\delta\right) + \left(1+\psi\right)\frac{\left(\mu-i\right)^2}{2\widetilde{\gamma}\sigma^2}\right]dt + \alpha\sigma dB_t,$$

[11] See also Imbs (2006), Dabušinskas, Kulikov, and Randveer (2012), Hnatkovska and Loayza (2004), and Miranda-Pinto (2016) for alternative approaches that also find a negative correlation between growth and volatility.

and model Q be the distorted model, (25):

$$\left(\frac{dk_t}{k_t}\right)^d = \left[\psi(i-\delta) + \left(1+\psi - \frac{2\vartheta}{\gamma+\vartheta}\right)\frac{(\mu-i)^2}{2\widetilde{\gamma}\sigma_k^2}\right]dt + \alpha\sigma dB_t,$$

Define p_P as

$$p_P = \text{Prob}\left(\ln\left(\frac{L_Q}{L_P}\right) > 0 \middle| P\right), \tag{38}$$

where $\ln\left(\frac{L_Q}{L_P}\right)$ is the log-likelihood ratio. When model P generates the data, p_P measures the probability that a likelihood ratio test selects model Q. In this case, we call p_P the probability of the model detection error. Similarly, when model Q generates the data, we can define p_Q as

$$p_Q = \text{Prob}\left(\ln\left(\frac{L_P}{L_Q}\right) > 0 \middle| Q\right). \tag{39}$$

Given initial priors of 0.5 on each model and that the length of the sample is N, the detection error probability, p, can be written as:

$$p(\vartheta; N) = \frac{1}{2}(p_P + p_Q), \tag{40}$$

where ϑ is the robustness parameter used to generate model Q. Given this definition, we can see that $1-p$ measures the probability that econometricians can distinguish the approximating model from the distorted model.

The general idea of the calibration procedure is to find a value of ϑ such that $p(\vartheta; N)$ equals a given value (for example, 10%) after simulating model P, (29), and model Q, (25).[12] In the continuous-time model with the iid Gaussian specification, $p(\vartheta; N)$ can be easily computed. Because both models P and Q are arithmetic Brownian motions with constant drift and diffusion coefficients, the log-likelihood ratios are Brownian motions and are normally distributed random variables. Specifically, the logarithm of the Radon-Nikodym derivative of the distorted model (Q) with respect to the approximating model (P) can be written as

$$\ln\left(\frac{L_Q}{L_P}\right) = -\int_0^N \overline{v}dB_s - \frac{1}{2}\int_0^N \overline{v}^2 ds, \tag{41}$$

where

$$\overline{v} \equiv v^*\alpha\sigma_k = \left(-\frac{\vartheta}{\gamma+\vartheta}\right)\left(\frac{\mu-i}{\sigma}\right). \tag{42}$$

Similarly, the logarithm of the Radon-Nikodym derivative of the approximating model (P) with

[12] The number of periods used in the calculation, N, is set to be 50, the actual length of the data we study.

respect to the distorted model (Q) is

$$\ln\left(\frac{L_P}{L_Q}\right) = \int_0^N \overline{v} dB_s + \frac{1}{2}\int_0^N \overline{v}^2 ds. \tag{43}$$

Using (38)-(43), it is straightforward to derive $p(\vartheta; N)$:

$$p(\vartheta; N) = \Pr\left(x < \frac{\overline{v}}{2}\sqrt{N}\right), \tag{44}$$

where x follows a standard normal distribution. From the expressions of $\overline{v}$, (42), and $p(\vartheta; N)$, (44), we can show that the value of p is decreasing with the value of ϑ because $\partial\overline{v}/\partial\vartheta < 0$. From (42) and (44), it is clear that the calibration of the value of ϑ is independent of both the elasticity of intertemporal substitution (ψ) and the discount rate (δ).

Following Turnovsky and Chattopadhyay (2003), we set $\alpha = 0.6$, $a = 0.3$, and $\delta = 0.04$ such that the capital share is 0.4, the long-run capital-to-output ratio is 3.3, and the mean net return to capital is 8 percent. Using the data set documented in Section 3.1., we set the parameter values for the processes of returns, volatility, and consumption as follows: $\mu = 0.07$, $r = 0.01$, and $\sigma = 0.156$. Figure 9 illustrates how DEP (p) varies with the value of ϑ for different values of γ. We can see from the figure that the stronger the preference for robustness (higher ϑ), the less the p is. For example, when $\gamma = 1.5$, $p = 0.293$ percent when $\vartheta = 1$, while $p = 0.219$ when $\vartheta = 2$.[13] Both values of p are reasonable as argued in AHS (2002), HSW (2002), Maenhout (2004), and Hansen and Sargent (Chapter 9, 2007).

Figures 10 illustrates how p varies with the value of ϑ when σ is increased from 0.1 to 0.2. This figure also shows that for different values of σ, the higher the value of ϑ, the less the p is. In addition, to calibrate the same value of p, less values of σ (i.e., more volatile stock market) lead to higher values of ϑ. The intuition behind this result is that σ and ϑ have opposite effects on $\overline{v}$. (This is clear from (42).) To keep the same value of p, if one parameter increases, the other one must reduce to offset its effect on $\overline{v}$. As emphasized in Hansen and Sargent (2007), in the robustness model, p is the deep model parameter governing the amount of model uncertainty, and ϑ reflects the effect of RB on the model's behavior.

Given (33), it is straightforward to show that the growth rate and volatility of real GDP can be negatively correlated when

$$\vartheta > \frac{1+\psi}{1-\psi}\gamma \tag{45}$$

for plausible calibrated robustness parameter ϑ and plausible values of γ and ψ. For example, when $\gamma = 1.5$, $\psi = 0.2$, and $\sigma = 15.6$ percent, we can calibrate that $\vartheta > 2.25$ by setting $p < 0.2173$

[13] Most empirical studies show that $\gamma \in [1, 5]$ are plausible values for the coefficient of relative risk aversion.

such that $\vartheta > \frac{1+\psi}{1-\psi}\gamma = 2.25$ always holds.[14]

3.3. Estimating the RB Parameter

The approach based on the detection error probability in the previous section provides an intuitive way to calibrate the degree of model uncertainty. Alternatively, we can estimate this parameter using the key growth equation in the RB model. For example, assuming the distorted model generates the data, we can estimate Equation (33) by regressing growth, g^d, on growth variance, Σ^2. Then, we can back out the value of ϑ from the regression coefficient which represents the value that is consistent with the data. The estimated value of ϑ can also be converted to the implied value of the DEP which measure the degree of model uncertainty implied by the data.

The regression uses the same data and follows the same two-stage procedure as described in Section 3.1 and Ramey and Ramey (1995). The results are reported in Table 2. It shows that a 1-percentage point increase in the variance of growth leads to a reduction in growth of 1.05 percentage points. Based on Equation (33), this elasticity implies

$$\left(1+\psi-\frac{2\vartheta}{\gamma+\vartheta}\right)\frac{\gamma+\vartheta}{2} = -1.05 \text{ or } \vartheta = \frac{2.1}{1-\psi}+\frac{1+\psi}{1-\psi}\gamma. \tag{46}$$

From (46), it is clear that ϑ will be negative if $\psi > 1$. Figure 11 illustrates how ϑ increases with ψ for given plausible values of γ. For example, by setting $\gamma = 1.5$ and $\psi = 0.2$, the uncovered value of ϑ is 4.87. $\vartheta = 4.87$ implies that the degree of model uncertainty, represented by the DEP, needs to be $p = 0.30$ to account for the observed relationship between growth and volatility in the data. Hansen and Sargent (2007) argue via introspection that values of p greater than 0.2 are "reasonable", although they are not clear about their justification.

3.4. Testing Models

The discussions in the above two subsections can be extended to test across different models to see which one is consistent with the data. As we have shown in the previous section, there are four types of equilibrium, depending on whether the approximating model or the distorted model generates the data, and whether the economy has a diversified or an undiversified equilibrium. In each of these four cases, the relationship between growth and volatility is different, as shown in Proposition 4. The crucial term which determines this relationship is a function of three parameters, the risk aversion parameter γ, the elasticity of intertemporal substitution parameter ψ, and the robustness parameter ϑ. By assigning reasonable values to these parameters, we can test if these models are consistent with the data regarding the negative growth and volatility relationship.

To focus on the effect of robustness, we set $\gamma = 1.5$ and $\psi = 0.2$. We first plug in values of the

[14]Note that for a given value of p, the calibrated value of ϑ is independent of EIS, ψ.

robustness parameter, ϑ, that correspond to reasonable detection error probability. As an example, Table 3 reports two calibrated values of ϑ corresponding to a detection error probability of $p = 0.22$ and $p = 0.29$, respectively, and the implied values of the key term in the growth equation which determines the relationship between growth and volatility. It shows that when $\vartheta = 1$, none of the four models can generate a negative correlation. When $\vartheta = 2$, only the distorted model in an undiversified equilibrium generates a negative correlation. This exercise gives us a sense how strong the degree for robustness needs to be to generate a negative correlation between growth and volatility. Following this idea, we can use the reduced-form equation in each model to estimate the value of the robustness parameter. That is, we can let data tell us how strong the degree of robustness needs to be in order to generate a similar negative relationship between growth and volatility in the data. These results are reported in Table 4.

Table 4 uses the key estimated coefficient in the volatility-growth equation, -1.05, in Table 2 and the model derived expression to solve for the corresponding value of ϑ in each model. The fourth column shows the corresponding detection error probability. We report the value of the detection error probability so we can check whether the model-implied value of ϑ makes sense. In other words, we don't want to just solve for a value of ϑ and claim the corresponding model matches the data; in contrast, we want the solved value of ϑ to imply a reasonable amount of uncertainty. Following this logic, the results in Table 4 shows only the distorted models generate the negative growth-volatility relationship with a plausible value of the robustness parameter. If the economy is in a diversified equilibrium, the distorted model can match the observed growth and volatility relationship under a detection error probability of $p = 0.15$. Similarly, if the economy is in an undiversified equilibrium, the distorted model can explain the data under a detection error probability of $p = 0.19$. In contrast, the approximating model cannot explain the data in either an diversified equilibrium or an undiversified equilibrium (the implied ϑ value is negative).

Our analytical results show that the assumption that $\psi < 1$ renders the approximating model unable to replicate the observed growth-volatility relationship. To explore the importance of this assumption, we set $\psi = 1.5$; Table 5 shows that only the undiversified equilibria can match the negative correlation, and both generate reasonable values for p. We have noted already our skepticism that the undiversified equilibrium is reasonable empirically. Comparing the two tables we conclude that the most reasonable environment has $\psi < 1$ and is governed by the distorted model in a diversified equilibrium.

4. International Integration under RB

In this section we extend the above benchmark closed-economy model to a multi-country economy. Following Obstfeld (1994), we now assume that there are N countries, indexed by $j = 1, 2, \cdots, N$, and the representative agent in country j has a coefficient of relative risk aversion γ_j, an elasticity of intertemporal substitution ψ_j, a discount rate δ_j, and a parameter governing the preference for robustness ϑ_j. We will generally suppose that preferences are homogeneous; however, for reasons we outline in the next section, that will imply that ϑ will generally not be the same across countries.[15] The interpretation we prefer is that preferences, which are represented by p, are still homogeneous, but that the resulting amount of uncertainty will vary according to the amount of risk (represented by ϑ). To completely understand the mechanisms of the model, however, we will consider heterogeneity in ϑ directly without necessarily assuming heterogeneity in σ.

In an integrated global equilibrium, there is a single risk free interest rate i^*. Country j's expected growth rate can be written as

$$g^a = \psi_j \left(i^* - \delta_j\right) + \left(1 + \psi_j\right) \frac{\left(\mu^* - i^*\right)^2}{2\widetilde{\gamma}_j \left(\sigma^*\right)^2}, \tag{47}$$

where $\widetilde{\gamma}_j = \gamma_j + \vartheta_j$, if the economy is governed by the approximating model, and

$$g^d = \psi_j \left(i^* - \delta_j\right) + \left(1 + \psi_j - \frac{2\vartheta_j}{\gamma_j + \vartheta_j}\right) \frac{\left(\mu^* - i^*\right)^2}{2\widetilde{\gamma}_j \left(\sigma^*\right)^2}, \tag{48}$$

if the economy is governed by the distorted model. If there exists risk free capital, we have $i^* = r$; if not, $i^* = \mu^* - \gamma^* \left(\sigma^*\right)^2 > r$.

Following Obstfeld (1994), the welfare gain from financial integration can be calculated as an equivalent variation: by what percentage must financial wealth be increased under financial autarky to leave the representative household indifferent to integration? Using (34), the equivalent variation for country j, Λ_j, can be written as

$$\Lambda_j = \left(\frac{m_j^*}{m_j}\right)^{1/\left(1-\psi_j\right)} - 1 = \left(\frac{2\psi_j\delta_j + \left(1 - \psi_j\right)\left(g_j^* + i^*\right)}{2\psi_j\delta_j + \left(1 - \psi_j\right)\left(g_j + i_j\right)}\right)^{1/\left(1-\psi_j\right)} - 1,$$

where m_j and m_j^* are the marginal propensity to consume before and after financial integration.

4.1. Symmetric Case

Consider the same situation discussed in Obstfeld (1994) in which all countries are diversified before integration and some countries still hold the risky asset afterward. In this case, all countries share the common interest rate r, before and after the integration. From (34), we can see that welfare is positively related to growth. International portfolio diversification encourages a global shift from relatively low return and low risk investments to high return and high risk investments.

We now consider a similar numerical example in Obstfeld (1994) and assume a symmetric two-

[15] We have studied the effect of heterogeneous ϑ in our other work, in particular Luo, Nie, and Young (2012, 2014).

country economy ($N = 2$) in which $r = 0.02$, $\mu_1 = \mu_2 = \mu = 0.05$, $\sigma_1 = \sigma_2 = \sigma = 0.1$, and the returns are uncorrelated. Preferences are the same in the two countries with $\gamma = 4$, $\psi = 0.5$, $\vartheta = 2$, and $\delta = 0.02$. Under financial autarky, the optimal share invested in the domestic risky portfolio in each country is $\alpha^* = \frac{\mu - r}{\widetilde{\gamma}\sigma^2} = 0.5$. Using Equations (47) and (48), we can compute the growth rates of individual countries. As shown in the upper panel of Table 6, the implied growth rates in the approximating model and the distorted model are 1.13 percent and 0.63 percent, respectively. Note that here the risk-free real rate of interest, i, is equal to r.

We now let the two countries financially integrate. Since the optimal global mutual fund is divided equally between the two risky assets, the mean rate of return μ is still 0.05 and the return variance σ^2 is $0.1^2/2$. Each country's total demand for the risky assets is now $\alpha^* = \frac{\mu - i^*}{\widetilde{\gamma}\sigma^2}$, where $\widetilde{\gamma} = \gamma + \vartheta$. If $i = 0.02$ we have $\alpha^* = 1$. That is, financial integration leads to a undiversified equilibrium in which risk free assets are no longer held. The intuition that the equilibrium interest rate remains the same after financial integration is that the reduction in the variability of wealth and the presence of robustness have opposite effects on precautionary saving and the effects are just cancelled out in equilibrium. As shown in the upper panel of Table 6, in the integrated equilibrium the implied growth rates in the approximating model and the distorted model are 2.25 percent and 1.25 percent, respectively. Under both the approximating and distorted models, growth has doubled due to integration because agents are no longer induced, via risk aversion and uncertainty aversion, to hold low-return riskless assets as a hedge against their risky asset returns. Furthermore, the welfare gain from financial integration measured by Λ in the approximating model and the distorted model are 33.88 percent and 19.58 percent of initial wealth, respectively. It is clear from the table that the growth rate and the welfare gain from financial integration are much lower in the distorted model than in the approximating model; evil nature selects a negative shift in the mean growth rate as the most destructive distortion. Figure 12 illustrates how the welfare gain varies with the value of EIS (ψ) under RB for different values of ϑ. It is clear from the figure that the welfare gain is always larger under the approximating model than under the distorted model for different values of ϑ and ψ. In addition, given ϑ, the welfare gain is increasing with ε. It is worth noting that the growth rates and the welfare gain in both cases are lower than that implied in the full-information Obstfeld model: $g = 1.687$ percent before integration, $g = 2$ percent after integration, and $\Lambda = 37.1$ percent.

To further explore the key difference between the standard Obstfeld model and our RB model, we do another numerical exercise by assuming that $\gamma = 2$ and holding all other parameters be as in the previous example. The reason for this assumption is to keep the effective coefficient of relative risk aversion the same between our model and the Obstfeld model. (Note that in our RB model, $\widetilde{\gamma} = \gamma + \vartheta = 4$.) Comparing the lower panel of Table 6 with the results reported in Obstfeld (1994), it is clear that the financial integration leads to the same growth improvement and welfare gains when the economy is governed by the approximating model, while these gains are proportionally larger when the economy is governed by the distorted model. Notice that one key difference between Examples 1 and 2 is that the economy moves from a diversified economy in which agents hold both risky assets and risk-free assets to an undiversified economy in which only risky assets are held. The implication of this change is that the equilibrium risk-free interest rate, i_t, ais higher in the new equilibrium. From the expression of the welfare gain, Λ_j, we can see that both an increase in growth and an increase in risk-free interest rate contribute to the welfare gain. In addition, the growth improvement can be different in an economy governed by the approximating model and an economy governed by the distorted model. Depending on the original growth rate before the financial integration and how much growth is improved after the financial integration, an economy could experience larger welfare gains under the distorted model.

In general, we can prove that if the economy remains in a diversified equilibrium after integration, the welfare gain is larger if the economy is governed by the approximating model than if it is governed by the distorted model.

Proposition 6. *Let $\psi < 1$. The welfare gain from financial integration, measured by Λ_j, declines with the degree of robustness in the diversified equilibrium after integration. In addition, for the same country, the welfare gain from financial integration under the distorted model is lower than that under the approximating model.*

Proof. See Appendix 6.3. ■

However, if the economy switches from a diversified to an undiversified equilibrium after financial integration, the relative size of the welfare gain depends on the actual change in growth rates.

4.2. Asymmetric Case

As another example, suppose we permit heterogeneity in ϑ. Specifically, we consider a case where $\vartheta_1 = 2$ and $\vartheta_2 = 4$. Under financial autarky, the optimal share invested in the domestic risky portfolio in the two countries are:

$$\alpha_1^* = \frac{\mu - r}{\widetilde{\gamma}_1 \sigma^2} = 0.5 \text{ and } \alpha_2^* = \frac{\mu - r}{\widetilde{\gamma}_2 \sigma^2} = 0.375,$$

respectively. As shown in Table 7, under financial autarky the implied growth rate in the two countries (g^a) are 1.125 percent and 0.84 percent, respectively, when the true economy is governed by the approximating model. In contrast, they are 6.25 percent and 0.28 percent, respectively when the true economy is governed by the distorted model. In other words, the country having stronger preference for robustness and facing greater model uncertainty experience lower economic growth under financial autarky when the true economy is governed by either the approximating model or the distorted model.

After we integrate the financial markets, the optimal global mutual fund is still divided equally between the two risky capital stocks; the mean and variance of returns to the global capital stock fund are μ_g and $\sigma^2/2$, respectively. The two countries' total demand functions for risky capitals differ because $\vartheta_1 \neq \vartheta_2$:

$$\alpha_1^* = 2\frac{\mu - i}{\widetilde{\gamma}_1\sigma^2} = 1 \text{ and } \alpha_2^* = 2\frac{\mu - i}{\widetilde{\gamma}_2\sigma^2} = 0.75.$$

In this case, we cannot construct the same equilibrium in which $\alpha_1^* = \alpha_2^* = 1$ as in the symmetric case; instead, country 2 holds 75 percent of assets in risky equities, divided equally between the two countries, and holds 25 percent in riskless capital while country 1 ceases to hold riskless capital completely. The growth rate and welfare gain of Country 1 after integration are the same as that obtained in the symmetric case in which $g^a = 2.25$ percent and $\Lambda = 33.88$ percent under the approximating model and $g^d = 1.25$ percent and $\Lambda = 19.58$ percent under the distorted model.

As shown in Table 7, for Country 2 with $\vartheta = 4$, in the integrated equilibrium, we have $g^a = 1.69$ percent and $\Lambda = 26.40$ percent under the approximating model, and $g^d = 0.56$ and $\Lambda = 9.12$ percent under the distorted model. That is, both growth rate and welfare gain are significantly increased after integration in both countries under either the approximating and distorted models. In addition, for the country with stronger preference for RB (Country 2), the welfare gain under the distorted model is much lower than that under the approximating model, i.e., Π_2 only accounts for 34.55 percent of Π_1.

In the above asymmetric case, given the same value of σ in both countries, stronger preference for robustness (i.e., higher value of ϑ) implies greater amount of model uncertainty ($\vartheta\sigma^2$). We consider another example in which we assume not only $\vartheta_2 > \vartheta_1$, but that $\sigma_2 > \sigma_1$. For example, let $\vartheta_1 = 2$, $\vartheta_2 = 4$, $\sigma_1 = 0.1$, and $\sigma_2 = 0.2$. Then we have

$$\alpha_1^* = \frac{\mu - r}{\widetilde{\gamma}_1\sigma_1^2} = 0.5 \text{ and } \alpha_2^* = \frac{\mu - r}{\widetilde{\gamma}_2\sigma_2^2} = 0.094.$$

Country 1 splits the portfolio evenly between risky and riskless assets, but does not split the risky assets evenly between capital stocks in each country (because $\sigma_1 < \sigma_2$, country 1's capital constitutes a larger fraction of each portfolio).

5. Conclusion

In this paper we have documented a negative relationship between average growth and average volatility in a cross-section of countries and showed that introducing model uncertainty due to a preference for robustness into an otherwise standard Obstfeld (1994) model has the potential to explain the negative relationship between the growth rate and the volatility of real GDP; in contrast, the basic model cannot replicate this relationship. We then use the model to estimate the welfare gains from international diversification. We find that gains are large and biased toward countries that have face low ambiguity. Our model could be extended to include other sources of risk, such as fiscal policy (Eaton 1981), and then to assess which countries gain from integration and how that is related to policy choices.

6. Appendix

6.1. Solving the Two-Asset Case in the Full-information Case

In the full-information case, the FOCs for consumption and portfolio choice are

$$\begin{aligned} c_t &= \delta^{\psi} J_k^{-\psi} \left[(1-\gamma) J\right]^{\frac{1-\gamma\psi}{1-\gamma}}, \\ \alpha_t &= -\frac{J_k (\mu - i)}{J_{kk} k_t \sigma_k^2}, \end{aligned}$$

respectively. Substituting these FOCs into the Bellman equation,

$$0 = \frac{\delta (1-\gamma) J}{1 - 1/\psi} \left[\left(\frac{c}{\left[(1-\gamma) J\right]^{1/(1-\gamma)}} \right)^{1-1/\psi} - 1 \right] + J_k r k_t - J_k c_t + J_k \alpha_t (\mu - i) k_t + \frac{1}{2} J_{kk} \alpha_t^2 k_t^2 \sigma^2,$$

yields the following ODE:

$$\begin{aligned} 0 = {} & \frac{\delta (1-\gamma) J}{1 - 1/\psi} \left[J_k^{1-\psi} \left[(1-\gamma) J\right]^{\frac{\gamma(1-\psi)}{1-\gamma}} \delta^{\psi-1} - 1 \right] + J_k r k_t - J_k J_k^{-\psi} \left[(1-\gamma) J\right]^{\frac{1-\gamma\psi}{1-\gamma}} \delta^{\psi} + \\ & J_k \left(-\frac{J_k (\mu - i)}{J_{kk} k_t \sigma^2} \right) (\mu - i) k_t + \frac{1}{2} J_{kk} \left(-\frac{J_k (\mu - i)}{J_{kk} k_t \sigma^2} \right)^2 k_t^2 \sigma^2. \end{aligned}$$

Conjecture that the value function is $J(k_t) = \frac{A k_t^{1-\gamma}}{1-\gamma}$ for some constant A. Dividing by J_k and k_t on both sides of the above ODE yields

$$0 = \frac{\delta}{1 - 1/\psi} \left(A^{\frac{1-\psi}{1-\gamma}} \delta^{\psi-1} - 1 \right) + \left(r - A^{\frac{1-\psi}{1-\gamma}} \delta^{\psi} \right) + \frac{(\mu - i)^2}{\gamma \sigma^2} - \frac{1}{2} \gamma \left(\frac{\mu - i}{\gamma \sigma^2} \right)^2 \sigma^2,$$

which implies that

$$A = \left\{ \delta^{-\psi} (1-\psi) \left[r - \frac{\delta}{1 - 1/\psi} + \frac{(\mu - r)^2}{2\gamma\sigma^2} \right] \right\}^{\frac{1-\gamma}{1-\psi}}.$$

Substituting it back to into $c_t = \delta^{\psi} J_k^{-\psi} \left[(1-\gamma) J\right]^{\frac{1-\gamma\psi}{1-\gamma}}$ yields the consumption function, (13), in the main text. Substituting the consumption function into the resource constraint gives (14) in the main text.

6.2. Solving the Two-Asset Case in the RB Case

Solving first for the infimization part of (26) yields:

$$v^*(k_t) = -\vartheta(k_t) J_k.$$

Substituting for $v^*(k_t)$ in the robust HJB equation gives:

$$\sup_{c_t,\alpha_t}\left\{\begin{array}{c}\frac{\delta(1-\gamma)J}{1-1/\psi}\left[\left(\frac{c}{[(1-\gamma)J]^{1/(1-\gamma)}}\right)^{1-1/\psi}-1\right]+ \\ J_k\left[(r+\alpha_t(\mu-i))k_t-c_t\right]+\frac{1}{2}J_{kk}\alpha_t^2k_t^2\sigma^2-\frac{1}{2}\vartheta(k_t)(\alpha_tk_t\sigma)^2J_k^2\end{array}\right\} \tag{49}$$

From (49), the FOCs for consumption and portfolio choice are:

$$\begin{aligned} c_t &= \delta^{\psi}J_k^{-\psi}\left[(1-\gamma)J\right]^{\frac{1-\gamma\psi}{1-\gamma}}, \\ \alpha_t &= -\frac{J_k(\mu-i)}{J_{kk}k_t\sigma^2-\vartheta(k_t)k_t\sigma^2J_k^2}, \end{aligned}$$

respectively.

Substituting these FOCs into the Bellman equation yields the following ODE:

$$0=\frac{\delta(1-\gamma)J}{1-1/\psi}\left[\left(\frac{\delta^{\psi}J_k^{-\psi}\left[(1-\gamma)J\right]^{\frac{1-\gamma\psi}{1-\gamma}}}{\left[(1-\gamma)J\right]^{1/(1-\gamma)}}\right)^{1-1/\psi}-1\right]+J_krk_t-J_kc_t-J_k\left(\frac{J_k(\mu-i)}{J_{kk}k_t\sigma^2-\vartheta(k_t)k_t\sigma^2J_k^2}\right)(\mu-r)k_t+$$
$$\frac{1}{2}J_{kk}\left(-\frac{J_k(\mu-i)}{J_{kk}k_t\sigma^2-\vartheta(k_t)k_t\sigma^2J_k^2}\right)^2k_t^2\sigma_k^2-\frac{1}{2}\vartheta\left[\frac{\mu-i}{(\gamma+\vartheta)\sigma^2}\right]^2k_t\sigma^2J_k,$$

where we assume that $\vartheta(k_t)=\frac{\vartheta}{(1-\gamma)J(k_t)}>0$. Conjecture that the value function is $J(k_t)=\frac{Ak_t^{1-\gamma}}{1-\gamma}$. Divided by J_k and k_t on both sides of the above ODE yields:

$$0=\frac{\delta}{1-1/\psi}\left(A^{\frac{1-\psi}{1-\gamma}}\delta^{\psi-1}-1\right)+\left(i-A^{\frac{1-\psi}{1-\gamma}}\delta^{\psi}\right)+\frac{(\mu-i)^2}{(\gamma+\vartheta)\sigma^2}-\frac{1}{2}(\gamma+\vartheta)\left[\frac{\mu-i}{(\gamma+\vartheta)\sigma^2}\right]^2\sigma^2,$$

which implies that

$$A^{\frac{1-\psi}{1-\gamma}}=\left\{\delta^{-\psi}\ (1-\psi)\left[i-\frac{\delta}{1-1/\psi}+\frac{1}{2}\frac{(\mu-i)^2}{\widetilde{\gamma}\sigma^2}\right]\right\}^{\frac{1-\gamma}{1-\psi}}.$$

Substituting it back to into $c_t=\delta^{\psi}J_k^{-\psi}\left[(1-\gamma)J\right]^{\frac{1-\gamma\psi}{1-\gamma}}$ yields the consumption function, (28), in the main text.

Under the approximating model, substituting the consumption function, (28), into the resource constraint gives the following expression for the expected growth rate:

$$g = \psi\left(i - \delta\right) + \left(1 + \psi\right) \frac{\left(\mu - i\right)^2}{2\widetilde{\gamma}\sigma^2}.$$

In contrast, under the distorted model, substituting (28) into the following evolution equation,

$$dk_t = \left[\left(\alpha\left(\mu - i\right) + r\right) k_t - c_t + \left(\alpha_t k_t \sigma\right)^2 v\left(k_t\right)\right] dt + \sigma\alpha k_t dB_t,$$

we have

$$\begin{aligned}
\frac{dk_t}{k_t} &= \left\{\alpha\left(\mu - i\right) + r - \left(1 - \psi\right)\left[i - \frac{\delta}{1 - 1/\psi} + \frac{1}{2}\frac{\left(\mu - i\right)^2}{\widetilde{\gamma}\sigma^2}\right] - \alpha_t^2 k_t^2 \sigma^2 \frac{\vartheta}{\left(1 - \gamma\right) J} J_k\right\} dt + \alpha\sigma dB_t \\
&= \left\{\frac{\left(\mu - i\right)^2}{\widetilde{\gamma}\sigma^2} + r - \left(1 - \psi\right)\left[i - \frac{\delta}{1 - 1/\psi} + \frac{1}{2}\frac{\left(\mu - i\right)^2}{\widetilde{\gamma}\sigma^2}\right] - \alpha_t^2 \sigma^2 \vartheta\right\} dt + \alpha\sigma dB_t \\
&= \left[\psi\left(i - \delta\right) + \left(1 + \psi - \frac{2\vartheta}{\widetilde{\gamma}}\right)\frac{\left(\mu - i\right)^2}{2\widetilde{\gamma}\sigma^2}\right] dt + \alpha\sigma dB_t
\end{aligned}$$

6.3. Proof of Proposition 6

We first prove that the welfare improvement after financial integration for country j, Λ_j, is a decreasing function of the degree of robustness, ϑ, under the diversified equilibrium:

$$\begin{aligned}
\Lambda_j &= \left(\frac{2\psi_j\delta_j + \left(1 - \psi_j\right)\left(g_j^* + i^*\right)}{2\psi_j\delta_j + \left(1 - \psi_j\right)\left(g_j + i_j\right)}\right)^{1/\left(1 - \psi_j\right)} - 1 \\
&\equiv \left(\frac{l + g_j^* + r}{l + g_j + r}\right)^{1/\left(1 - \psi_j\right)} - 1,
\end{aligned} \tag{50}$$

where $l \equiv \frac{2\psi_j\delta_j}{1 - \psi_j}$, and we have used the fact that $i = r$ in the diversified equilibrium.

For convenience, we drop all subscripts in the growth equations. In the diversified equilibrium, the growth rates under the approximating model and the distorted model are given by

$$g^a = \psi\left(r - \delta\right) + \frac{\left(1 + \psi\right)\widetilde{\gamma}}{2}\Sigma^2 \tag{51}$$

and

$$g^d = \psi\left(r - \delta\right) + \left(1 + \psi - \frac{2\vartheta}{\gamma + \vartheta}\right)\frac{\widetilde{\gamma}}{2}\Sigma^2, \tag{52}$$

respectively. Using the expression for Σ, we can rewrite these equations as

$$g = p + q^i(\vartheta)\frac{(\mu - r)^2}{\sigma^2}, (i = a, d),$$

where $p = \psi(r - \delta)$, $q^a(\vartheta) = \frac{(1+\psi)}{2(\gamma+\vartheta)}$, and $q^d(\vartheta) = \left(1 + \psi - \frac{2\vartheta}{\gamma+\vartheta}\right)\frac{1}{2(\gamma+\vartheta)}$.

Notice that the effect of the financial integration is to change country-specific σ and μ to the equilibrium σ^* and μ^*, which is independent of the degree of robustness ϑ^j in country j. Define $m = \frac{(\mu-r)^2}{\sigma^2}$ and $m^* = \frac{(\mu^*-r)^2}{\sigma^{*2}} \equiv h \cdot m$. Without loss of generality, we assume $h > 1$ which means growth rises after financial integration. Then we can rewrite growth before financial integration as

$$g = p + q^i(\vartheta)m$$

and growth after financial integration as

$$g = p + q^i(\vartheta)hm.$$

Substituting the above expressions into (50), we have

$$\begin{aligned} \Lambda_j &= \left(\frac{l + p + q(\vartheta)mh + r}{l + p + q(\vartheta)m + r}\right)^{1/\left(1-\psi_j\right)} - 1 \\ &\equiv \left(h - \frac{(h-1)s}{s + q(\vartheta)m}\right)^{\chi} - 1 \end{aligned} \tag{53}$$

where $s \equiv l + p + r$, $\chi = 1/\left(1 - \psi_j\right)$, and therefore we have

$$\frac{\partial \Lambda_j}{\partial \vartheta} = (\chi - 1)\left(h - \frac{(h-1)s}{s + q(\vartheta)m}\right)^{\chi-1}\frac{(h-1)sm}{(s + q(\vartheta)m)^2}q'(\vartheta).$$

It is easy to see $q'(\vartheta) < 0$. In addition, if $\psi_j < 1$, $\chi > 1$. Thus, $\frac{\partial \Lambda_j}{\partial \vartheta} < 0$. Similarly, we have

$$\frac{\partial \Lambda_j}{\partial q} = (\chi - 1)\left(h - \frac{(h-1)s}{s + qm}\right)^{\chi-1}\frac{(h-1)sm}{(s + qm)^2} > 0. \tag{54}$$

As $q^a > q^d$, we have $\Lambda_j^a > \Lambda_j^d$.

References

[1] Aghion, Philippe and Abhijit Banerjee (2005), *Volatility and Growth*, Oxford University Press.

[2] Anderson, Evan W., Lars Peter Hansen, and Thomas J. Sargent (2003), "A Quartet of Semi-groups for Model Specification, Robustness, Prices of Risk, and Model Detection," *Journal*

of the European Economic Association, 1(1), 68-123.

[3] Bansal, Ravi, and Amir Yaron (2004), "Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles," *Journal of Finance* 59, 1481-1509.

[4] Benigno, Pierpaolo and Salvatore Nistico (2012), "International Portfolio Allocation under Model Uncertainty," *American Economic Journal: Macroeconomics* 4(1), 144-189.

[5] Campbell, John and Luis M. Viceira (2002), *Strategic Asset Allocation: Portfolio Choice for Long-Term Investors*, Oxford University Press.

[6] Campbell, John (1999), "Asset Prices, Consumption, and the Business Cycle," , in Taylor, John B. and Michael Woodford (eds.), *Handbook of Macroeconomics* Volume 1, Chapter 19, 1231-1303.

[7] Campbell, John, George Chacko, Jorge Rodriguez, and Luis M. Viceira (2004), "Strategic Asset Allocation in a Continuous-Time VAR Model," *Journal of Economic Dynamics and Control* 28(11), 2195-2214.

[8] Cagetti, Marco, Lars Peter Hansen, Thomas J. Sargent, and Noah Williams (2002), "Robustness and Pricing with Uncertain Growth," *Review of Financial Studies* 15, 363-404.

[9] Chen, Hui, Nengjiu Ju, and Jianjun Miao (2014), "Dynamic Asset Allocation with Ambiguous Return Predictability," *Review of Economic Dynamics* 17(4), 799-823.

[10] Corsetti, Giancarlo (1997), "A Portfolio Approach to Endogenous Growth: Equilibrium and Optimal Policy," *Journal of Economic Dynamics and Control*, Elsevier 21, 1627-1644.

[11] Dabušinskas, Aurelijus, Dmitry Kulikov, and Martii Randveer (2012), "The Impact of Volatility on Economic Growth," Eesti Pank Working Paper 7/2012,

[12] Devereux, Michael B. and Gregor W. Smith (1994), "International Risk Sharing and Economic Growth," *International Economic Review* 35(3), 535-50.

[13] Djeutem, Edouard and Kasa, Kenneth (2013), "Robustness and Exchange Rate Volatility," Journal of International Economics 91(1),

[14] Duffie, Darrell and Epstein, Larry G (1992), "Asset Pricing with Stochastic Differential Utility," *Review of Financial Studies* 5(3), 411-36.

[15] Eaton, Jonathan (1981), "Fiscal Policy, Inflation, and the Accumulation of Risky Capital," *Review of Economic Studies* 48(3), 435-45.

[16] Eozenou, Patrick (2008), "Financial Integration and Macroeconomic Volatility: Does Financial Development Matter?" MPRA Paper 12738.

[17] Epaulard, Anne and Aude Pommeret (2003), "Recursive Utility, Growth, and the Welfare Cost of Volatility," *Review of Economic Dynamics* 6(3), 672-684.

[18] Epaulard, Anne and Aude Pommeret (2005), "Financial Integration, Growth, and Volatility," IMF Working Paper 05/67.

[19] Epstein, Larry G. and Stanley E. Zin (1989), "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework," *Econometrica* 57(4), 937-969.

[20] Epstein, Larry G. and Martin Schneider (2010), "Ambiguity and Asset Markets," *Annual Review of Financial Economics* 2(1), 315-346.

[21] Farmer, Roger E.A. and Amartya Lahiri (2006), "Economic Growth in an Interdependent World Economy," *Economic Journal* 116(514), 969-990.

[22] Gilboa, Itzhak and Schmeidler, David (1989), "Maxmin Expected Utility with Non-unique Prior," *Journal of Mathematical Economics* 18(2), 141-153.

[23] Gourinchas, Pierre-Olivier and Olivier Jeanne (2006), "The Elusive Gains from International Financial Integration," *Review of Economic Studies* 73(3), 715-741.

[24] Gourinchas, Pierre-Olivier and Jonathan A. Parker (2002), "Consumption over the Life Cycle," *Econometrica* 70(1), 47-89.

[25] Guvenen, Fatih (2006), "Reconciling Conflicting Evidence on the Elasticity of Intertemporal Substitution: A Macroeconomic Perspective," *Journal of Monetary Economics* 53(7), 1451-1472.

[26] Hall, Robert E, (1988), "Intertemporal Substitution in Consumption," *Journal of Political Economy* 96(2), 339-57.

[27] Hansen, Lars Peter and Thomas J. Sargent (2007), *Robustness*, Princeton University Press.

[28] Hansen, Lars Peter, Thomas J. Sargent, and Thomas D. Tallarini, Jr. (1999), "Robust Permanent Income and Pricing," *Review of Economic Studies* 66(4), 873-907.

[29] Hansen, Lars Peter, Thomas J. Sargent, and Neng Wang (2002), "Robust Permanent Income and Pricing with Filtering," *Macroeconomic Dynamics* 6(1), 40-84.

[30] Hnatkovska, Viktoria and Norman Loayza (2004), "Volatility and Growth," World Bank Policy Reseach Paper WPS3184.

[31] Hoxha, Indrit, Sebnem Kalemli-Ozcan, and Dietrich Vollrath (2013), "How Big Are the Gains from International Financial Integration?" *Journal of Development Economics* 103(C), 90-98.

[32] Imbs, Jean (2007), "Growth and Volatility," *Journal of Monetary Economics* 54(7), 1848-1862.

[33] Kasa, Kenneth (2006), "Robustness and Information Processing," *Review of Economic Dynamics* 9(1), 1-33.

[34] Klibanoff, Peter, Massimo Marinacci, and Sujoy Mukerji (2005), "A Smooth Model of Decision Making under Ambiguity," *Econometrica* 73(6), 1849-1892.

[35] Ju, Nengjiu and Jianjun Miao (2012), "Ambiguity, Learning, and Asset Returns," *Econometrica* 80, 559-591, 2012.

[36] Liu, Jun, Jun Pan, and Tang Wang (2005), "An Equilibrium Model of Rare-Event Premia and Its Implication for Option Smirks," *Review of Financial Studies* 18(1), 131-164.

[37] Lucas, Robert E., Jr. (1990), "Supply-Side Economics: An Analytical Review," *Oxford Economic Papers* 42, 293-316.

[38] Luo, Yulei and Eric R. Young (2010), "Risk-sensitive Consumption and Savings under Rational Inattention," *American Economic Journal: Macroeconomics* 2(4), 281-325.

[39] Luo, Yulei, Jun Nie, and Eric R. Young (2012), "Robustness, Information-Processing Constraints, and the Current Account in Small-Open Economies," *Journal of International Economics* 88(1), 104-120.

[40] Luo, Yulei, Jun Nie, and Eric R. Young (2014), "Robust Control, Informational Frictions, and International Consumption Correlations," *European Economic Review* 67, 1-27.

[41] Maenhout, Pascal (2004), "Robust Portfolio Rules and Asset Pricing," *Review of Financial Studies* 17, 951–983.

[42] McGrattan, Ellen R. (1998), "A Defense of AK Growth Models," *Federal Reserve Bank of Minneapolis Economic Quarterly* 22(4), 13-27.

[43] Mendoza, Enrique G., Vincenzo Quadrini, and José-Víctor Ríos-Rull (2009), "Financial Integration, Financial Development, and Global Imbalances," *Journal of Political Economy* 117(3), 371-416.

[44] Merton, Robert C. (1971), "Optimum Consumption and Portfolio Rules in a Continuous-Time Model," *Journal of Economic Theory* 3(4), 373-413.

[45] Miranda-Pinto, Jorge (2016), "Sectoral Connectivity, Volatility, and Downturns: Lessons from Cross-Country Data," manuscript, University of Virginia.

[46] Obstfeld, Maurice (1994), "Risk-Taking, Global Diversification and Growth." *American Economic Review* 84, 1310-1329.

[47] Ramey, Garey and Valerie A. Ramey (1995), "Cross-Country Evidence on the Link between Volatility and Growth," *American Economic Review* 85(5), 1138-1151.

[48] Rebelo, Sergio (1991), "Long-Run Policy Analysis and Long-Run Growth," *Journal of Political Economy* 99(3), 500-521.

[49] Schularick, Moritz and Thomas M. Steger (2010), "Financial Integration, Investment, and Economic Growth: Evidence from Two Eras of Financial Globalization," *Review of Economics and Statistics* 92(4), 756-768.

[50] Smith, William T. (1996), "Taxes, Uncertainty, and Long-term Growth," *European Economic Review* 40, 1647-1664.

[51] Turnovsky, Stephen J. and Chattopadhyay, Pradip (2003), "Volatility and Growth in Developing Economies: Some Numerical Results and Empirical Evidence," *Journal of International Economics* 59(2), 267-295.

[52] Vissing-Jørgensen, Annette and Orazio P. Attanasio (2003), "Stock-Market Participation, Intertemporal Substitution, and Risk-Aversion," *American Economic Review* 93(2), 383-391.

Growth and Volatility by Country, 1962-2011

Country	Mean Growth	S.D. Growth	Country	Mean Growth	S.D. Growth
Argentina	1.46	5.32	Malawi	1.60	6.91
Australia	2.04	1.71	Malaysia	3.96	3.92
Austria	2.51	1.97	Malta	4.49	4.33
Bangladesh	1.55	4.04	Mauritius	3.01	5.04
Barbados	1.87	4.22	Mexico	1.83	3.49
Belgium	2.36	1.97	Mozambique	2.01	5.03
Bolivia	0.69	3.82	Nepal	1.44	2.72
Botswana	5.99	5.88	Netherlands	2.25	2.03
Brazil	2.46	3.65	New Zealand	1.23	2.72
Canada	2.05	2.09	Niger	-1.09	6.06
Chile	2.43	5.10	Norway	2.55	1.83
Colombia	1.98	2.16	Pakistan	2.47	2.06
Congo, Dem. Rep.	-2.11	5.90	Panama	3.31	4.38
Costa Rica	2.23	3.21	Paraguay	1.66	3.97
Cyprus	3.87	5.85	Peru	1.30	5.16
Denmark	1.95	2.39	Philippines	1.39	3.05
Dominican Republic	3.12	4.80	Portugal	2.93	3.38
Ecuador	2.04	4.16	Senegal	-0.11	4.18
El Salvador	1.34	3.50	Sierra Leone	0.31	7.57
Fiji	1.70	4.46	Singapore	5.15	4.25
Finland	2.57	3.21	South Africa	1.02	2.50
France	2.21	1.97	Spain	2.68	2.73
Germany	2.21	2.09	Sri Lanka	3.44	2.31
Ghana	0.81	4.56	Sweden	2.07	2.27
Greece	2.55	4.22	Switzerland	1.35	2.17
Guatemala	1.37	2.43	Syria	2.10	8.86
Honduras	1.04	3.12	Taiwan	5.56	2.94
Hong Kong	4.58	4.24	Tanzania	1.75	3.19
Iceland	2.47	4.00	Thailand	4.74	4.94
India	3.15	3.36	Togo	0.15	5.65
Iran	1.05	10.25	Trinidad & Tobago	2.34	4.97
Ireland	3.24	3.32	Tunisia	3.28	4.39
Israel	3.17	6.27	Turkey	2.65	3.98
Italy	2.32	2.74	Uganda	1.19	4.48
Jamaica	0.47	3.71	United Kingdom	1.96	2.20
Japan	3.73	5.11	United States	2.04	2.13
Jordan	0.95	6.38	Uruguay	1.68	4.23
Kenya	0.71	2.82	Venezuela	0.62	5.40
Korea, Republic of	5.80	3.61	Zambia	-0.05	4.86
Lesotho	2.79	6.39	Zimbabwe	0.17	7.13

Source: Penn World Tables and author's calculations.

Figure 1. *List of Countries*

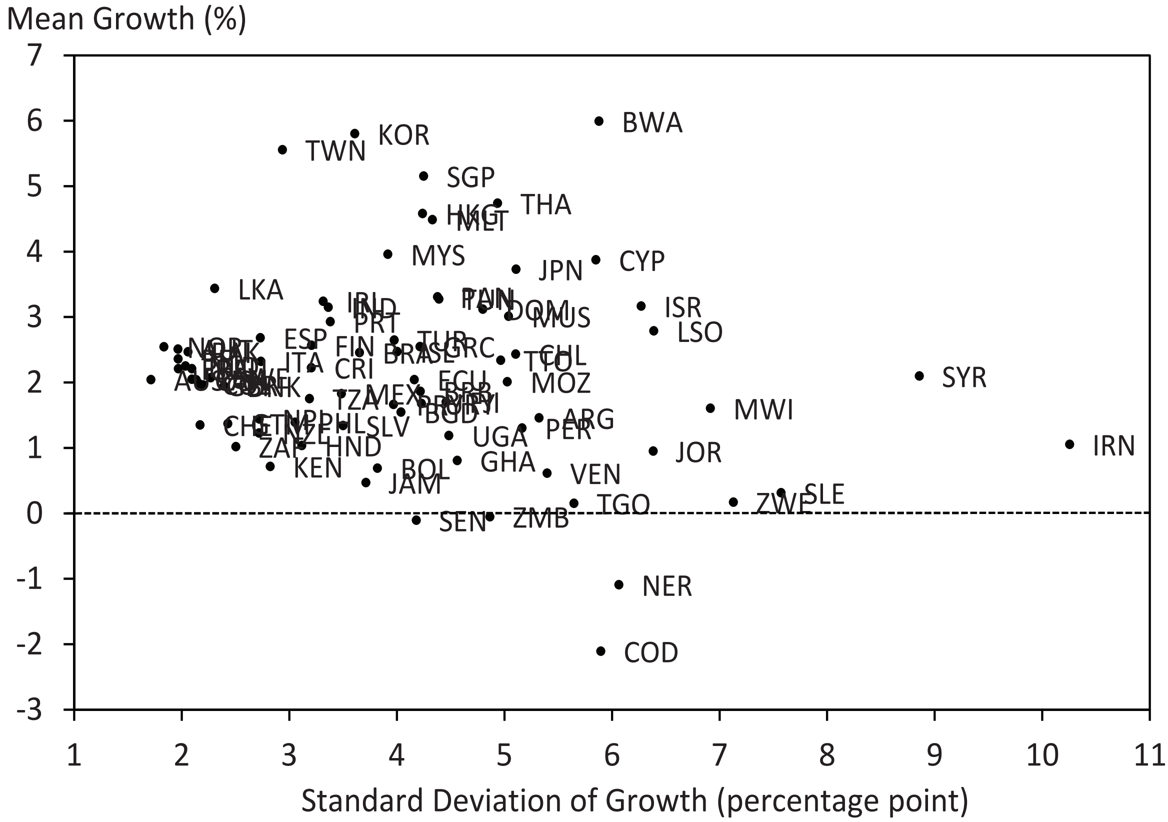

Figure 2. Growth and Volatility of Per-Capita GDP across Countries (1962-2011)

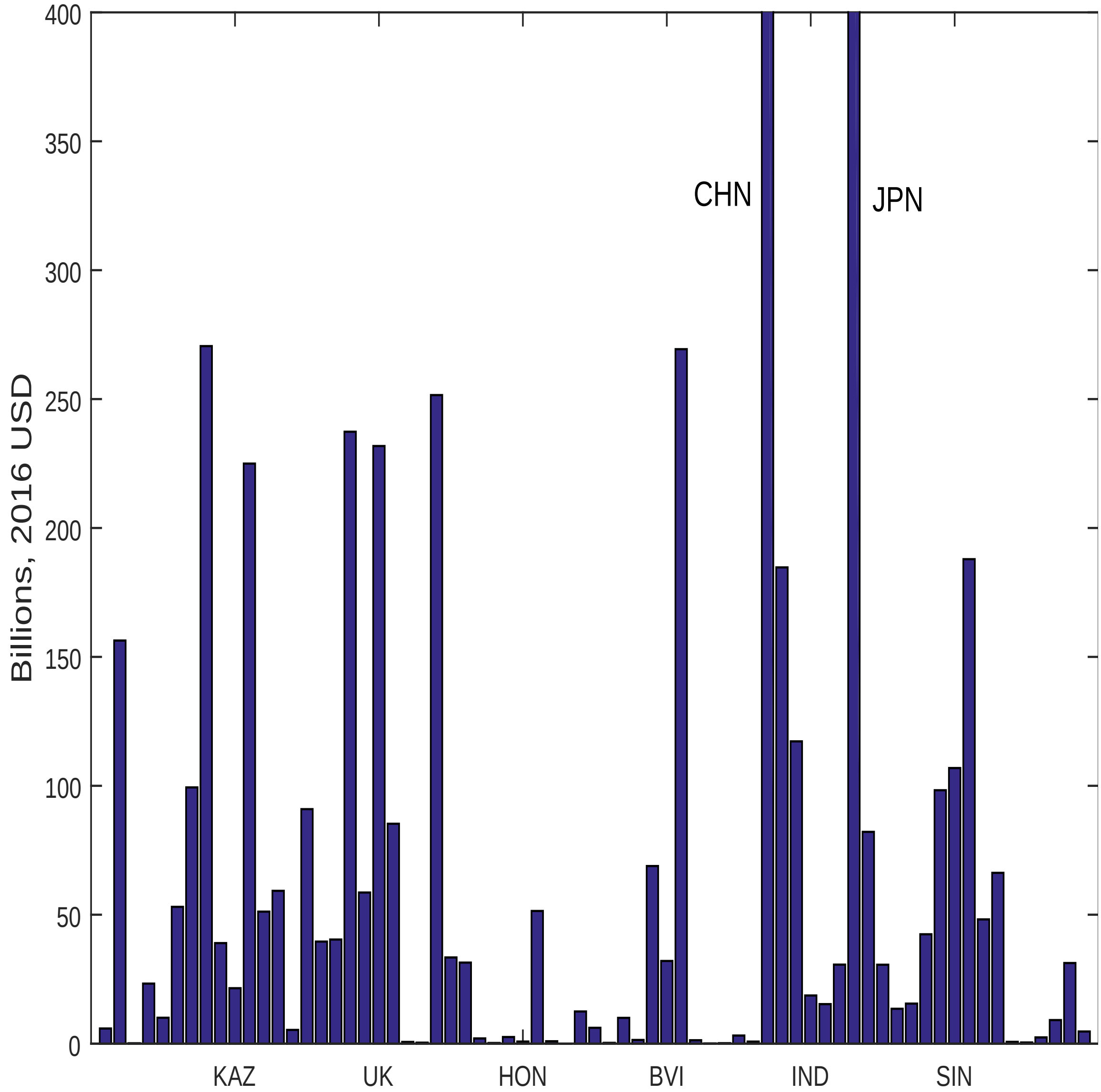

Figure 3. *Distribution of US Treasury Debt*

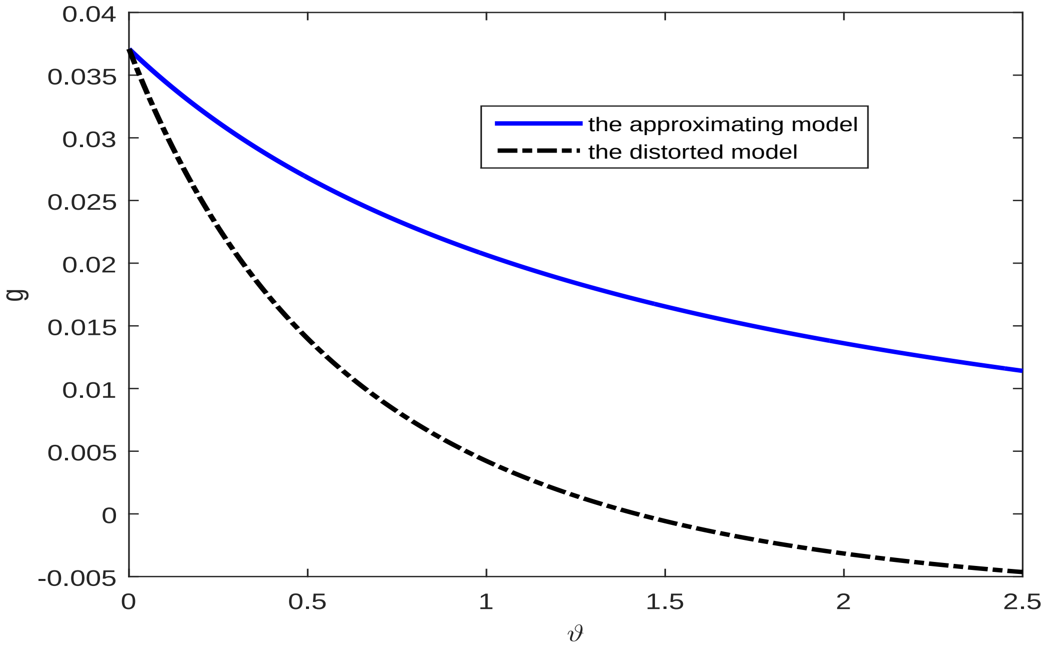

Figure 4. *Effect of RB on Growth in the Diversified Equilibrium*

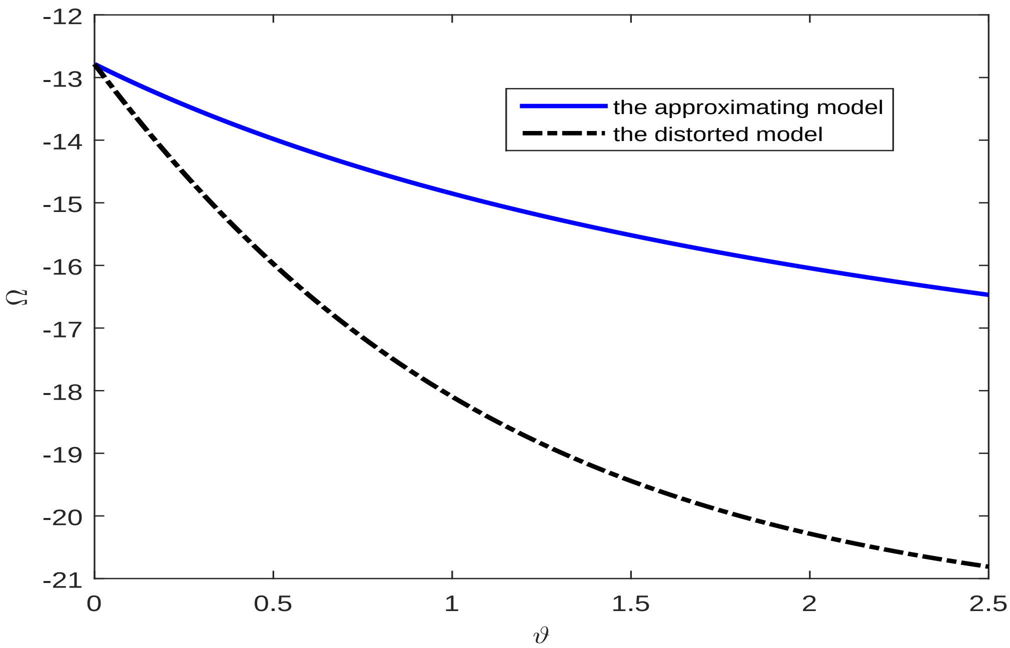

Figure 5. *Effect of RB on Welfare in the Diversified Equilibrium*

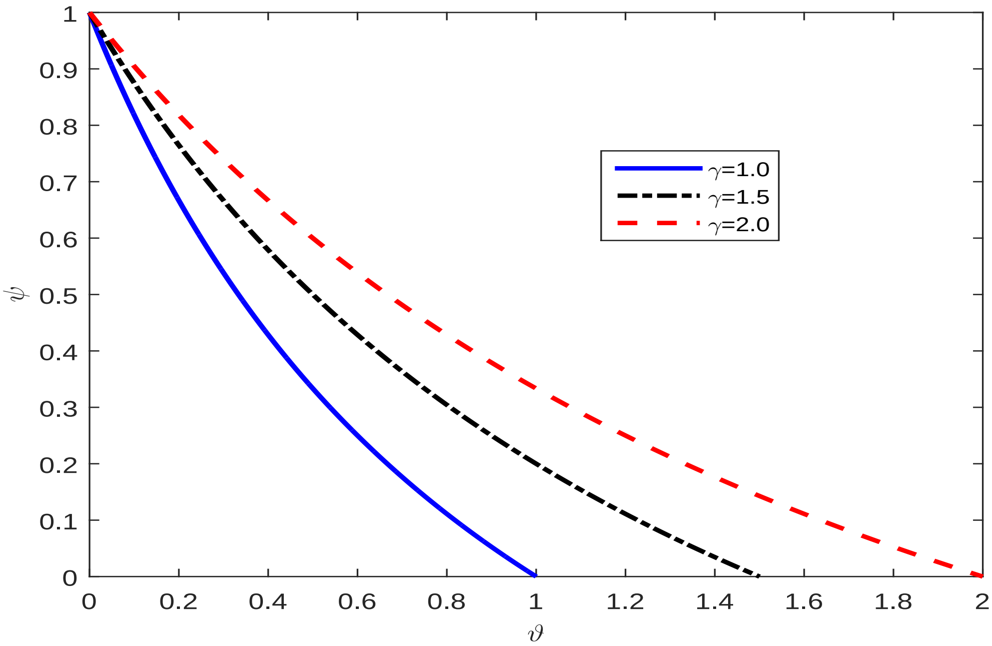

Figure 6. *Relation between EIS and RB*

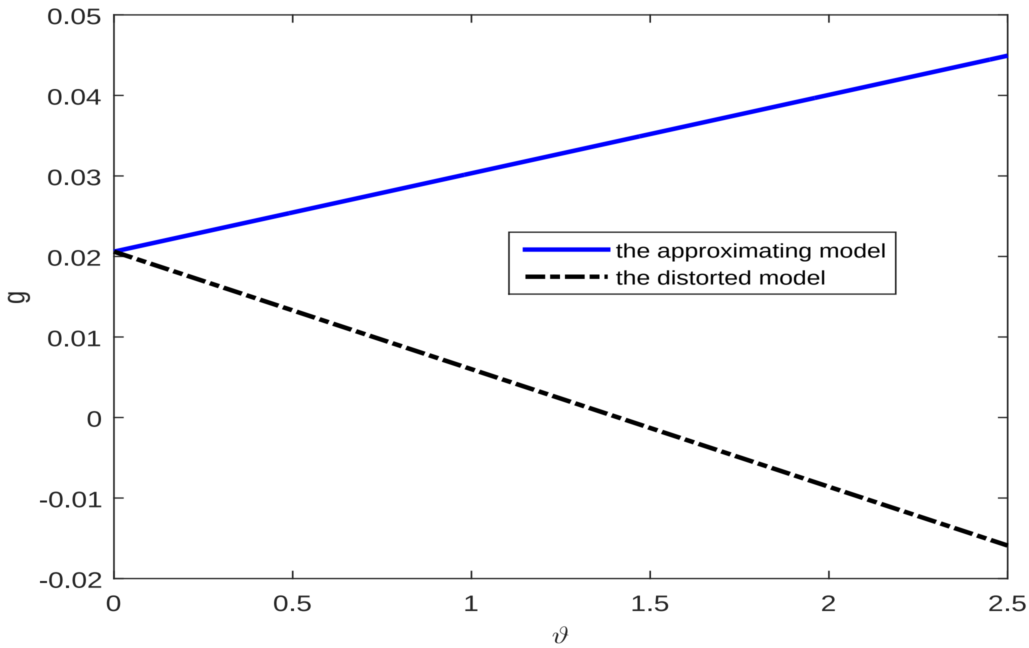

Figure 7. *Effect of RB on Growth in the Undiversified Equilibrium*

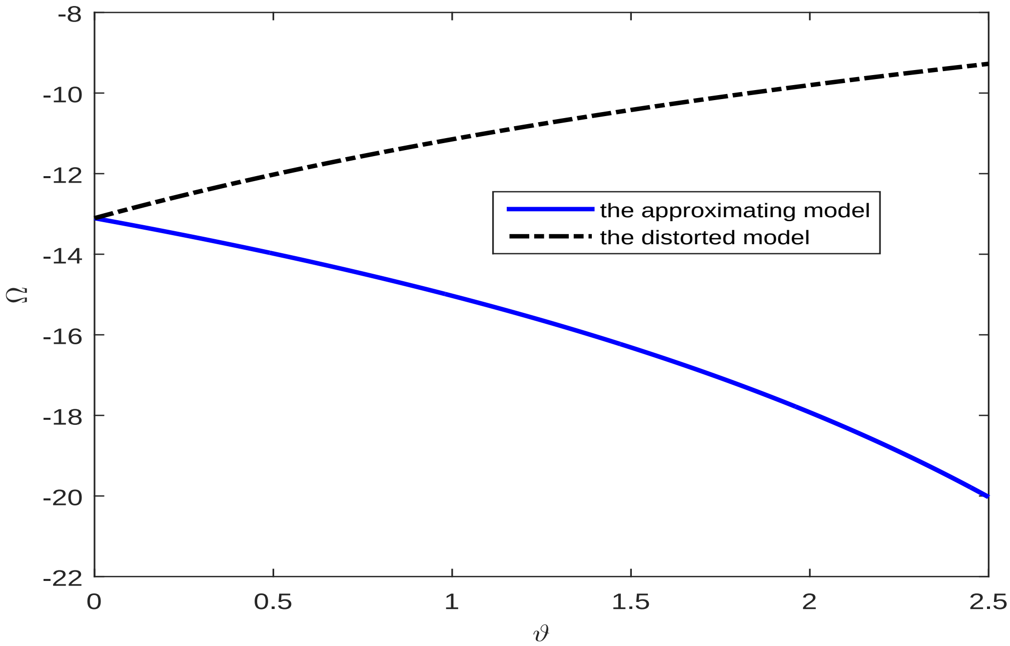

Figure 8. *Effect of RB on Welfare in the Undiversified Equilibrium*

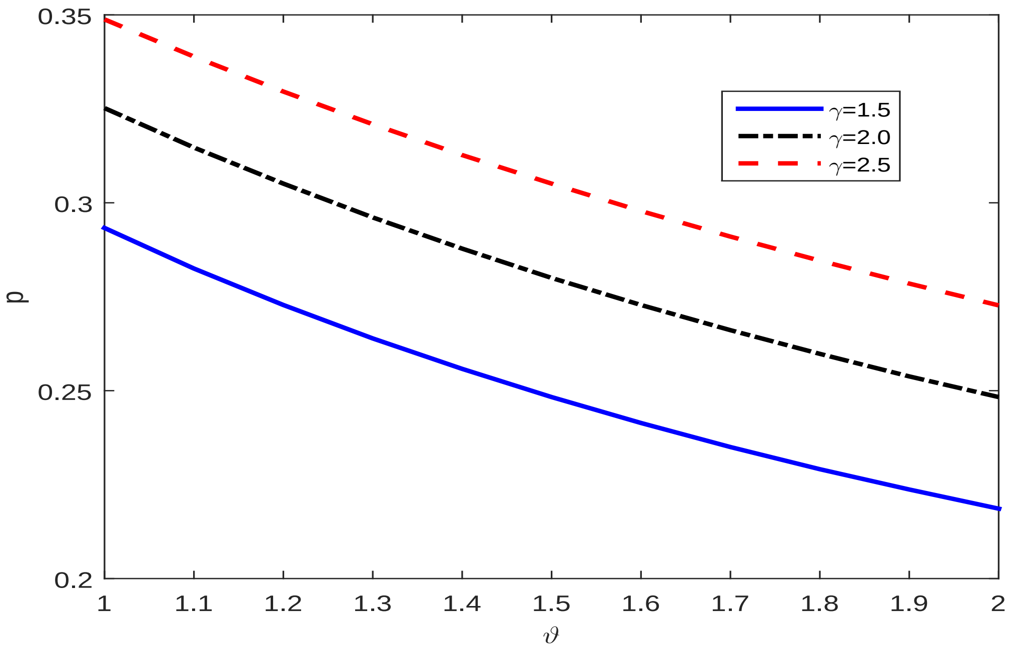

Figure 9. *Relationship between ϑ and p*

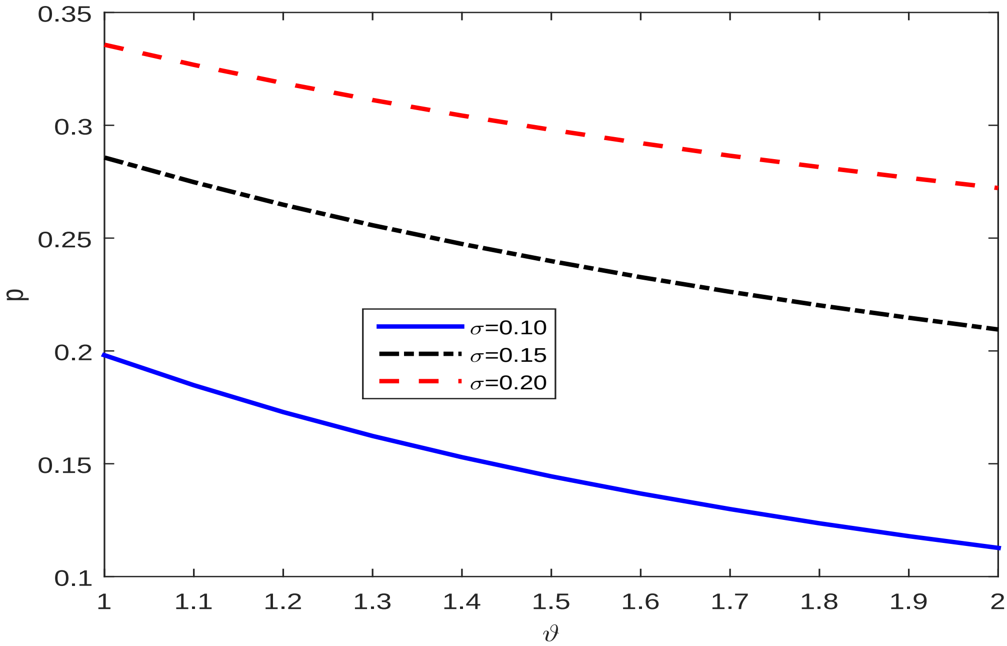

Figure 10. *Relationship between ϑ and p*

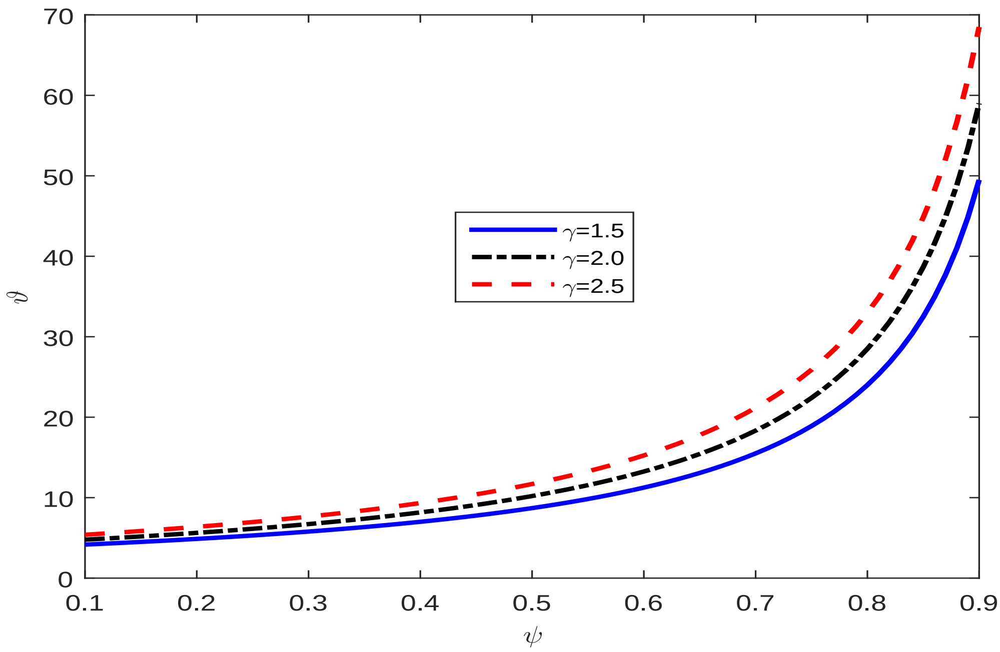

Figure 11. *Inferred Values of ϑ for Given Values of ψ and γ*

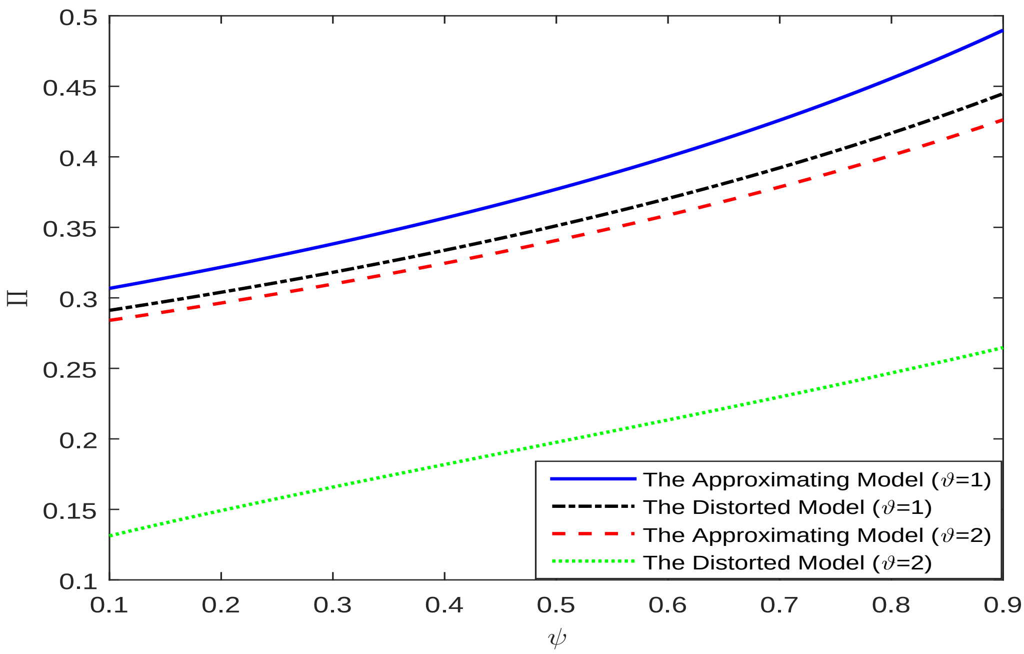

Figure 12. *The Welfare Gain from Intergration under RB*

Table 1. *Regression Results on the Effects of Volatility on Per-Capita GDP Growth (1962-2011)*

Independent Variables	Regression Coefficient	*p*-value
Volatility	-0.095	0.038
Investment Share	0.068	0.000
Human Capital	0.003	0.053
Initial Per-Capital GDP Level	-0.003	0.004
Constant	0.024	0.000

Table 2. *Regression Results on the Effects of Σ^2 on g^d (1962-2011)*

Independent Variables	Regression Coefficient	p-value
Variance, Σ^2	-1.047	0.018
Investment Share	0.068	0.000
Human Capital	0.003	0.069
Initial Per-Capital GDP Level	-0.002	0.007
Constant	0.021	0.000

Table 3. *Comparing Models' Predictions on the (g, Σ) relationship*

Type of Equilibrium	Coefficient	$\vartheta = 2$ $(p = 0.22)$	$\vartheta = 1$ $(p = 0.29)$
(Diver., Appr.)	$\frac{(1+\psi)\widetilde{\gamma}}{2}$	2.1	1.5
(Diver., Dist.)	$\left(1+\psi-\frac{2\vartheta}{\gamma+\vartheta}\right)\frac{\widetilde{\gamma}}{2}$	0.1	0.5
(Undiv., Appr.)	$\frac{1}{2}(1-\psi)\widetilde{\gamma}$	1.4	1
(Undiv., Dist.)	$\frac{1}{2}\left(1-\psi-\frac{2\vartheta}{\gamma+\vartheta}\right)\widetilde{\gamma}$	-0.6	0

Table 4. *Model Implied value of ϑ ($\psi = 0.2$)*

Type of Equilibrium	Coefficient	ϑ	p
(Diver., Appr.)	$\frac{(1+\psi)\widetilde{\gamma}}{2}$	-3.25	n.a.
(Diver., Dist.)	$\left(1+\psi-\frac{2\vartheta}{\gamma+\vartheta}\right)\frac{\widetilde{\gamma}}{2}$	4.87	0.15
(Undiv., Appr.)	$\frac{1}{2}(1-\psi)\widetilde{\gamma}$	-4.13	n.a.
(Undiv., Dist.)	$\frac{1}{2}\left(1-\psi-\frac{2\vartheta}{\gamma+\vartheta}\right)\widetilde{\gamma}$	2.75	0.19

Table 5. *Model Implied value of ϑ ($\psi = 1.5$)*

Type of Equilibrium	Coefficient	ϑ	p
(Diver., Appr.)	$\frac{(1+\psi)\widetilde{\gamma}}{2}$	-2.34	n.a.
(Diver., Dist.)	$\left(1+\psi-\frac{2\vartheta}{\gamma+\vartheta}\right)\frac{\widetilde{\gamma}}{2}$	-11.70	n.a
(Undiv., Appr.)	$\frac{1}{2}(1-\psi)\widetilde{\gamma}$	2.70	0.17.
(Undiv., Dist.)	$\frac{1}{2}\left(1-\psi-\frac{2\vartheta}{\gamma+\vartheta}\right)\widetilde{\gamma}$	0.54	0.42

Table 6. *Welfare Gains after Integration under RB (Symmetric Case)*

Growth Rate $(g, \%)$	Approximating Model	Distorted Model
Example 1 ($\gamma = 4, \vartheta = 2$)		
Before Integration	1.13	0.63
After Integration	2.25	1.25
Welfare Gain (Π, % of wealth)	33.88	19.58
Example 2 ($\gamma = 2, \vartheta = 2$)		
Before Integration	1.69	0.56
After Integration	2.00	1.00
Welfare Gain (Π, % of wealth)	37.06	48.72

Table 7. *Welfare Gains after Integration under RB (Asymmetric Case)*

Growth Rate $(g, \%)$	FI-RE	Approximating Model	Distorted Model
Country 1 ($\vartheta = 2$)			
Before Integration	1.69	1.13	0.63
After Integration	2.00	2.25	1.25
Welfare Gain (Π, % of wealth)	8.22	33.88	19.58
Country 2 ($\vartheta = 4$)			
Before Integration	1.69	0.84	0.28
After Integration	2.00	1.69	0.56
Welfare Gain (Π, % of wealth)	18.34	26.40	9.12